THE STRENGTH OF CONCRETE UNDER COMPRESSIVE LOAD

A thesis
submitted in partial fulfilment
of the requirements for the Degree
of
Doctor of Philosophy

by

Laurence Gaerty

University of Canterbury

1984

ABSTRACT

A background literature survey pertinent to the behaviour of plain concrete under short-term compressive load is presented. This yields little to suggest that any consistent "understanding" of the overall strength/deformation/failure topic prevails in the mainstream of current thought.

The rationale of a generalised hierarchical model of material behaviour, the basis for which was first suggested some fifteen years ago is examined in considerable depth. The model itself is thoroughly unconventional in the context of typical modern views and actually challenges many commonly-held precepts. Philosophical arguments are advanced on behalf of its more radical implications. Its consistent predictive potential as regards the load-induced behavioural traits exhibited by real concrete systems is demonstrated.

ACKNOWLEDGEMENTS

In formally acknowledging the support I have received from within the Department of Civil Engineering, University of Canterbury, special thanks are due to Professors H.J. Hopkins and R. Park (past and present Heads, respectively) for their maintaining positive encouragement during a protracted period of study and writing. As supervisor, the latter showed a degree of tolerance in the face of successive delays for which I will always be appreciative.

Many of the concepts embodied within this work owe their very existence to the fertile mind of F.J. Grimer, Building Research Establishment, Garston, U.K. I am ever grateful for his having imparted these to me.

Finally, the substantial contribution of my wife, Carolynne, can not be overstressed. Beyond the act of typing this thesis (of which all too numerous "final" versions ultimately suffered a complete denegration of status to become heavily amended drafts), she has given me immense support, both practical and moral, over a long and often arduous period. Without her understanding, patience, and encouragement, this work could never have come to fruition.

CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

On Monday, when the sun is hot
I wonder to myself a lot:
'Now is it true or is it not
That what is which and which is what?

On Tuesday, when it hails and snows,
The feeling on me grows and grows
That hardly anybody knows
If those are these, or these are those.

A.A. Milne *(Winnie-the-Pooh)*

1.1. BACKGROUND

Any doctoral thesis necessarily reflects, to a certain extent at least, something of the earlier educational background and related experience of the candidate who presents it. This dissertation is no exception, although, in the manner of its presentation, it is probably influenced thereby to a greater degree than most. For this reason, a few comments on pertinent background influences would seem apposite.

With regard to overall outlook, the writer would readily admit that he conforms in no small measure to the descriptive tag, "physical thinker". As such, his basic attitude to the topic of mathematics is somewhat old-fashioned when viewed in the light of most current trends; i.e. he sees mathematics quite simply as an extremely useful tool which serves the prime purpose of providing a means to an end, rather than as the only possible source of rational understanding or even as the end itself. Whether or not this prevailing state of mind, which was well established by the end of his secondary schooling, was essentially created and developed by others, such as past teachers, or occurred instead through processes of natural inclination and self-determined logical preference, is strictly beyond supposition; however, its very existence undoubtedly shaped the writer's final decision on that form of tertiary education he then chose to follow. Thus, despite a liking for both elementary physics and chemistry, the pragmatic "real-world" approach, which, at the time, appeared to the writer to be naturally endemic to the Civil Engineering profession and its associated disciplines, held far greater appeal than the seeming remoteness of "pure" science at its higher levels.

It was during his undergraduate days as an engineering student that the writer's interest in the load-induced behaviour and failure of materials in general, and of concrete in particular, was initially fostered. This interest grew, virtually by default, through the emergence of a nagging personal dissatisfaction with certain almost standard would-be "explanations" of the typical concrete failure mode in simple compression, tendered by both recognised authoritative textbooks and instructors alike. One specific incident remains clearly in the writer's memory. Since this was possibly the crucial "nucleus" from which his "gel" of discontent ultimately grew, culminating some ten years later in the assembly of the present work, its relation here is not inappropriate. The incident referred to occurred during a routine laboratory class in which an over-reinforced concrete beam was being tested to destruction within a third-point flexural loading rig. The academic (A) in charge of the demonstration drew the attention of the assembled group of students to the first signs of the beam's distress. At this point a simple question was raised by one particularly observant onlooker (B) - a fellow student of the writer. From memory, the ensuing dialogue followed a path similar to that shown below.

A: *Gentlemen, you will note that crushing of the concrete above the neutral axis has begun at one point within the constant-moment zone.*

B: *Why do you call it "crushing"? It looks more like lots of splitting to me.*

A: *That "splitting" as you describe it is what we term crushing.*

B: *Why? Splitting and crushing are quite different things, aren't they?*

A: *Gentlemen, you are here to learn engineering, not to quibble about semantics or worry about etymology.*

The questioner quickly wilted under the superior vocabulary of the academic and the latter's obvious annoyance with the non-technical nature of the enquiry. No one (including the writer) felt well disposed to continue the matter further. Case dismissed!

On reflection, and from a strictly pedantic viewpoint, the academic was of course quite correct. There is no conflict, providing that the definition of "crushing" as being synonomous with multiple splitting, when applied to the failure of concrete in uniaxial compression, is kept distinctly separate from its more usual meaning in other contexts. The "peculiar" rationale underlying such a stance is, however, of extremely dubious overall merit, due principally to the embarrassing fact that simple triaxial loading regimes can be devised for concrete and other "brittle" materials which do produce "crushing" in the normal sense of the word; within the context of brittle materials this generally implies extensive, directionally random, bulk disintegration, partial reduction to fine fractions, etc. Almost needless to say, the considerable inherent potential for descriptive confusion and fundamental misunderstanding immediately concomitant to the adoption of a "special" meaning for an otherwise common word is often realised to the full. As merely an aid to a form of loose description, a blanket implementation of the term "crushing" is, at best, tolerably inaccurate; conversely, as a "suitable" basis for any consistent degree of pertinent understanding, it can have no real justification whatsoever! In many ways the apparent reticence which still lingers on in concrete terminology with regard to accepting a sensible nominal recognition of the multiple splitting phenomenon, so characteristic of uniaxial compressive failure, is strangely reminiscent of another passage from A.A. Milne's classic children's story quoted earlier:

"Hallo, Rabbit," he (Pooh) said, "is that you?"

"Let's pretend it isn't," said Rabbit, "and see what happens."

Subsequent to his graduation, the writer found gainful employment within the field of concrete research. (In the best interests of objectivity and fair reporting, it must be stated that this immediate post-graduate situation actually owed very much more to a fortuitous combination of circumstance than it did to any burning personal desire or real positive commitment on the part of the writer to become involved then in such activity. The strong degree of motivation which underlies the present treatise had not, as yet, been fully generated.) For a time, the pressing need to assimilate relevant new information on concrete and gather associated knowledge in somewhat unfamiliar surroundings largely suppressed those seeds of doubt planted earlier; accordingly, their erstwhile seeming importance partially diminished. However, it was shortly after this initial period of environmental reorientation that the writer became aware of the existence of a novel and quite unorthodox approach to the load-induced response of material systems; this new treatment not only recognised prima facie the multiple splitting (or cleavage) mode of concrete failure under applied uniaxial compression, but indeed also offered an apparently rational model of systematic behaviour to account for the phenomenon itself. Bearing in mind that which has already been intimated with regard to personal outlook, coupled with the inherent dependence of the behavioural model referred to upon a limited number of relatively simple physical notions, the almost instant intellectual appeal which the new approach held for the writer should not come as any great surprise. Discussions with the originator of the model, F.J. Grimer (a senior colleague under whom the writer was privileged to work for a short while), proved to be both stimulating and enlightening. As a result, the writer - a most willing pupil - soon became a staunch proponent of its conceptual merit, joining a school of fellow dissidents which probably numbered no more than a handful.

Like most unorthodox theories which exhibit a marked departure from the norms of prevailing thought, the new approach to "understanding" the behaviour of materials under load virtually demands an element of positive belief from those who, in the face of all standard convention, would choose to accept and follow it. In the writer's case, this degree of implicit faith (by no means blind) in the basic suitability and logical sense of Grimer's hierarchical model was initially born of a personal dissatisfaction with the ultimate suitability of its mainstream alternatives, fostered by the intellectual appeal of its simple physical concepts, and then effectively reinforced through observing the attitudes of both its principal adherents and its sternest critics. (The model itself was not widely known at the time, having received little in the way of formal publicity, and so neither of the opposing "camps" - firmly committed for or against - could then claim significant numerical strength. At present, the overall situation is probably very similar. However, the powerful inertial resistance to radical improvisation characteristically embodied within the scientific status quo - an established "faith" of the conformist variety with much inherited dogma - is such that the associated potential for a preponderance of glib criticism of the seemingly bizarre, bordering on immediate and totally unreasoned dismissal, is almost inevitably bound to be high.) With the passage of time, the writer's inner confidence as regards the fundamental applicability of the new approach has grown steadily. Any progressive gain of confidence is, of course, a mental process which invariably relies to some extent on the feedback principle. Thus, that initial element of faith in the suitability of the hierarchical model, held on the part of the writer, created a personal state of mind which was particularly conducive to investigating the possible vulnerability of certain conventional precepts he had once automatically taken for granted; the encouraging discovery that such a line of intellectual inquiry did in fact raise some seemingly valid points of specific issue, essentially highlighting the somewhat arbitrary nature of the conventional would-be "obvious", then acted to generate further confidence, which, in turn, promoted a corresponding willingness to extend the logical questioning, and so on.

In 1971 the author accepted a lecturing position at the Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, and thus foresook the "safe" close company of his fellow dissidents. (On reflection, this return to a University environment, albeit on the other side of the world and in a quite different role, was perhaps a most ironic twist in the light of earlier events.) It was then the writer decided that he should attempt the production of a doctoral thesis which would formally examine the rationale of the new approach, and, by doing so, hopefully bestow upon it some measure of the credibility and academic respectability he felt it deserved. Hence this work.

1.2 SCOPE

Any model, strictly by virtue of definition, must have limitations as regards its true reflection of "reality". At first sight, and in the context of currently prevailing tenets, Grimer's hierarchical model of material behaviour under applied load would seem to have more than its fair share! This treatise aims to show, to the contrary, that many of these apparent limitations are not in fact imposed by a consistent logical "understanding" of reality itself, but rather by an unwarranted general assumption that the fundamental notions underlying the more conventional view are somehow uniquely privileged, and thus totally incontrovertible.

The writer is well aware that the proposition of radical alternatives in any field of thought, for which the prevalent theories thereof are, by and large, considered both secure and "appropriate", invariably generates a degree of what can only be termed "emotional" resistance. The recorded history of scientific development through the ages illustrates this typical human aspect of unreasoned mental inertia forcibly and often. (Notable precedents include the round/flat Earth controversy, the Earth as the centre of the Universe dispute, etc.) However, despite this knowledge of the associated risks involved, he has deliberately seen fit to challenge certain features of that "normal" dogma inherent to the conventional view of material behaviour. There was, of course, another and indeed much "softer" presentational option available - that of merely introducing the hierarchical model as a simple behavioural analogy of somewhat limited usefulness. This soft alternative was rejected by the writer as having little to recommend it other than expediency; taking into account those pertinent background influences alluded to in the previous section, its adoption would have amounted, in effect, to a quite misleading denial of his own opinions, a most unwilling compromise to unnecessary ambivalence, and at least a partial sacrifice of personal academic integrity.

Much technical information of both a quantitative and qualitative nature already exists in relation to the physical characteristics of concrete systems and their responsive behaviour under applied load. Chapter II reviews this briefly, with particular reference to the effects of compressive loading regimes. Chapter III follows up the review with a "state-of-the-art" type summary, examining the many different theories and associated models which have been proffered to "explain" those load-induced behavioural traits previously described.

The form, style, and syntax of Chapters II and III are, of course, largely prescribed by the strictly conventional vocabulary of the appropriate background literature from which both derive their immediate relevance and sustained notional emphasis. However, various aspects of this conventional vocabulary, and also of the normal "understanding" it functions to represent, later become the subject of somewhat closer scrutiny in Chapter IV which deals with an alternative approach to strength-related concepts, based ostensibly on Grimer's hierarchical model. Chapter IV touches upon numerous facets of the new approach, from the initial foundations of its philosophical rationale, through the potentially problematic areas of its apparently radical implications, to certain more obvious examples of its possible usefulness. The range of seemingly <u>consistent</u> applicability so demonstrated extends from an "explanation" of gravity and electrostatic effects to the more mundane (but no less significant) "prediction" of concrete failure modes under a wide variety of loading regimes. (The immediate ramifications of the hierarchical model are indeed many and diverse!)

CHAPTER II

A BRIEF REVIEW OF THE SHORT-TERM STRENGTH AND BEHAVIOURAL ASPECTS OF CONCRETE IN COMPRESSION

In relating behaviour to the nature of the material it is useful to have a mental picture of the relative sizes of its constituents. If concrete is imagined to be enlarged ten thousand times .... the overall range of the characteristic dimensions is from the height of the Eiffel Tower to the diameter of a cobweb filament.

R.P. Johnson(1)

2.1. FOREWORD

Various levels of discrimination can be exercised in the description of materials and their properties depending on the context in which this is required. Unfortunately the term concrete belongs to one of the most general classifications which relates to groups or families of materials having some common traits. As such, it takes its place beside such terms as metal, plastic, and timber. Within any one of these groupings a wide range of materials with dissimilar internal structures and composition and markedly different properties may co-exist. This is especially true of the concrete family, where the common traits are so loosely defined that, even in practical terms, an almost infinite number of combinations of suitable constituent materials is possible. Therefore, although some generalisations can be made, many statements regarding the properties of concrete must be qualified to make allowance for such potential diversity.

2.2. NATURE OF CONCRETE SYSTEMS

Concrete is a heterogeneous multi-phase material consisting of hydrated cement paste, unhydrated cement, aggregate particles, water, and air, the volume fractions of which depend on the initial mix proportions, the methods of mixing and placing, the moisture and temperature conditions of curing or storage, and the age of the concrete.

The physical and chemical properties of the cement paste or gel which surrounds the unhydrated cement particles will obviously depend on the type of cement used. A comprehensive description of hydraulic cements and their products of hydration can be gained from a number of sources[(2-4)], and is beyond the scope of this present study. However, it is worth noting that most of the work cited in this chapter relates to concrete systems which incorporate ordinary Portland cement (A.S.T.M. Type I). Cement paste consists of cement gel and a semi-continuous network of capillary pores which may be saturated, partly water-filled, or dry, depending on the availability of moisture. In turn, the gel consists of a structurally complex matrix of hydration products, permeated by minute water-filled gel pores. Two types of cement paste porosity can therefore be distinguished.

Aggregates are graded, particulate, intrinsically porous materials which, for most structural applications, generally range in size from a minimum of around 150 microns to a maximum of 20-40 mm, although larger aggregate sizes find application in mass concrete work. For reasons of convenience and practicality, materials above and below a size of approximately 5 mm are classified as coarse and fine aggregates respectively, but no particular significance can be attached to the level of distinction. Many aggregates are naturally occurring; e.g. river-run sands and gravels. Some are produced by crushing natural bulk rock materials to the required sizes, and others by completely artificial processes; e.g. expanded shale. Aggregates can therefore differ greatly in chemical composition, internal structure, homogenuity, density, porosity, strength, stiffness, shape and surface texture. In any consideration of the behavioural aspects of aggregates within concrete systems it is important to differentiate between those which are stiffer and stronger than the hydrated cement paste phase and those for which the reverse applies. With reasonably mature cement pastes the former category includes most normal and high density aggregates, while the latter is usually reserved for lightweight aggregates, which tend to be somewhat weaker and less stiff as a direct result of their relatively high porosity. Different aggregate types, their properties, and characteristics have been covered in some detail by Neville[(5)].

To produce concretes of sufficient workability the quantity of water incorporated in the initial mix is generally well in excess of that required for chemical combination with the cement. The overall water demand, which can be expressed in terms of the total water-cement ratio (w/c), depends on the aggregate-cement ratio (a/c), the grading of the aggregate, and the level of workability required. If the aggregate fractions enter the mix in an unsaturated condition these will absorb or adsorb part of the mixing water, the remainder being available to the cement. Despite the logical simplicity of the effective or net w/c concept which results from a recognition of such interaction, its practical assessment can be complicated by any bleeding of the freshly compacted concrete which may occur, and by the indeterminate within-the-mix absorption characteristics of many aggregates[6-7]. The effective w/c and the degree of hydration control the capillary porosity of the cement paste phase[8]. Any absorption which occurs in freshly compacted concrete prior to setting and hardening lowers the effective w/c of the paste around the aggregate particles, thus tending to decrease the local paste porosity, but this aspect is offset by the formation of small bubbles as the air within the aggregate pores is displaced by water[9]. The situation as described is, however, rarely stable since most fresh concrete is also prone to a form of sedimentation known as water-gain[10, 11]. By this action, certain amounts of free water, together with any air bubbles resulting from inadequate compaction, deliberate entrainment, or absorption, tend to rise slowly towards the top surface as cast and become accumulated on the lower surfaces of the aggregate particles - the accumulation under any one piece of aggregate being a function of its size and shape[12].

All physical parameters associated with concrete systems (e.g. w/c, a/c, aggregate grading, air content, etc.) are spatially distributed and therefore subject to local variations as a consequence of initial mixing and placing methods, the inherent variability of the constituent materials, and subsequent interactive processes such as those mentioned above. Local variations increase heterogenuity, and must lead to at least some degree of anisotropic behaviour.

The use of microscopic and X-ray techniques has shown that, even in an unloaded condition, hardened concrete contains a system of flaws and fine microcracks[13-19], almost all of which exist at the interfaces between coarse aggregate particles and the fine mortar matrix. Some of these flaws and bond cracks are thought to be caused by segregation and bleeding of the fresh mix[20] and others by volume changes which occur in the cement paste during setting and hardening[21]. If an unlimited amount of free water is available during hydration the paste expands, Conversely in an environment devoid of free water shrinkage occurs. Changes in the temperature and moisture condition of hardened paste can induce similar dimensional instability. Theoretical and experimental model studies(22, 23) have indicated that these volume changes are of sufficient magnitude to give rise to tensile and shear stresses at the mortar-coarse aggregate interfaces capable of partially breaking down the bond there, thus producing microcracks. It would appear that micro-cracking within the mortar phase is generally not prevalent in the unloaded state, and that the mortar-coarse aggregate interfaces are zones of potential weakness in many concrete systems.

2.3. FACTORS INFLUENCING THE SHORT-TERM COMPRESSIVE STRENGTH OF CONCRETE

2.3.1. Introduction

The research efforts of individual workers, small groups, and large organisations throughout the world over a period of many years has yielded vast amounts of data on the varying properties of different concrete systems; much of this is related to factors influencing compressive strength. Some of the more important factors are listed below, although it should be stressed that their relative importance can vary considerably from one concrete system to another:

- cement quality, type, composition, and fineness;
- aggregate quality, type, size, shape, grading, surface texture, stiffness, and absorption characteristics;
- water quantity and quality;
- degree of compaction;
- curing and storing regime;
- age, moisture condition, and temperature at time of test;
- testing details.

Further comment on these factors will be restricted to areas of general relevance to the study in hand.

2.3.2. Cement, Aggregates, and Water

The qualitative effects of varying the relative proportions of cement and water on the strength of mortars and concretes have probably been known since ancient Egyptian and Roman times. Smeaton(24), in discussing experiments on the cementitious materials to be used for the construction of the Eddystone Lighthouse completed in 1759, mentioned *"using as little water as possible"* with what would now be described as hydraulic limes. However, it was not until the late 19th and early 20th centuries that experimentally verified quantitative statements were proposed. In 1892 Feret(25, 26) included the relative volumes of cement and water in a functional relationship which attempted to correlate the compressive strengths and densities of various mortars. A few years later, Zielinsky(27) published a systematic study of the effect of water-cement ratio on the strength of mortar, but his efforts and the contributions of others have generally become eclipsed by the work of Abrams(28, 29), who put forward his famous "law" in 1918. This empirical rule takes the form,

$$\text{compressive strength, } S = k_1 k_2^{-(w/c)} \qquad \text{..... 2.1}$$

where k_1 and k_2 are experimentally determined constants, implicitly dependent on the age of the concrete, curing regime, specimen size, etc., and w/c is the water-cement ratio.

In Abrams' own words(28),

> *The equation expresses the law of strength of concrete so far as the proportions of materials is concerned. It is seen that for given concrete materials the strength depends only on one factor - the ratio of water to cement.*

The uniqueness or otherwise of the relationship between the compressive strength and water-cement ratio for concrete has been shown(30-38) to depend on the particular interpretation placed on the phrase, *"for given concrete materials"*. If this is taken to include the aggregate, size and grading, thereby making w/c the only variable, the rule appears to hold reasonably well provided that the mix in the plastic state can be classified as workable. On the other hand, Abrams and many who came after, excluded any effects of aggregate size and grading on concrete strength. Indeed, Abrams' own results appear to justify his statement(28),

> *.... the size and grading of the aggregate .... are no longer of any importance except in so far as these factors influence the quantity of water required to produce a workable mix.*

However, other results to be discussed later in this section indicate that such a general interpretation is both invalid and potentially misleading.

It has been repeatedly established by experiment that the presence of voids can greatly influence the mechanical behaviour and strength of all engineering materials. Various relationships between the relative strengths and deformational properties of individual materials and their degree of porosity have been proposed; those suggested for concrete will be reviewed in Chapter III. With a fully compacted hardened cement paste the capillary porosity is a function of the water-cement ratio and the degree of hydration(8). In practice, incomplete compaction and/or deliberate air entrainment will increase the relative volume of voids. (Whereas Abrams' equation(28) only alludes to fully compacted concretes, Feret's expression(25), relating mortar strength to density, does include a term to allow for the proportional volume of any entrapped air.) A comprehensive experimental programme by Powers and Brownyard(39) indicated that the compressive strengths of mortars and cement pastes could be effectively correlated with a number of parameters indicative of porosity. The suggested functional relationships, based on a fixed specimen size, were independent of mix proportions and the age at test. Powers(40) subsequently defined the generalised gel-space ratio, X_F, perhaps the most well known measure of cement paste porosity.

$$X_F = \frac{\text{gel volume}}{\text{gel volume + capillary pores + air voids}}$$

The F subscript differentiates the general value from a previously defined(41) gel-space ratio, X, which did not include air voids. Conceptually, the gel-space ratio is a fundamental rather than a strictly practical parameter since its evaluation requires prior knowledge of the water-cement ratio, the air content, and the hydration characteristics of the cement, together with an experimental determination of the degree of hydration achieved at any particular time. Powers showed(8) that for individual cements, the compressive strength, S, of paste or mortar was proportional to the cube of the generalised gel-space ratio, X_F, within very close limits. The following expression was therefore proposed:

$$S = k_3(X_F)^3 \qquad \ldots\ldots \; 2.3$$

The form of the equation implies that k_3, a constant depending on the cement, is an upper limit of possible strength corresponding to a gel-space ratio of unity, and that strength is controlled by the quality of the gel, in terms of porosity, not the quantity. This second implication will be discussed further in Chapter III. The "characteristic" gel strength, k_3, was found to vary with the chemical composition of the cement, cements with low tricalcium aluminate contents giving the highest strengths. It is important to note that the gel volume term in the expression for X_F includes the volume occupied by the minute gel pores. Under normal mixing and curing conditions most cement gels have a porosity of around 28%[8].

Powers himself[42] had earlier reported work by Abrams which resulted in neat paste strengths of more than twice the typical k_3 values, through the use of high pressure moulding and an extremely low water-cement ratio of 0.08 by weight. The inapplicability of equation 2.3 to the products of high pressure techniques is, however, understandable since these not only reduce the capillary porosity but also affect the size and extent of the gel pores. Wischers[43] has proposed another equation, relating the compressive strength, S, of cement paste to the volume fraction of solid materials present;

$$S = k_4 \ (V_S/V)^n \qquad \ldots\ldots \ 2.4$$

where k_4 and n are constants. The volume of solid materials, V_s, present in any volume of paste, V, includes all hydrated and unhydrated material; the equation has been shown to hold reasonably well for pastes produced by either conventional or high pressure techniques.

A considerable amount of research data exists on the various methods used to produce high strength cement pastes: see, for example, the references of Lawrence[44] and Bajza[45]. Without exception, the beneficial effects on both the compressive and tensile strength afforded by these techniques can be attributed to the low final porosities achieved. Roy and Gouda[46] have recently obtained overall paste porosities below 2%, by utilising a combination of high temperature and pressure during the moulding process. The compressive strength of such pastes was in excess of 600 MPa. Although the results of this and other similar work have relatively few short-term practical implications, they do give valuable insight into the nature of cement paste strength, or weakness, and the importance of physical bonding. The experiments of Sereda, Feldman, and Swenson[47] are particularly pertinent. Hydration products formed by mixing cement with large quantities of water over a period of time were subsequently compacted under pressure. These materials were then found to have the same strength and deformational properties as normal hardened cement pastes of the same porosity. Packing density of the solid fractions would thus appear to be the major factor governing the strength of cement paste, although the work of Yudenfreund et al.[48] has indicated that strength is not insensitive to pore size distribution.

In normal concrete production, high strength is achieved mainly through keeping the water-cement ratio, and hence the overall porosity, as low as possible. To ensure adequate workability chemical admixtures are often used. A complete understanding of the manner in which concrete properties are affected by these water-reducers does not exist; however, strength gains in excess of those expected from a simple lowering of the water-cement ratio are frequently claimed. Other methods have been advocated to increase concrete strength[49, 50] but these find little general application. Such methods include seeding of the mix with hydrated cement, high speed slurry mixing, and revibration of the concrete after some degree of setting and hardening has taken place. The water-cement ratio not only affects strength but also controls the kinetics of the hardening process[51, 52]. As the water-cement ratio increases, both the initial hardening rate and the deceleration of hardening rate with time become less. Thus, a concrete with a low water-cement ratio has a relatively higher early strength but lower subsequent strength gain than a similar concrete with a higher water-cement ratio.

For many years the role of aggregates in concrete systems was seriously underestimated. The water-cement ratio "law" of Abrams undoubtedly contributed towards the general view of normal aggregates as cheap inert fillers which increased the stiffness of the cement paste, restrained shrinkage, and decreased the cement requirement, thus increasing the economic potential of concrete. Abrams' pronouncement excluded the effects of aggregate shape, size, stiffness, surface texture, grading and quality on concrete strength, and relegated such factors to considerations of workability. Although the rule was repeatedly shown to hold within reasonable limits for structural concrete mixes with graded normal aggregates up to 40 mm maximum aggregate size - one obvious reason for its endurance - it was not without its critics. An excellent history of the dissent has been given by Gilkey[33], who in summarising his paper, states that,

> *On the basis of a substantial accumulation of evidence from a variety of sources, the pronouncement as made is not valid and can be seriously in error for unusual gradings in which the maximum sizes and/or amounts of aggregate depart substantially from the "structural" type of mixture.*

Powers' attitude is interesting here. Thus, although an early critic(53) of the assertion that concrete strength was not affected by aggregate size and grading, he maintained that his own proposed relationships for the strength of mortars(8) were independent of mix proportions. The dominant effect of the water-cement ratio in controlling the compressive strength of concrete systems has never been questioned, only its uniqueness. Taken literally, Abrams' rule implies that the strength of a cement paste is the same as that of a mortar, or a concrete with the same water-cement ratio provided that all three are workable mixes in the plastic state. Although it is often difficult to produce workable pastes and mortars having the same water-cement ratio, similar problems need not arise with a comparison of individual mortar and concrete strengths. Much of Gilkey's "evidence" was based on the results of such comparisons. However, his conclusions regarding the relative strength effects of fine and coarse aggregates were not in total agreement with the results of others. For example, according to Gilkey(33) concrete strength decreases as the aggregate-cement ratio increases, but according to Hughes and Chapman(36), who also reviewed a wide range of publications a few years later, the reverse should apply. Unfortunately, no generalised statement on aggregate strength effects could ever be justified from the experimental data in existence. Different workers(30-38), using different aggregates in different concrete systems, and testing different specimen types have produced relatively different results. Thaulow(54) suggested that the differences between specimen types could explain the conflict of results between investigators in the U.S. and their counterparts in Europe, especially in the U.K. Hughes and Bahramian(37) confirmed that the high frictional effects associated with the standard compression test for concrete cubes magnified the strength differences brought about by changing aggregate proportions. They also maintained, however, that many of the U.S. investigations had not allowed for absorption in an adequate manner. With the benefit of hindsight it can now be seen that the conclusions of Gilkey and those of Hughes and Chapman, mentioned earlier, are not totally inconsistent. As early as 1930 Collier(55) had shown that, when all other factors were held constant, there was an optimum sand content for maximum concrete strength. Others[32, 34] have since confirmed this. Ishai[56] has reported mortars made with a certain sand as having up to twice the compressive strength of the original cement paste. The optimum or maximum critical sand content is affected by both aggregate type and grading. The work of Hughes and Bahramian[37] indicated that, for the particular mixes studied, a minimum critical sand content could also be identified. They suggested that when the sand content was reduced below this value, microcracking at the mortar-coarse aggregate interfaces increased as a result of inadequate fine aggregate restraint on cement paste shrinkage, leading to a decrease in concrete strength. Hughes and Ash[57] subsequently found that this minimum critical sand content depended on aggregate grading, type, and overall mix proportions. Strength is also affected by the volume fraction of coarse aggregate, and maximum critical values have been observed[58]. It appears that these values can vary greatly with aggregate type, grading, and maximum size and be effectively zero in some cases. Among others, Te'eni[59] has shown that with some mortar systems, the inclusion of small amounts of coarse aggregate causes a strength decrease, but that this trend may be reversed when a certain volume fraction is exceeded, indicating the further possibility of a minimum critical coarse aggregate content. The existence of a number of critical mix parameter values would certainly explain many of the conflicting conclusions arrived at by earlier investigators.

Different strength-controlling effects, resulting from the inclusion of normal aggregates in cement paste and mortar systems, have received qualitative recognition but a rational quantitative assessment still appears remote. The aggregate amount and size gradation is important as this will affect the workability of the fresh mix and the maximum packing density. If the paste does not fill the spaces between aggregate particles the ensuing porosity will have a detrimental effect on strength. Concrete and mortar mixes with sand contents in excess of the maximum critical value invariably exhibit this particular form of porosity[51, 60]. It has not been suggested, however, that loss of density through packing incompatibilities is necessarily associated with the maximum critical coarse aggregate content for concrete. Other effects can also exert some influence.

Concrete is often described in terms of a three phase model. At the first level of discrimination the material is considered as a mortar matrix surrounding the coarse aggregate particles; in turn the mortar is considered as a cement paste matrix, including air, water, and unhydrated cement in which the fine aggregate fraction is distributed. Most concretes contain continuously graded aggregates which makes the distinction between fine and coarse material somewhat arbitrary; however, the three phase model is more useful for general descriptive purposes than a simple two phase model which considers concrete in terms of cement paste and aggregate only and takes no account of size distribution. As hard inclusions in a soft matrix, normal aggregate particles act as "stress-raisers" but, since they are also bonded to the matrix to some extent, they may serve to arrest or control cracks and hence contribute their strength and rigidity to the composite of which they are an integral part[60]. Depending on the degree of interaction between the aggregates and the matrix, an individual aggregate particle can exert a net disruptive influence or offer effective reinforcement. Swami and Rao[58] have implied that overall disruption occurs when the maximum critical coarse aggregate content is exceeded. Bond strength, especially that between coarse aggregate particles and a mortar matrix, has been the subject of many studies[61-65]. Most experimental work has involved determinations of the tensile or shear resisting properties of isolated matrix-aggregate interfaces. While the results of such tests are particularly sensitive to the testing techniques employed, a number of important factors have been established. Bond strength has been shown to depend to varying degrees on aggregate type, stiffness, size, shape, and surface texture, on the matrix strength and stiffness, and on casting and curing details. Bond surfaces are the consequence of chemical and/or physical interactions between the aggregate particles and the matrix. With chemically inert aggregates all bonding is of a physical nature resulting from almost solid-to-solid contacts (van der Waals forces) and mechanical interlocking. The existence of matrix-aggregate contact bonding was demonstrated as early as 1887[66]. More recent experimental and theoretical studies by Wittman[67] and others have indicated that the actual contact involves a very small proportion of the available aggregate surface area. It is interesting to note that the same van der Waals forces contribute significantly to the strength of hardened cement pastes[67, 68], especially when these are produced under pressure. Most aggregates used in practice also exhibit some form of chemical bonding with the matrix. The strength of these bonds and the effects of chemical interaction depend on specific aggregate and matrix characteristics. A comprehensive description of various aggregate-matrix interactions has been given by Swamy[69].

The importance of bond strength[70, 71] and the potential reinforcing action of aggregates[72-75] have been verified by a number of workers. Although many of the experiments have involved concrete mixes with unusual aggregates, such as glass marbles, plastic-, polymer-, and rubber-coated materials, graded cement clinker, and crushed partly hydrated cement paste, the results are extremely relevant to a general understanding of the role of bond forces in composite systems. Studies on more usual aggregate concretes have been less illuminating. For example, compare the different conclusions reached by Kaplan[31] and Bennet et al.[76] regarding the relative strength-affecting significance of aggregate surface texture and elastic modulus. However, these results and other phenomena such as the high strengths associated with crushed aggregate concretes below a certain water-cement ratio[77], and the anomolous behaviour of very rich mixes which show a loss of strength after an initial curing period[78], can be explained rationally in terms of the varying influence of different parameters on bond quality. The concept of a minimum critical sand content, mentioned previously, may be particularly relevant to the last example cited.

Normal aggregate mixes with the same water-cement ratio decrease in overall porosity with increasing aggregate-cement ratio provided adequate workability can be maintained and that packing incompatibilities do not occur. For lightweight aggregate mixes the reverse applies and overall porosity appears to be the main strength-controlling factor although others have been identified[79-81]. No reinforcing action can be expected from lightweight aggregate particles since these act as soft, weak inclusions in a relatively hard, strong matrix.

2.3.3 Testing Details

As is common for other materials, the mechanical properties of concrete are influenced to some extent by aspects of the testing methods used in their determination. Variations in test procedures, test equipment, specimen size and geometry, etc., can account for significant differences in the measured values of nominally similar parameters[82-90]. Thus, although it is often convenient to consider that a property such as concrete compressive strength is a unique intrinsic material characteristic, there is no real justification for this assumption.

Attempts to compare results from different sources are further complicated by the dependence of measured concrete properties on prevailing moisture and temperature conditions(68, 90).

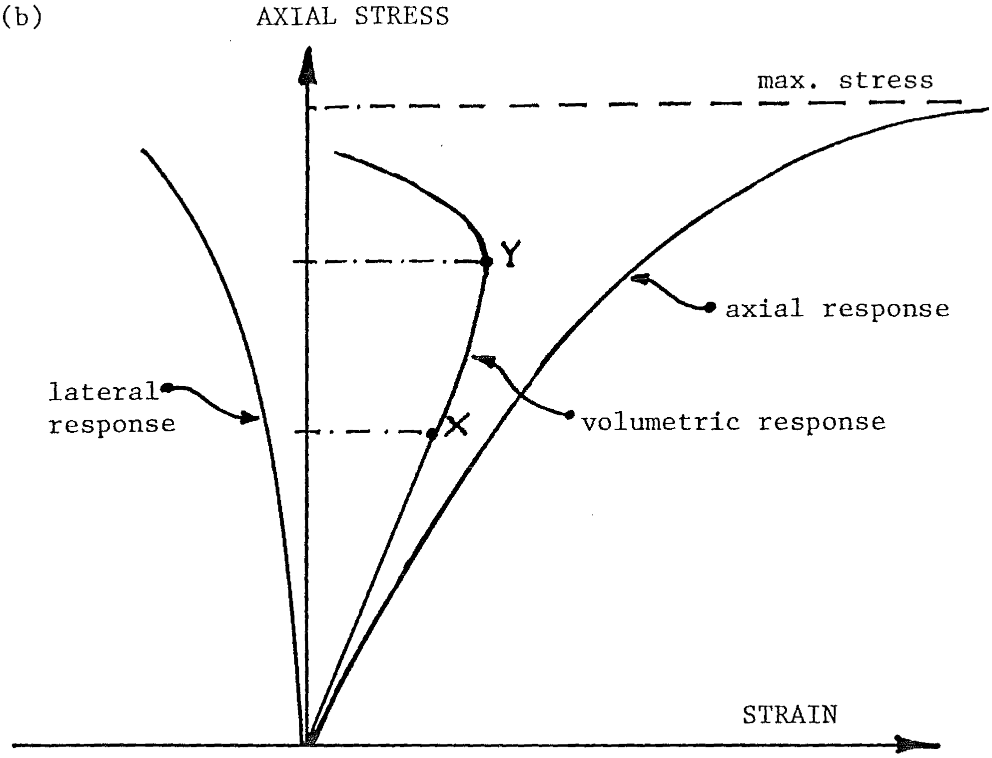
Existing standards only cover a few simple prescribed methods of strength determination. As a result, various non-standard techniques have been employed in different studies. Hobbs(91) has drawn attention to the complex conditions associated with many of these test procedures, the indeterminate nature of which can make rational interpretation of the final results extremely difficult. Unfortunately, a failure to recognise the role of secondary effects would appear to cast serious doubts on the validity of the analytical assumptions used and the conclusions reached by a number of investigators. Even the prescribed "simple" tests differ in complexity since these are not standardised on a global basis. The variations in specific test values, determined in accordance with the standardised procedures of different countries or organisations can be quite considerable. When concrete is tested in a compression machine between hard, stiff platens, restraining frictional forces are induced at the ends of the specimen, which can increase the applied nominal stress required to produce failure. The contribution of these end-effects towards overall stability decreases as the ratio of specimen height/width increases. Thus, the apparent strength of concrete cubes is generally significantly higher, in a statistical sense, than that of either cylinders or prisms with a height/width ratio of 2, although the actual strength differences involved are also sensitive to other factors such as moisture content, aggregate grading, and relative strength levels; differences of up to 30% have been reported. Even allowing for the effects of specimen geometry, the B.S.I.(92) and A.S.T.M.(93) measures of concrete compressive strength, based on the load bearing capacities of 100 mm or 150 mm cubes, and 300 mm x 150 mm diameter cylinders respectively are not strictly equivalent since different casting procedures, curing environments, and loading regimes are employed for each.

2.4. THE BEHAVIOUR OF CONCRETE IN COMPRESSION

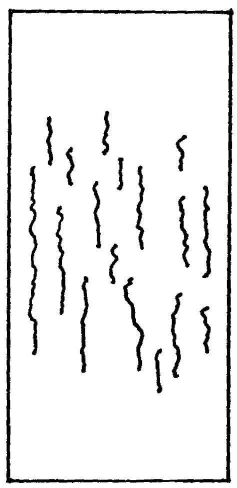
2.4.1 Uniaxial Compression

Typical short-term "stress-strain" curves for concrete under continually increasing axial compression are shown in Figure 2.1(a).

(b)



(b)



FIGURES 2.1: Response of Concrete to Uniaxial Compression

(a) Typical stress-strain curves

(b) Multiple splitting characteristics

For any particular concrete, the exact form of the curves and the limiting values depend on specimen dimensions, test machine characteristics, the rate of loading, and the methods employed for strain measurement[88, 94]. Failure loads tend to increase and failure strains decrease with increasing rates of load application[88], although failure loads are far less sensitive than ultimate strains within the range of loading rates normally considered for concrete testing. The stresses and strains represented are average values for a volume of material considerably larger than any of the constituent particle sizes. As concrete is a heterogeneous, frequently anisotropic, composite material large variations in the magnitude and nature of local stresses are to be expected. A number of experimental studies[17, 95, 96] have indicated the complexity and range of typical internal strain distributions associated with the "simple" compression testing of concrete.

The formation and subsequent propagation of microcracks are now generally recognised as important factors in any consideration of the behavioural aspects of concrete under compressive load. It was noted previously that some limited degree of microcracking is already present in most unloaded concrete systems as a direct result of earlier interactions at mortar-coarse aggregate interfaces, and that these bond areas are often sources of potential weakness. Several different experimental techniques[13-19] have been used to observe, both directly and indirectly, the development of microcracks within concrete specimens as external compressive stress is applied. Three types of microcracks have been identified:

(1) interfacial, or bond cracks, generally in the region of large aggregate particles;
(2) mortar cracks, in the paste-fine aggregate matrix;
(3) continuous cracks, formed by mortar cracks bridging two or more bond cracks.

Bond cracks within the mortar phase have not been widely reported; mortar cracks appear to frequently pass through fine aggregate particles[69, 70], which trend agrees with other work[62] indicating that aggregate-matrix bond strength increases with decreasing particle size.

In the initial stages of loading, microcracks are initiated in areas of high tensile strain concentration, which are thereby relieved so that the cracks do not propagate. As further load is applied the microcracks begin to increase in number, size, and extent, but are still stable in that they require additional load for further propagation. At a later stage severe continuous cracking can be detected[15, 97, 98]; many of the cracks are now unstable and will continue to propagate whether or not additional load is applied. With increasing load the whole internal system becomes disrupted, macrocracks appear on the surface of the specimen, and collapse takes place.

The stress level at which the stable propagation of microcracks first begins will not be unique to the entire volume of concrete because of local variations in behaviour. However, an average stress level, the "initiation stress"[70], can be inferred from the point at which the longitudinal or volumetric stress strain curves first exhibit a deviation from earlier trends, point X in Figure 2.1(a). This has been termed the "discontinuity point" by K. Newman[99] and suggested as a lower bound definition of concrete failure, for design purposes[100, 101]. The initiation stress and the fatigue strength of concrete are of a similar magnitude. Factors influencing the initiation stress include aggregate quantity, size and grading, aggregate-matrix bond strength[102], and rate of load application[99]. The lateral tensile strain at discontinuity decreases with increasing aggregate content[100, 101].

Measured values of the stress at which unstable crack propagation occurs vary slightly with the methods of detection employed[100], but also appear sensitive to the factors listed above[70]. The "critical stress"[105] corresponding to the onset of dilation, point Y in Figure 2.1(a), is commonly used as a good estimate, and has been associated with the long-term strength of concrete[106]. It has also been claimed, however, that the onset of dilation gives an underestimate of long term strength and that this can best be related to an "incremental Poissons' ratio" concept[107].

Local strain distributions are affected by the relative stiffness, spacing, and size of the aggregate particles[17, 95, 96], and the presence of voids[19, 108], but favourable conditions for the initiation of microcracks will also depend on the distribution of strength characteristics within any one small region. Although bond cracks appear to predominate at all stages of loading[13, 18], the eventual failure of the composite is brought about by the initiation and subsequent development of mortar cracks. In the early stages, within the mortar phase, these tend to grow parallel to the direction of applied compression. However, the progress of cracks which propagate further will be affected by the physical presence of aggregate particles. Within normal aggregate concrete the mortar cracks will generally be forced to deviate around the larger of these. In some areas, deviation can be achieved through an interaction with previously formed bond cracks in the interfacial zones. In other regions where local bond is unimpaired, an individual particle may effectively restrain areas of the surrounding matrix and thus cause a mortar crack to deviate, or even stop, before reaching the particle itself. An examination of concrete specimens after compressive collapse often reveals the presence of mortar "cones" on isolated coarse aggregate particles[104] indicative of this latter action. Multiple stable crack growth and repeated deviations absorb energy and produce irrecoverable internal deformations throughout the material, which therefore behaves in a quasi-ductile manner. If the microcracks are less in number or are able to propagate without deviation, more brittle behaviour is displayed. This can be seen in the stress-strain curves of mortars, cement pastes[58, 69, 109], and lightweight aggregate concretes[110]. Cement pastes are particularly brittle and show no onset of dilation during compression. Although the quasi-ductile behavioural characteristics of normal aggregate concretes usually increase with increasing aggregate content, overall failure strains tend to decrease, as a result of the progressively more complex internal loading system and higher local strains induced[58]. The complexity of the matrix-aggregate interaction can be judged from the following general observations of behaviour in uniaxial compression:

(1) cement paste failure strains generally appear to increase with increasing strength[109];

(2) concrete failure strains generally appear to decrease with increasing strength[111].

Since paste stiffness and strength are highly correlated, and paste strength exerts a considerable influence on the strength of concrete, it can be seen that factors other than the aggregate-matrix stiffness differential must also contribute significantly to the heterogenous nature of concrete.

The final form of concrete failure under increasing compressive load depends on the boundary conditions between the specimen under test and the source of compression. When frictional end-restraint is effectively eliminated, multiple splitting or cleavage, approximately parallel to the direction of applied compression, as shown in Figure 2.1(b), is most often observed, although ultimate failures by way of a subsequent shear mechanism have been reported[112] under such

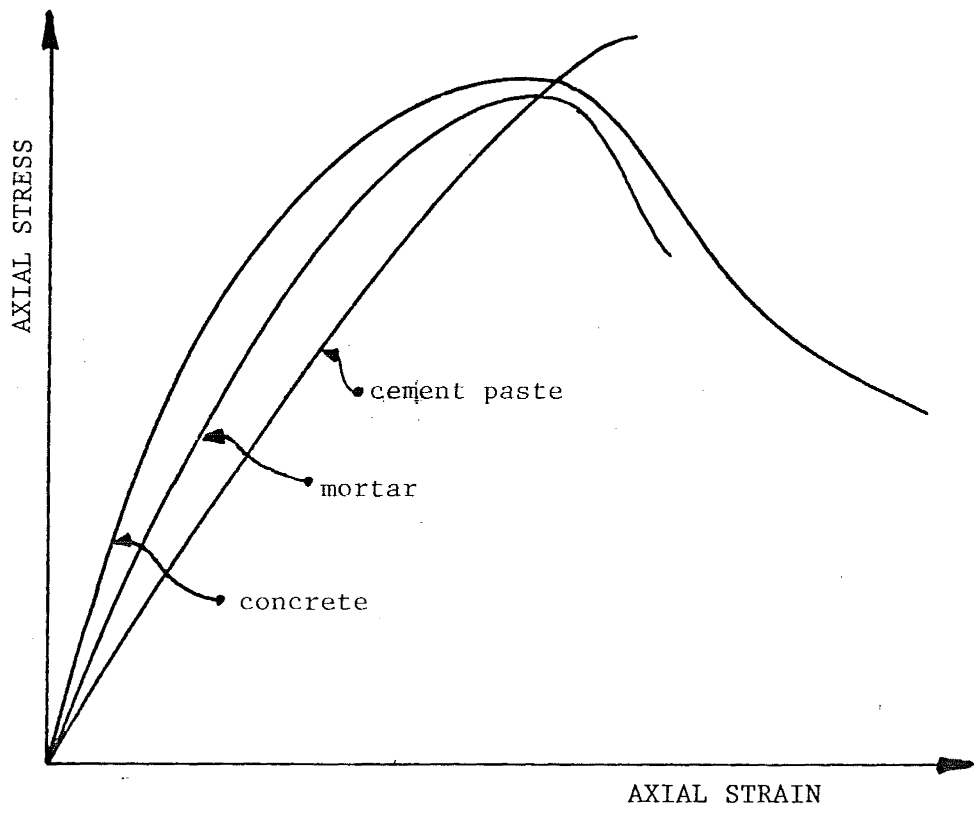


FIGURE 2.2: Axial Stress-Strain Response for Concrete, Mortars, and Cement Paste

conditions.

If the compressive deformation, rather than the load, is increased monotonically and if the testing arrangement offers adequate restraint, the stress-strain curves for cement pastes, mortars, and concretes can be continued beyond the point of maximum applied stress. Complete longitudinal compressive stress-strain curves for a typical normal aggregate concrete, its equivalent mortar, and corresponding cement paste are shown in Figure 2.2. Testing details[87], and the stiffness of test facilities can influence the form and limiting values of the curves; the strain corresponding to maximum load is especially sensitive to the rate of deformation[106], increasing as the latter decreases. The previously mentioned increase in quasi-ductile material characteristics associated with the inclusion of fine and coarse aggregate fractions in cement paste systems can be clearly seen, and is further illustrated by the varying abilities of concrete, mortar, and paste to sustain deformation beyond maximum load. In strain-controlled test systems (uniform strain rate) the available energy for crack propagation increases at a decreasing rate until the point of maximum compressive stress is attained, and then decreases continuously as the load-carrying capacity of the material falls. This regime encourages a far greater multiplicity of stable crack growth than a load-controlled system (uniform load rate) which provides the specimen with a progressively increasing source of external and internal strain energy up to the point of maximum applied compressive stress. Although the eventual failure mode which ultimately manifests itself is similar for both test procedures, the extent of internal specimen disruption prior to maximum load is likely to be quite different in each case. Thus, a typical concrete specimen, subjected to a strain controlled compressive test, will not generally exhibit any signs of severe surface macrocracking until the strain corresponding to maximum load is exceeded. In the falling branch of the curve, mechanical interlocking of aggregates and matrix and frictional effects can make substantial contributions to overall stability.

The limiting values of stress-strain curves inferred from the behaviour of concrete specimens subject to eccentric compression[113] are often significantly different from those of concentrically loaded specimens, tested under the same average conditions. However, eccentric compression induces a strain gradient and, as testing proceeds, different rates of strain necessarily prevail within the specimen. Since strain rate, which controls the growth and stability of microcracking[99], is known to affect the stress-strain curves of concrete in concentric compression[106], the somewhat "different" behaviour exhibited in the eccentric loading case is therefore understandable[113]; the influence which the presence of a strain gradient can exert on the growth and likely distribution of microcracks has been demonstrated experimentally [114]. Similar considerations are pertinent to the results of "pure" bending tests; however, with plain concrete specimens, the ultimate failure thereof is generally an obvious consequence of relative weakness in tension[115]. (It is worth noting that the tensile strength of concrete appears to be controlled by many of the factors influencing compressive strength[116, 117], matrix-aggregate bond strength being especially important. "Complete" axial tension stress-strain curves, similar in form to those of concrete in axial compression, have been obtained; the growth of local microcracking prior to the attainment of maximum load and subsequent failure by a single cleavage mode has been observed[118, 119].) In "simple" bending situations, longitudinal stress gradients are also induced as a result of the prevailing variation of bending moment. Tests on tapered prisms[120] have indicated that concrete compressive strength increases in the presence of such gradients, due to the stabilising action of "neighbouring" lower-stressed areas.

2.4.2 Biaxial and Triaxial Compression

Many investigators[16, 34, 36, 104, 112, 124-136] have attempted to study the behaviour of concrete in biaxial and triaxial compression using various combinations of hydraulic and/or mechanical loading techniques. Some general behavioural characteristics can be inferred from this work, but many of the reported "conclusions" are unfortunately clouded by a distinct element of uncertainty. Quantitative results and exhibited failure modes are extremely sensitive to the precise manner of loading and to the nature of any secondary stresses thereby induced. Thus, although experimental strength data would often seem to indicate a quite significant difference between the load carrying capacity of concrete in biaxial and uniaxial compression under approximately plane strain conditions, both strength measures - and their difference - are exaggerated somewhat by frictional boundary effects[112, 133].

The opening of microcracks in a concrete specimen subject to biaxial compression tends to occur in the direction of the unstressed axis, resulting in eventual fracture by extensive parallel cleavage[104] or combined cleavage and shear[112]. If one of the applied biaxial stresses is of a relatively low magnitude, failure modes similar to those in uniaxial compression are commonly observed[104]. The action of a two-dimensional compressive stress system effectively limits the extent of possible bond cracking[133]; large aggregate particles examined after failure frequently show no loss of bond with characteristic planar "haloes" of surrounding mortar matrix[104].

The degree of overall restraint offered by the loading regime of a triaxial compression test can, in some circumstances, encourage the multiple growth of stable microcracking[100], the growth patterns themselves being influenced by the direction, or directions, of least applied stress[128]. However, under relatively high confining pressures, internal cracking may be almost completely inhibited; this results in eventual local breakdown through the onset of crushing rather than regional separation[135]. With increasing three dimensional restraint, the specimen under test can sustain progressively higher loads before overall failure takes place, by some cleavage, shear, crushing, or combined failure mode; exactly which failure mode (if any) ultimately predominates is controlled by both the relative magnitudes of the applied loads and the physical nature of those boundary conditions imposed on the specimen by the testing regime itself.

More detailed discussion of individual results and of other studies relating to combined stress states will be given later in this work.

CHAPTER III

EQUATIONS, MODELS, AND THEORIES

The writer feels that the strength of concrete under biaxial tension is more than that under uniaxial tension. The writer has shown this by developing a mathematical model for concrete.

K.T. Krishnaswamy[137]

3.1. FOREWORD

Synge[138] has defined a "pygmalion syndrome" from which, he suggests, most scientists suffer. This affliction takes the form of an inability to differentiate between a model of behaviour and reality. It would appear that concrete technologists are no less immune to the complex.

3.2. CONCRETE AS A MULTI-PHASE MATERIAL

Many models have been proposed[139-154] to predict the elastic moduli of a composite material in terms of the properties and volume fractions of its constituents. There are, however, numerous difficulties associated with the realistic modelling of complex materials and simplifying assumptions must be made. Consider, for example, a two phase model consisting of coarse aggregate particles in a mortar matrix, or of fine aggregate particles in a cement paste matrix. Even if the individual phases could be considered as homogeneous, isotropic, and linearly elastic, a rigorous analysis would require a knowledge of the shape, size, and spatial distribution of the inclusions before the field equations could be determined and the solutions matched at the phase boundaries. Despite the probalistic nature of concrete systems the statistical continuum theories[146, 147] do not offer much hope of a practical solution. Concrete is not a strictly contiguous granular material[148] at the primary levels of discrimination, in the sense that the aggregate inclusions are rarely involved in extensive surface

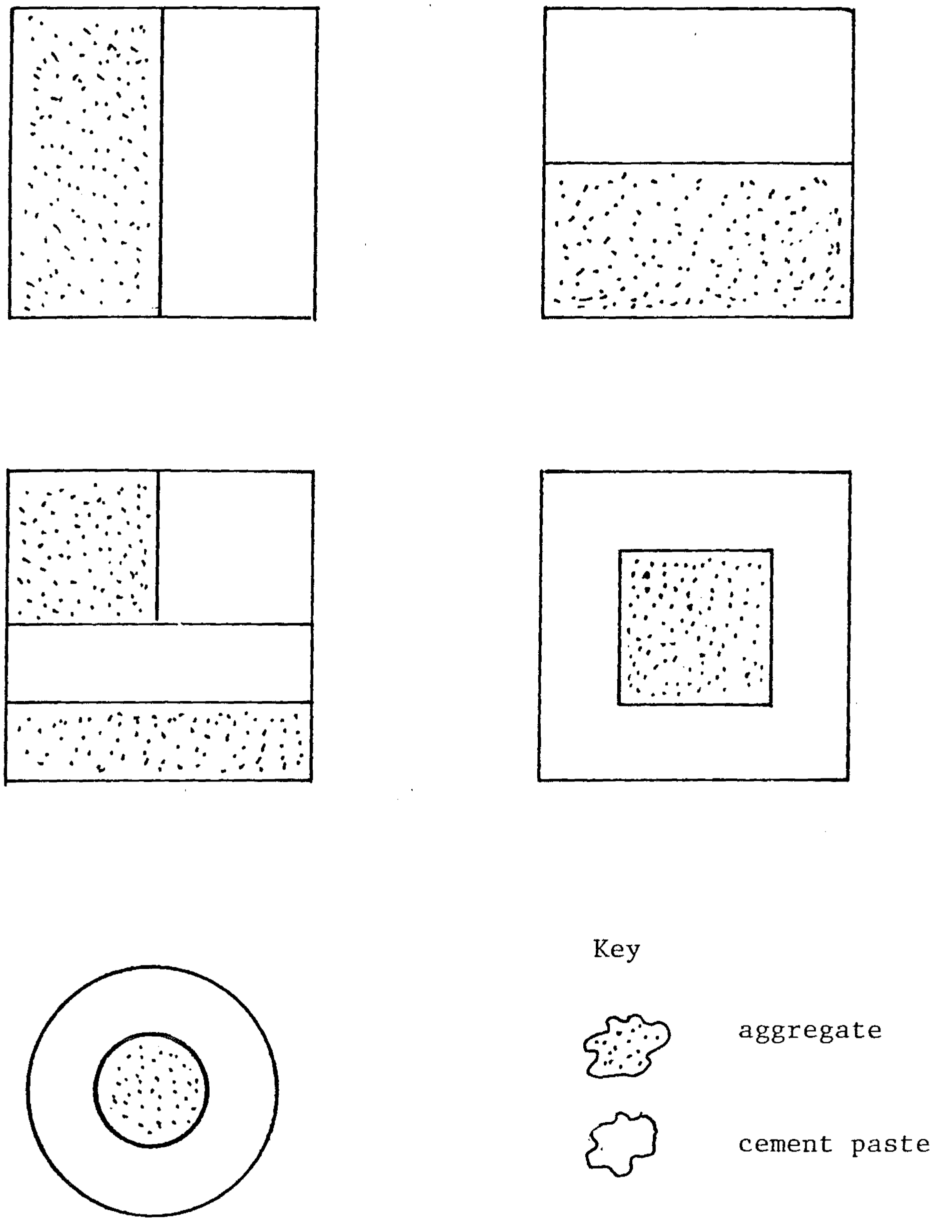


FIGURE 3.1: Two-Phase Elastic Models

contact. Most mathematical models therefore treat concrete as a heterogeneous continuum and, in deriving the effective elastic moduli, ignore such interactions, although inclusion surface effects must exist to some degree and will tend to increase with increasing aggregate content[59].

The extended variational principles of elastic strain energy have been used[143] to bound the effective moduli of idealised general multiphase continua having no specific internal geometry, but the realistic application of the resulting formulations necessarily involves further simplifications and assumptions. A more common alternative approach is to first simplify the structure of the material and give this a particular physical form before undertaking an analysis. Several examples of such two phase models which have been employed for previous concrete studies are shown in Figure 3.1, with particular reference to the action of an external compressive stress, σ. The models range in complexity and overall applicability, the composite Young's modulus, E, for uniaxial compression being the most frequently modelled parameter. The various boundary values derived from the more general applications of the elastic continuum theories can be associated with particular models[91]. Simple exponential and power relationships, involving empirical constants have been suggested[59, 153, 154], to describe the changing elastic properties of concrete with increasing aggregate content. The work of Te'eni[59] is particularly interesting in this context since it attempts to take some account of different aggregate characteristics. Using data from a number of sources, Te'eni demonstrated that an equation of the form,

$$E = E_m e^{aV_k} \qquad \text{..... 3.1}$$

, where E and E_m are the compressive elastic moduli of concrete and matrix, respectively, V_k is the volume fraction of aggregate, and _a_ is an empirical constant predicted E within closer limits than any of the general theoretical or abstract elastic model expressions. Consideration was then given to the implied boundary value of E for an aggregate volume fraction of unity. Since the continuum theories generally ignore any effects of aggregate size and the possibilities of surface contact interactions, these tend to predict a boundary value equivalent to the elastic modulus of the individual aggregate particles. Te'eni showed that equation 3.1 offered a similar prediction only for the dynamic modulus, the experimental determination of which involves extremely low applied stresses[155]. For the higher applied stresses, and hence the greater likelihood of surface contact effects associated with the physical measurement of the static modulus, the implied boundary value was invariably less than the equivalent modulus of the individual particles and decreased with decreasing aggregate size (increasing inclusion surface area), as would be expected.

The role of aggregates in controlling or influencing concrete compressive strength is extremely complex and variable, and is beyond the scope of the effective modulus models, these being concerned principally with behaviour under relatively low stresses, to which the assumptions of linear elasticity can be applied with some confidence. As aggregate content increases, critical areas within a concrete system are subjected to stresses and strains beyond the local elastic limits at progressively lower values of externally applied compressive load[58,156], although this need not necessarily be reflected by a decrease in overall strength. However, the presence of air voids is known to affect both the initial stiffness and final strength of all concrete systems in a much more consistent manner, and a number of studies[151, 157-163], theoretical and experimental, have been devoted to these actions.

Unfortunately, the rationale and implicit assumptions of the mathematical and quasi-physical models for the elastic moduli of general two phase materials frequently break down when applied to air void inclusions. Even the small deformation elastic solutions relating specifically to porous materials[164-166] are only strictly valid for relatively low porosities. Numerous simple empirical formulae[152, 158, 167-170] have been proposed which show varying degrees of correlation with experimental data; some such as the exponential form equivalent to equation 3.1 do not, however, comply with the boundary condition of zero elastic modulus with complete porosity.

The effects of porosity on concrete strength have not received extensive theoretical study, although Hasen[161] has suggested a functional relationship, based on a simple geometrical model, which fits available data reasonably well. An equation derived by Schiller[171] for general porous materials, and an extension[160] of the Mackenzie model[164] originally formulated to predict the elastic constants of such media, have been applied to the compressive strength characteristics of cement pastes with some success. Concrete compressive strength, being a manifestation of many complex local interactions, is more sensitive than the initial elastic modulus to increases in porosity, since the latter is primarily influenced by a reduced effective load-bearing area. Using the compressive strength of fully compacted concrete as a convenient reference value, Popovics[158] has proposed an empirical expression for the relative compressive strength, S_{rel}, of concrete containing air voids, of the form,

$$S_{rel} = e^{-a_1 V_1 - a_2 V_2} \qquad \text{..... 3.2}$$

, where V_1 and V_2 are the entrapped and entrained air contents, respectively, and a_1 and a_2 are constants. This formula, although predicting a finite strength for complete porosity, does recognise the possibility of different magnitudes of effect arising from different types of voids. In practice, such differences can be quite marked; for example, incomplete consolidation results in an uneven distribution of irregularly shaped pores, whereas deliberately entrained air is generally well dispersed throughout the volume of the cement paste matrix as a discontinuous system of minute bubbles. Other less discriminating exponential equations[59, 91, 167], power functions[8, 43, 159, 172], and linear relationships[173] have been suggested to describe the porosity dependence of concrete, mortar, and cement paste strength. Thus, for example, Wischers' paste strength equation[43], mentioned in Chapter II,

$$S = k_4 (V_s/N)^n \qquad \text{..... 2.4}$$

, which can also be written in terms of porosity, p,

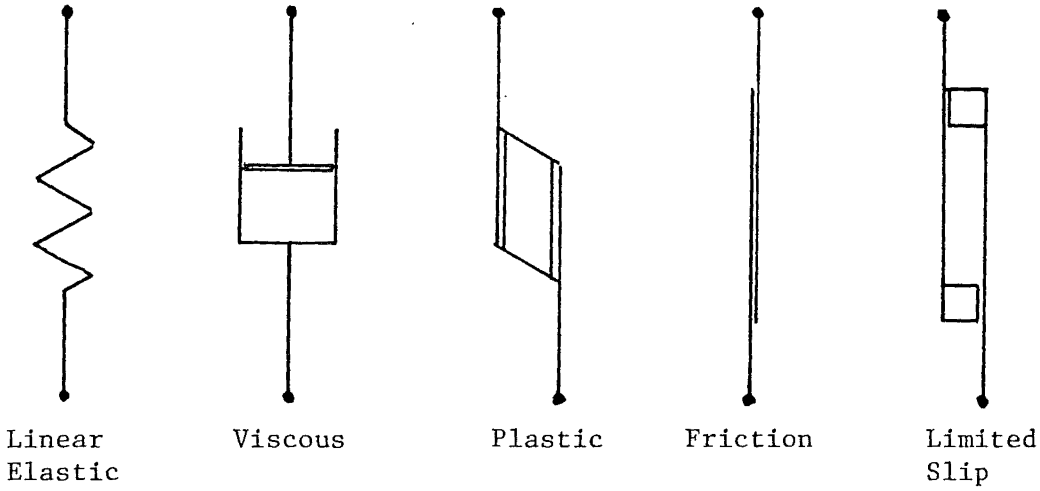
$$S = k_4 \ (1-p)^n \qquad \text{..... 3.3}$$

, makes no distinction between capillary pores, gel pores, and air voids. The power functions have the advantage that, unlike many of the exponential equations, these correctly predict zero strength for a completely porous material. The paste or mortar strength equation suggested by Powers[8],

$$S = k_3 (X_F)^3 \qquad \text{..... 2.3}$$

, is only of a similar form to those described above if complete hydration of the cement has taken place, since it ignores the presence of any unhydrated material. Two pastes with the same generalised gel-space ratio could have quite different overall porosities. The dependence of strength on overall porosity indicated by other work[43-48, 172, 174]

Elements



Combination Examples

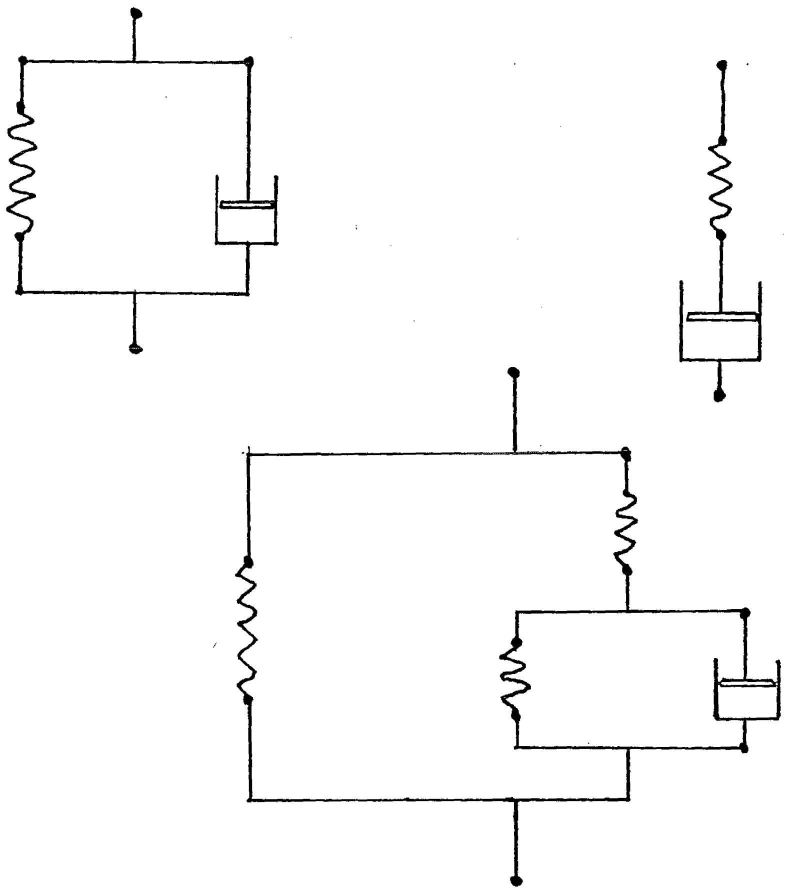


FIGURE 3.2: Rheological Models (Elements and Combinations)

would therefore appear to cast serious doubts on the underlying rationale of equation 2.2. Although pore size distribution has been shown[48,90] to affect the strength and behavioural characteristics of cement pastes and concretes, there are obvious practical difficulties involved in any attempt to make a general quantitative experimental assessment of this action.

3.3 THE RHEOLOGICAL MODELS

The time-dependent behaviour of materials under applied loads or displacements can be simulated by combining various idealised rheological elements, representing separate sources of elastic, viscous and plastic deformation[175].

A number of simple basic elements and suggested models for concrete are shown in Figure 3.2. The constants which characterise the individual elements, and hence influence the behavioural aspects of any particular model, are generally inferred from the results of actual tests on the real material. (It is difficult to attach unique associative "meaning" to the quasi-physical internal structure of a rheological model; different combinations of different numbers of component elements can be made to display similar deformational traits through a "suitable" choice of element constants.)

While the rheological models have found greatest application in the field of creep and relaxation studies[176, 177], they can also be employed to describe strength characteristics and predict the form of stress-strain curves under short-term loading[179, 180]; such is achieved by incorporating specific limiting values of load-carrying capacity or deformation within the general properties of individual elements. Where viscous elements are included in any rheological model combination the general differential equation describing overall behaviour will include terms relating to the time derivatives of local displacements. A variety of mathematical solutions will therefore be possible, each depending on the precise nature of the boundary conditions employed. For example, stress-strain expressions derived for circumstances of a constant rate of strain application will differ in form from those obtained by modelling a constant rate of load application; of course, both will reflect the prevailing value of "control rate" seen to be involved.

3.4 FRACTURE MECHANICS

There can be little question that the theories following on from Griffith's[18] original work on the growth of cracks in a perfectly brittle material under tension, have been able to give valuable insight into the internal behaviour of concrete systems under externally applied stresses.

Although the heterogeneous nature of concrete defies simple idealisation and hinders the use of the more quantitative aspects of fracture mechanics, the basic concepts of crack initiation and propagation are extremely powerful. An excellent review of theories and concepts relating to concrete and other similar materials has been given by Radjy and Hansen[182].

Cracks are generally assumed to begin whenever a prevailing local tensile stress exceeds a certain value, the theoretical strength, at some point within the material. Even in situations where the applied loads are compressive, areas of high tensile stress concentration, resulting from internal structural actions, can be inferred from both experimental observations[17, 95, 96] and refined analytical techniques (161, 183, 184). In any normal concrete system aggregate particles, pores, flaws, pre-existing cracks, and other manifestations of heterogenuity at different levels of discrimination may all contribute towards this effective stress conversion. However, the local conditions suitable for the initiation of a crack in a particular region may not be those necessary for extensive propagation.

If, during the early stages of crack development, a point is reached where the locally available strain energy which would be released by further crack movement falls below that required to create the additional new surfaces involved, overcome any associated frictional effects, and produce the necessary amounts of inelastic deformation, the crack will become stable and remain immobilised until the specific energy demand is met. The relief of peak stresses and their subsequent redistribution may therefore enable multiple slow crack growth to occur before the energy balance favours unstable propagation.

Although a mixture of concepts has been described above, the argument of available energy can also be applied to actual crack initiation. Local fracture can therefore be visualised from either a stress or an energy viewpoint and this is reflected in the separate, but typically complementary, approaches of different theoretical developments[60]. A number of fracture parameters have been defined and proposed as material constants. For example, the stress intensity factor, K, which is used to effectively specify the stress and displacement fields in the vicinity of a flaw tip, is given a critical value, K_c; conversely, from an energy viewpoint, the critical strain energy release rate (energy per unit area of fracture) for crack propagation is designated as G_c, the fracture toughness. Many attempts[185-192] have been made to obtain measures of these and other fracture parameters for cement pastes, mortars and concretes from tests involving notched or pre-cracked specimens in either direct tension or flexure. Unfortunately, the significance of the equations employed and the relevance of the underlying assumptions, especially when these include homogenuity, linear elasticity, and negligible crack growth prior to fracture, are often questionable. On the basis of experimental results, Kesler et al.[192] have stated that the concepts of linear elastic fracture mechanics are not directly applicable to such tests. Within a concrete specimen under applied compression, the internal stress states and energy distributions are even more complex and less amenable to simplifying assumptions. This does not, however, invalidate the use of the general principles of fracture mechanics. A qualitative model[108, 193] to describe both stable and unstable crack propagation in a heterogeneous material, based on simple linear-elastic strain energy release formulations but incorporating a non-linear energy demand to account for different regional values of fracture toughness, has been suggested for concrete. Although this is still an over-simplification, it has considerably more conceptual merit than many of the quantitative homogeneous models.

The additional energy demand, or increased fracture toughness[60], arising from the work dissipated by local inelastic deformations, differentiates cracks in concrete from those in an ideal brittle material which only require energy for the creation of new surfaces. Any loading condition or internal structural action which results in an increase in local energy demands will tend to decrease the likelihood of ideal brittle behaviour by encouraging multiple crack growth. However, in discussing overall effects, differences in available energy must also be borne in mind. Consider, for example, the action of normal aggregate particles within a concrete system under uniaxial compression. Even at nominally low values of applied load, high tensile and shear stresses induced by the presence of the coarse aggregate fraction can cause some degree of cracking(58) but, since the available energy for crack propagation is also low, this remains stable. At higher loads the particles act as physical barriers and cause the cracks to deviate, thereby increasing surface and frictional energy demands. With equivalent cement pastes and mortars, cracks are initiated at relatively higher values of applied load when available energy is greater, and are therefore less stable; the cracks which do propagate are free to move without substantial deviation, and more brittle behaviour than that associated with the original concrete is observed. It is important, however, to re-emphasise that brittleness in this context refers to the shape of the stress-strain curve and to the extent of pre-failure cracking or internal disruption, and not necessarily to the limiting values of load or deformation. The stress-strain behaviour of cement pastes is a particular case in point; here, failure loads, equivalent strains, and brittle characteristics all generally tend to increase(59) as the capillary porosity decreases (with either decreasing water-cement ratio or increasing degree of hydration). Porosity lowers compressive strength but improves the potential for quasi-ductile behaviour by increasing the amount and extent of early cracking; while the pores act as stress raisers(158) or converters(108) they may also serve to blunt crack tips(194) and arrest local propagation. Tensile and flexural strengths, being controlled to a far lesser extent by multiple crack growth, are therefore relatively less sensitive to changes in porosity(158).

The link between available external strain energy and the stability of crack propagation has been repeatedly established(85, 108, 185). For example, Glucklich(108) has demonstrated that localised rapid crack propagation, consistent with near-ideal brittle behaviour, can be induced in a concrete specimen under uniaxial compression by testing this in series with a coiled spring. Conversely, concrete subjected to slowly increasing compressive or tensile strain in a stiff testing machine will behave in a quasi-ductile manner, and sustain deformations well beyond the point of maximum load. Despite the misleading adjective "true" which is sometimes applied to the complete stress-strain curves obtained from stiff testing arrangements, the form of these curves is sensitive to the stiffening methods employed, to the prevailing rates of strain, and to the size of specimen tested. As specimen size increases, progressively more brittle behaviour (as described previously) can be expected[196] since, for any nominal value of external stress,the available internal and external energy for crack propagation is a function of the quantity of material under load. The effects of specimen size on failure values depend on the degree to which overall specimen stability is controlled by individual crack movement; tests for compressive strength which produce multiple internal crack growth before ultimate collapse will thus be less sensitive than those involving direct tension, flexure, or splitting where eventual failure is extremely localised. Varying degrees of size effect have been reported[85, 195-201] with regard to compressive strength. It is known[198-200] that secondary factors such as differential rates of curing and drying can serve to magnify apparent changes. Statistical concepts of strength and size will be reviewed in the next section.

The quantitative models of fracture mechanics are not generally applicable to the failure of concrete at a phenomenological level because of the difficulties associated with any attempt to define precise rational criteria by which this may be judged, especially when multiple crack growth, crack system interactions, and progressive internal disruptions are involved. Analytical techniques can be used to determine the orientation of critical flaws, having specific geometrical characteristics, which will first provide the prerequisite local stresses for flaw extension or the initiation of branch cracking within an idealised material under prescribed external loading, and subject to particular boundary conditions[202]. The common assumptions of homogenuity and linear elasticity enable the prediction of preferred directions of individual crack growth from the critical flaws on the basis of maximum strain energy release[203], without having to account for any changes in specific energy demand. Some models[204, 205] have been adapted to include crack propagation through a creep mechanism, by incorporating a time-dependent elastic modulus. Others[19, 206] recognise the possible closure of cracks or flaws in areas of prevailing compression; these "friction cracks" can then sustain normal and shearing stresses across and along their surface boundaries. Although the Griffith approach[202] only considers crack initiation and propagation in terms of separation through high induced tensile stresses (Type I fracture), crack movement by sliding action resulting from high local shear stresses (Type II fracture) may also occur. The effects of different flaw spacings on the peak values of internal stress distributions and subsequent crack interactions have received limited experimental and analytical[19, 205, 207] study. Despite the obvious heterogenuity of concrete and the probabilistic nature of flaw, stress and strain energy distributions, the idealised models of fracture mechanics do predict microcrack patterns within the mortar phase similar to those observed under actual test conditions. For example, crack growth parallel to the direction of applied loading in a uniaxial compression test is consistent with considerations of local stress and available strain energy. It is worth noting that, within the matrix of a heterogeneous composite material, high local tensile and shear stresses can theoretically be induced in regions where no real "flaws" as such exist. Since the cracking theories are concerned principally with local fracture rather than with overall failure it has been suggested[106] that these can best be related to the concepts of "discontinuity" proposed by K. Newman[99].

3.5 THEORIES AND MODELS OF FAILURE

Various criteria have been formulated to predict the relative magnitudes of external stresses or strains under which some defined point of material breakdown will occur. Reviews of the classical theories of failure, relating mainly to the ultimate strength or yield point of linearly elastic, homogeneous, isotropic materials, can be found in the works of Timoshenko[208] and Nadai[209]; although based on the concepts and axioms of applied mechanics, these can often be viewed as simply different expressions of mathematical empiricism. Nevertheless, numerous attempts have been made to extend or modify the theories to describe the failure of concrete at the macroscopic level in quantitative terms.

As would be expected from an essentially empirical background, similarities frequently exist between the final forms of different failure criteria. Consider, for example, the well known criterion which assumes some functional dependence, f_1, of the octohedral shear stress, τ_o, on the value of octohedral normal stress, σ_o, at "failure", defined by maximum load-bearing capacity;

$$\tau_o = f_1 (\sigma_o) \qquad \ldots\ldots 3.4$$

Both τ_o and σ_o can be expressed independently of the third invariant, I_3, of the stress tensor. If the value of the third invariant has any influence on the failure conditions under a general loading system, it would therefore affect the function f_1. For the special case of a biaxial stress state, where I_3 is zero for all load combinations, linear and parabolic functions have been suggested(131, 210). However, the results of Rosenthal and Glucklich(131) required two separate linear equations, the applicability of each being restricted by the sign of the second invariant, I_2, of the stress tensor. Other work involving triaxial ($I_3 \neq 0$) stress states has further indicated(28) that, for any particular concrete, f_1 is not unique but depends on the type of external loading conditions employed. Criteria involving all three invariants of the stress tensor have been proposed(211, 212), but these have not been applied extensively. Equation 3.4 can also be written in terms of the root mean shear stress, τ_{av}, and the mean normal stress, σ_{av}, which equals σ_o.

$$\tau_{av} = \frac{3}{\sqrt{15}}\,\tau_o = \frac{3}{\sqrt{15}}\,f_1(\sigma_{av}) \qquad \ldots\ldots\ 3.4a$$

If f_1 is a linear function, equation 3.4a becomes,

$$\tau_{av} = a + b\sigma_{av} \qquad \ldots\ldots\ 3.5$$

where $\underline{a}$ and $\underline{b}$ are constants. This may be regarded as a generalisation of Coulomb's internal friction equation(207);

$$\tau = c + \mu\sigma \qquad \ldots\ldots\ 3.6$$

where τ and σ are values of shear and normal stress on a critical internal plane at the point of slippage, and c and μ are the cohesion and coefficient of friction, respectively. Frictional theories developed by Rowe(213), in the field of soil mechanics, to describe inter-particle slippage in a granular material under axial compression, σ_1, and all-round lateral pressure, σ_3, in terms of effective stresses, have been applied by Gardner(130) to the behaviour of concrete in an identical "cylindrical-triaxial" test situation. The resulting failure criteria, which express σ_1 as a linear function of σ_3, are similar to the empirical expressions proposed by Considere(214) and Richart et al.(124). Equations 3.5 and 3.6 are not completely analogous since the latter ignores any effect of the intermediate principal stress, σ_2. The role of σ_2 as a part-determinant of concrete failure appears slight(132).

However, it does exert some influence[100]. Equation 3.6 predicts a linear envelope to the limiting Mohr circles of individual biaxial or triaxial stress states. To account for the curved envelopes implied by experimental results, non-linear relationships, based on the generalised Mohr theory, have been suggested[215]. Among others, Paul[216] has questioned the significance of the internal friction concept when the normal stress component is tensile.

Equation 3.4 originally evolved from the Huber-Hencky-Von Mises plasticity criterion[175] for ductile metals, involving the postulate of constant distortional or shear strain energy, U_D, at yield,

$$U_D = \text{constant} \qquad \ldots\ldots 3.7$$

where

$$U_D = \frac{3}{4G}(\tau_o)^2 \qquad \ldots\ldots 3.7a$$

and G is the elastic shear modulus. Since experiments showed that the strength of concrete, and other similar "brittle" materials, is significantly affected by the mean normal stress, the constant was replaced by a function of σ_o.

$$U_D = f_2(\sigma_o) \qquad \ldots\ldots 3.8$$

The underlying rationale of equations 3.4 - 3.8 precludes material breakdown under isotropic tension and implies a basic mechanism of failure by shear. Although concrete failure modes under some loading conditions are of this nature at a phenomenological level, cleavage modes are also commonly observed. To reconcile these differences, and enable a prediction of the actual failure mode, a number of theories[131, 216-218] specify two governing criteria of failure. The conditions for cleavage fracture generally allude to some concept of limiting tensile stress after Lamé, Clapeyron, and Rankine, or extensional strain, after St.Venant. For example, K. Newman's[99] two part criterion of discontinuity is based on equation 3.8 together with a limitation, ε_d, on the maximum extensional strain, ε_1, which depends on the mean normal stress, σ_o, the volume fraction of coarse aggregate particles, V, and the maximum particle size, D.

$$\varepsilon_1 \leqslant \varepsilon_d = f_3(\sigma_o, V, D) \qquad \ldots\ldots 3.9$$

Under short-term loading regimes, well-compacted concrete systems tend to display bulk elastic properties up to the discontinuity point and hence equation 3.9 can be converted to a criterion of stress by using the appropriate elastic constants. As applied stresses are increased beyond this point, inelastic deformations take place, the effective value of G in equation 3.7a changes, and the precise significance of equation 3.8 at ultimate failure, or at maximum load, becomes a matter of conjecture. If, alternatively, equation 3.4 is accepted empirically in its own right, rather than as being directly related to elastic shear strain energy, then this need not necessarily be rejected on such grounds. Newman has suggested[99] that for a criterion of ultimate strength, U_d in equation 3.8 be replaced by the deviatoric component, W_D, of the total work of deformation, and that equation 3.9 be altered to provide either a distortion or dilation limit. It is important, however, to note that since W_D includes dissipated work the former proposal is not founded on the same concepts of available elastic energy which permeate the general approaches of Huber and Hencky[175] and which are partially analogous to the ideas of fracture mechanics.

Failure criteria expressed specifically in terms of stress can be represented as lines or curves in two dimensional coordinate systems, or as surfaces in three dimensional stress space, experimental results being represented by points in both cases. For situations of general loading, the adoption of a two dimensional failure envelope necessarily involves some loss of information. To what extent the effect of this "loss" is critical would seem to depend on the particular test being modelled and the actual quantities being compared. Thus, for example, empirical formulations which ignore the intermediate principal stress can often be successfully "fitted" to triaxial data but do not always agree well with biaxial results; conversely, the octohedral stress relations obtained from biaxial tests can not be applied with any confidence to triaxial stress states. Three dimensional failure surfaces may, however, be considered in terms of particular planes. For example, in principal stress space, failure curves can be constructed in the planes of constant mean stress, σ_o, normal to the axis of the space diagonal, and in the Rendulic planes containing the space diagonal and one of the principal stress axes. Various empirical forms of these curves have been suggested for design purposes[199, 219, 220].

The general convex shape of failure surfaces in principal stress space is often emphasised. In the biaxial tension-compression region, however, the failure curves of some concretes and mortars exhibit distinct concavity[104, 221]. This fact has been taken by some[100] as an apparent violation of a stability postulate due to Drucker[222] but, as the postulate concerned was specifically formulated in terms of materials which are not strain-rate sensitive, such an inference is hardly justified. Similarly, the postulates of convex failure surfaces due to Palmer et al.[223] are limited to certain behavioural classes of materials to which concrete apparently conforms for some loading cases but not for others; these workers have concluded that concave surfaces are not unreasonable if inelastic deformations and disruptions of the internal material microstructure occur before the onset of macroscopic failure. The observed concavity of the stress envelope in biaxial tension-compression has been modelled by Paul[215] using a modified Coulomb-Mohr theory which only includes the action of internal friction when the normal stress component is compressive. Paul has also produced more general failure surfaces[224], having both convex and concave regions, for materials which are sensitive to the value of mean normal stress. Limited amounts of experimental data[228] imply that the failure surface of concrete is not necessarily open-ended and that breakdown under very high isotropic compression may take place; however, no attempt appears to have been made to include a condensation limit in any macroscopic theory of failure.

Experimental results, and their interpretations by different workers, vary considerably. Failure modes at the phenomenological level are extremely sensitive to testing details and the boundary conditions of loading; combined or transition modes are common[100, 104, 129, 210]. Doubt often surrounds the calculated values of average principal stresses especially when these are derived on the basis of elastic analysis. Even when the nominal principal stresses are applied directly uncertainties may still exist regarding induced secondary effects. The sequence of loading in biaxial and triaxial tests is important[128, 134] since failure is the result of sub-macroscopic interactions which are not generally controlled by the principles of linear superposition; i.e. different behaviour can be expected when individual loads are applied sequentially in large increments from that when either small increments or continuous proportional loading techniques are employed. During some forms of testing, such as those involving combined torsion and direct stress, sequential loading produces a rotation of the principal axes of stress and strain[129]. If load-induced anisotropic behaviour should occur, the axes of each may not always coincide. Failure surfaces in principal strain space have been used in other fields[225] but are relatively uncommon in concrete research[226], this despite the extensive measurement and recording of deformations which has been undertaken in association with testing programmes. However, both practical and conceptual difficulties arise with the strain approach at the phenomenological level if cracking and failure do not occur simultaneously.

Very few of the macroscopic failure theories or empirical formulations relating to average values of applied stress provide a rational framework within which the mechanics of material breakdown can be described. A logical alternative, in view of the generally progressive nature of concrete failure, is to transfer limiting criteria to the internal structural level and then treat overall behaviour in terms of combined local effects and interactions. This approach, although founded on a conceptually "realistic" basis, necessarily requires that some form of simplification or idealisation be applied to the material itself before any quantitative aspects can be examined.

Brandtzaeg[105], for example, considered a statistically representative "unit volume" of an ideal material, in a cylindrical-triaxial compressive stress state, to be composed of a large number of component elements, each of which possessed specific parallel planes of weakness, and hence a preferred direction of slip (governed by the Coulomb-Mohr internal friction theory): all possible orientations of planes of weakness were assumed to be distributed randomly throughout the many non-isotropic elements. This model, which he subsequently applied to "explain" the behaviour of concrete[124], largely evolved from the implications of prior experimental and analytical work due to Boker[227]. (Having studied the breakdown of marble under various combinations of external compressive stress, Boker had concluded that local anisotropy, due to a random distribution of internal weaknesses, exerted considerable influence on those ultimate behavioural traits exhibited.) The basic rationale of Brandtzaeg's approach might be summarised as follows. After the initial linearly elastic stages of loading, during which period the individual elements of the model are subjected to an ever increasing uniform stress distribution, a point is reached where the limiting shear stress is first attained on the planes of weakness of one particular element. The immediate potential for extensive "plastic" deformation is countered, however, by the local restraining action of physically sound neighbouring elements, within which secondary tensile stresses are thus induced as a consequence of the "contained thrust".

A continued increase in external loading produces further local plasticity and stress re-distribution throughout the body of the material as the critical shear stress is reached on the planes of weakness of progressively more individual elements. Brandtzaeg derived mathematical expressions relating the principal overall deformations to the level of applied stresses, based upon an assumption of equal deformation for all elements, both elastic and plastic, at any particular loading stage. Failure was taken to occur by a "splitting" action when induced tensile stresses attained a critical value causing separation of the material elements. The combined shear-tension concepts are of course consistent with the failure modes commonly observed in uniaxial, biaxial, and triaxial tests and have since been used by others(228) to describe the debonding and subsequent wedging action of coarse aggregate particles within loaded concrete systems at the internal structural level. Brandtzaeg's ideas have been extended recently by Taylor(229) to include more general types of loading, time dependent effects, and several sources of internal weakness, each having a range of limiting parameter values to simulate the heterogeneous nature of concrete; computed failure envelopes show reasonable agreement with experimental results, but are obviously sensitive to the limiting values chosen, independent estimates of which are not always available.

The Griffith theory of fracture mechanics relating to the stress raising action of critical flaws, and modifications of this to allow for crack closure and subsequent frictional and normal load transfer, apply to strictly local behaviour and hence may be used as a basis from which to predict the relative strengths of idealised materials. It has been shown(100, 230, 231) that both the original and modified criteria appropriate thereto closely resemble the corresponding expressions of the generalised Coulomb-Mohr theory. The resulting theoretical fracture envelopes in relative stress space are quite similar in form to experimental failure surfaces or curves(100, 232), even although it is generally unlikely that local fracture and overall failure will occur simultaneously. However, there is a reasonable amount of experimental evidence(100, 104) to suggest that typical failure and "discontinuity" envelopes are also somewhat similar in appearance.

A recognition of the important role and the particular characteristics of microcracks in concrete systems has led to a number of analytical studies(133, 233, 234) involving structurally simplified model concretes, subjected to uniaxial, biaxial and triaxial stress states. The hypothetical materials consist of specific arrangements of idealised, hard, coarse aggregate particles within a soft mortar matrix. (With careful preparation, specimens of real material conforming to this description can, in fact, be "created".) The subsequent analyses, based upon equilibrium and compatibility considerations, generally incorporate several criteria to account for the different possible manifestations of local failure. Emphasis is usually placed on the potential weakness of the aggregate-matrix bond regions which are assumed to fail under some combination of tension and shear, or compression and shear, controlled by generalised forms of the Coulomb-Mohr equation; limiting values of tensile and compressive stress within the matrix are used to predict mortar cracking and local crushing, respectively. As a result of the progressive nature of the failure process envisaged, the models behave in a non-linear manner, even when linear properties are assumed for the constituents. The application of finite element techniques has produced patterns of internal distruption and trends of relative strength values under different external loading regimes, which are consistent with those of experimental studies.

Several structural analogies have been suggested to describe the failure of concrete under applied compressive stress states. An idealisation due to Reinius[(235)] treats the hydration products of the cement paste phase as bars of low tensile strength. Baker's[(236, 237)] lattice structure synthesises the stress-raising and stress-converting action (the "thrust ring" effect) of hard coarse aggregate particles, the overall properties of the model being dependent on the values of strength and stiffness chosen for the various lattice members. Uppal and Kemp[(238)] considered a partially cracked concrete specimen in either uniaxial or flexural compression as an inter-connected two dimensional assemblage of rigid bending and tension elements, which becomes unstable at a critical buckling load, equivalent to the maximum stress; beyond this point, if the test is strain controlled, individual tension and bending elements fracture in a progressive manner as a result of excessive lateral deformations.

All measures of material strength are subject to inherent variations and can therefore be treated as statistical quantities. Numerous simplified versions of the mathematically complex probabilistic continuum strength theories[(239)] have been proposed; these consider any representative finite volume of a particular material to be statistically equivalent to some system of component elements having variable strength.

The overall failure characteristics displayed by the simple statistical models are extremely sensitive to the actual system envisaged. Equally stressed elements arranged in series give rise to the "weakest link" concepts, where failure of a single element results in a total breakdown of the system. The associated probability of failure is therefore governed by the underlying strength distribution and the number, "r", of elements involved.

An empirical distribution due to Weibull[240] has been used extensively; the resulting formulations predict that, as specimen sizes decrease, mean strengths and the scatter of these should increase, but that the magnitude of such a size effect decreases with an increase in the volumetric concentration of material weaknesses or flaws. In contrast, a parallel arrangement of elements, each carrying the same load while intact, leads to the classical "bundle" concepts which imply that all "n" elements must yield before overall failure occurs, although the maximum load bearing capacity will generally be attained prior to this point. Each of the n parallel elements can, however, be considered as a series combination of r sub-elements controlled by the weakest link[241, 242]. The predicted behaviour of series-parallel systems, including the sign and magnitude of any size effect, depends on the form of strength distribution adopted and the relative values of r and n. When a Weibull distribution is assumed for the sub-elements in series, then, for constant values of r, the mean strength increases rapidly with increasing n, provided n is small, but becomes independent of n when this is large. If n is held constant the mean strength decreases with increasing r. The expected variability of strength, in terms of the standard deviation, increases with increasing n but decreases with increasing r. Both the weakest link and bundle theories have been applied to concrete strength results with varying degrees of success. (Use of the former with regard to the failure of concrete in uniaxial compression must be seen as a somewhat indiscriminate application.)

The progressive failure (degenerative response) characteristics manifest by concrete specimens under simple test conditions are well-suited to "explanation" via statistical models[58, 243-249]. However, it is generally the case that such models embody little associative meaning as to the significance and/or nature of underlying (component) strengths. Thus, the depth of understanding ultimately provided is fairly minimal. The lack of a sustained phenomenological background tends to be especially obvious in the context of uniaxial compression.

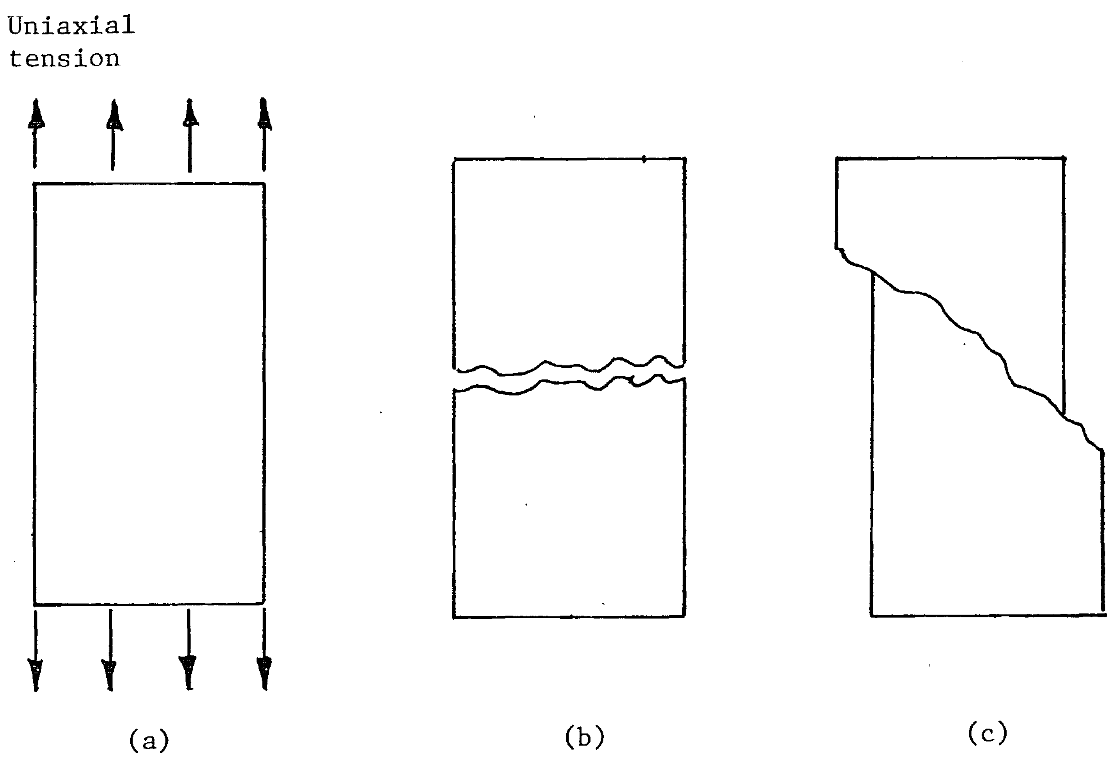
CHAPTER IV

A "NEW" MODEL - SOME OLD IDEAS REVISITED

.... some element of irrationality about materials lingers in us all....
J.E. Gordon(250)

4.1 INTRODUCTION

Conceptual interpretations and would-be predictions of material behaviour, including failure, under different forms of applied loading have evolved with the passage of time in the combined light of accumulated direct experience, intuition, assumptions, and forms of consequential reasoning founded principally in the axiomatic domains of classical mechanics and its associated field theory. Axioms, whether quantitative or qualitative, may generally be viewed as the smallest defined links in various implicit or assumed causal chains; i.e. these represent the basic constitutive elements which combine to provide the intellectual framework for any theory, "explanation", or attempted rationalisation of related phenomena. Since strength parameters, whether at a macroscopic or sub-macroscopic level, are more often than not given an axiomatic connotation, the nature of strength itself is rarely furnished with any element of derived "understanding". However, all theories, including those of failure, must have some defined starting points and it might well be argued that an extremely low conceptual origin is liable to be unwarranted unless an effective link from the most fundamental to the highest level of discrimination can be positively established. Thus, for example, an intimate knowledge of the biology and organic chemistry of herbs and spices is unlikely to improve the chef's culinary abilities or greatly influence his choice of seasonings. In a similar manner, the design engineer is not generally concerned with the physics of elementary particles or with the current view of the atom since these aspects, although relevant in the overall context of material science, would usually appear somewhat redundant to his own particular information requirements. If it were presently



FIGURES 4.1: Brittle Material Subject to Tension

(a) The test

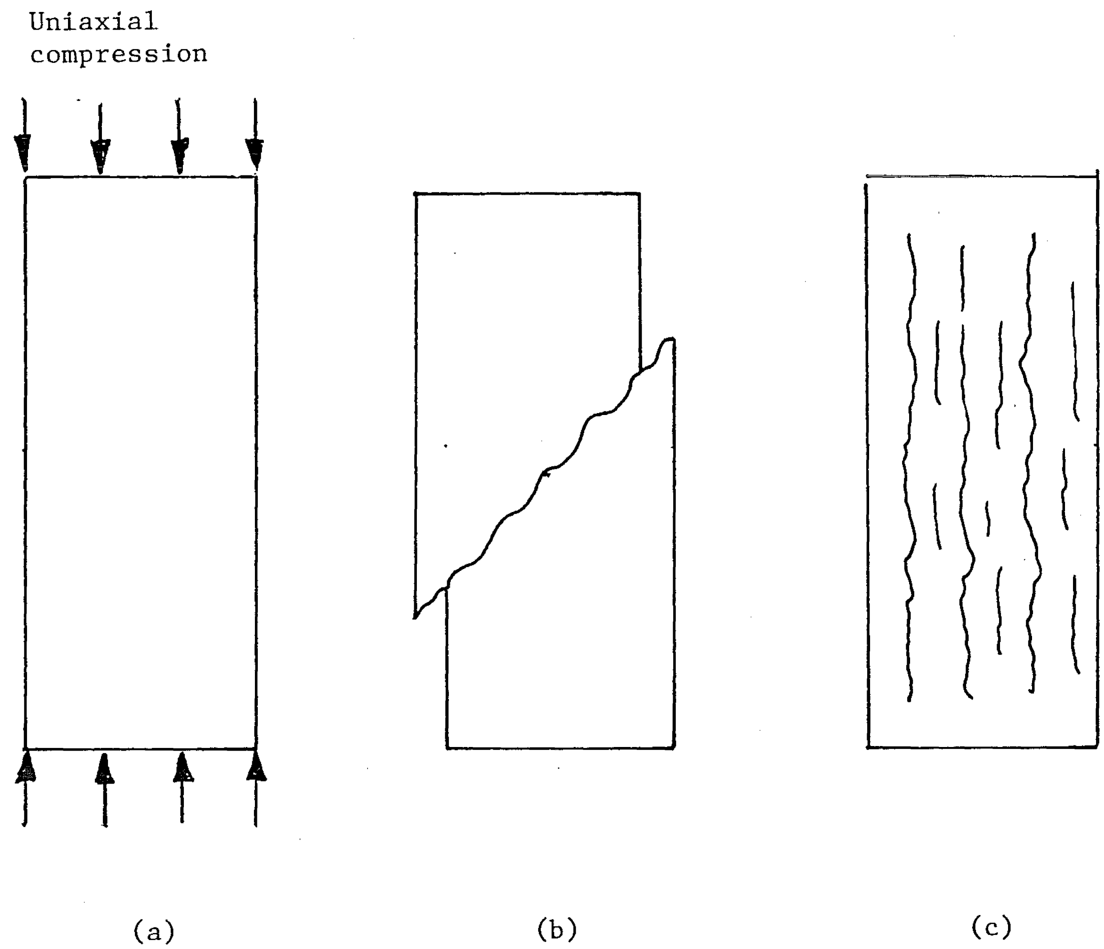
(b) Failure via separation

(c) Failure via slip

possible to effectively predict or even realistically describe the failure characteristics of materials under general forms of loading including (or in terms of) observed behavioural traits derived from simple testing regimes, the practical need for any philosophical reinterpretation of prevailing strength concepts would not exist. Unfortunately this is not the case: an objective review of most available literature reveals that the important topic of generalised material strength is either conveniently by-passed as being beyond the scope of accepted analytical techniques, or presented as a disjointed combination of possible hypotheses interlaced with seemingly unrelated examples of applied empiricism and a myriad of special cases and obvious exceptions covering both particular materials and certain load combinations.

Consider a "brittle" material specimen subjected to a short-term uniaxial tension test as shown in Figure 4.1(a). To avoid any differences between maximum and ultimate load carrying capacities it will be assumed that the tension increases monotonically. The failure mode illustrated in Figure 4.1(b) is consistent with the common, essentially intuitive, inherited notions of the nature of tensile force. In colloquial terms, the material is "pulled apart" when the intensity of the applied loading reaches a certain value defined as the ultimate tensile strength. The raison d'etre of the failure process is, however, unexplained unless recourse is made to conceptual tautology. Of course, even an elementary analysis of internal equilibrium requirements, involving the axiomatic properties of force, reveals the presence of shear stresses and, although all materials will eventually be pulled apart under increasing applied tension, incipient failure by an individual (Figure 4.1(c)) or multiple slip mechanism, rather than by direct separation alone, can easily be envisaged. Thus another aspect of strength in tension must be recognised, which may be simply demonstrated as being equally applicable to the ultimate failure of any brittle material containing planes of weakness or to the plastic yielding of ductile materials. It is worth noting that the terms "strength" and "weakness" are totally complementary in a relative sense, and are therefore fundamentally indistinguishable at a common level of internal discrimination; a realistic description of one which invokes the other necessarily demands an effective differentiation between particular levels.

Despite the possibility of two basic failure modes - direct separation and relative slip - the behaviour of materials in a simple



FIGURES 4.2: Brittle Material Subject to Compression

(a) The test

(b) Failure via slip

(c) Failure via multiple cleavage

tension test does not appear to offer many practical probelms of interpretation. However, when the applied forces are compressive as shown in Figure 4.2(a) conceptual difficulties may well arise if the traditional intuitive approach is adopted. Whilst the yielding of ductile materials and the failure of brittle materials containing planes of weakness occurs by either single (Figure 4.2(b)) or multiple internal shearing actions "as expected", the multiple cleavage patterns (Figure 4.2(c)) associated with many other brittle materials defy the more primitive forms of mechanistic logic. With the benefits of hindsight, the original view of a simple compression test, a product of abstract thought rather than of experience*, now appears somewhat akin to the schoolboy connundrum of the irresistable force and the immovable object: unlike the physical consequences of a simple tension test, in which the remaining fragments after separation undergo no further degradation, a continued application of uniaxial compression was considered to result in a progressive "crushing" of the material and all of its constituents, thus ignoring the various possible structural alterations which can follow on from localised internal breakdown. Fortunately, in the context of uniaxial compression, the term "crushing strength" has almost been eradicated from the field of concrete technology, but misleading references to crushing modes of failure are still remarkably common. It is now generally (but by no means universally) accepted that the multiple cleavage fracture patterns observed in uniaxial compression with most concretes, and with various other nominally brittle materials, are not the induced manifestations of secondary test effects but that apparent shear failures frequently are, although for many years the opposite inference was widely drawn. The physical appearance of a specimen after failure in a multiple cleavage mode suggests some form of lateral tensile mechanism but, from the accepted macroscopic viewpoint, no tensile stresses exist. However, in association with axial contraction, lateral extensional strains do occur: these <u>constitute</u> the Poisson's ratio <u>effect</u>, per se, and are not a mysterious consequence of the same as is often implied. Almost any measured or calculated parameter can be (and has been) designated as a criterion of failure for an individual test regime, the usefulness of any particular criterion being related to its versatility of application in different test situations. The criterion of limiting lateral "tensile" strain or deformation in a uniaxial

---

* Compare Galileo's mistaken appreciation of beam bending[251].

compression test is unobjectionable in the conceptual context of an effect but lacks any real causal significance consistent with related experience. Thus, for example, the "tensile" strain associated with a continued increase in ambient temperature does not generally result in material disintegration. Within the underlying philosophy of Newtonian mechanics it is extremely difficult to conceive of bulk separation in the absence of force. (Whilst the complementary but logically ordered ideas of cause and effect might appear philosophically redundant in some spheres of intellectual endeavour, these are still very much part of the fabric of engineering science, being at the basis of "physical understanding" and of confidence in "rational design" procedures.)

Considering the conceptual inadequacies of the traditional macroscopic approach when applied to the "simple" uniaxial compression of brittle materials suhh as concrete, it is hardly surprising that the information derived from such a loading regime combined with that from a uniaxial or indirect tension test has generally appeared insufficient to provide a satisfactory basis for a theory of failure under more complex states of stress. The apparent success of some modified forms of the classical failure theories has almost invariably been achieved through the use of additional test results and the dextrous juggling of arbitrary functions and empirical formulations. The fitting of actual data virtually eliminates any element of theoretical prediction, especially with regard to failure modes.

For all materials, in either tension or compression, the importance of internal structure[252-255] has been repeatedly established: for example, the recognised existence of dislocations, their possible movement and multiplication, and their interaction with grain boundaries furnishes a rational visualisation of yielding and strain-hardening phenomena in ductile materials. The absence of macroscopic tension in a uniaxial compression test is therefore not necessarily reflected at the lower levels of discrimination where, as was discussed in the previous chapters, local heterogenuity and/or anisotropy may exert considerable influence on the internal stress distribution. Numerous plausible descriptions of the uniaxial "compressive" failure characteristics typical of concrete-like materials have been offered which account for the process of macroscopic disintegration in terms of progressive regional breakdown through the action of various shear mechanisms and/or induced tension effects, but which still present the underlying concepts of intrinsic strength as primary axioms; the physical role played by coarse aggregate particles in the mechanics of concrete failure is often accentuated despite the generally similar failure modes exhibited by mortars and cement pastes. It is important to note that since the terms "homogenuity" and "isotropy" only strictly pertain in limit to idealised continua, assumptions of these properties for real materials or phases and constituents thereof, although often justifiable on the practical grounds of simplicity and negligible error for certain non-critical bulk behavioural considerations, can rarely survive close scrutiny in the context of strength.

In the sections which follow, a strikingly different but yet quite simple model of material behaviour and failure under applied external loading systems will be examined. It will be seen that by treating the phenomenon of composite strength as an elementary consequence of structural interaction and stability this approach is founded on a lower conceptual basis than has been described above, and that paradoxically (in view of the commonly assumed complexity of failure in uniaxial compression) the model requires a critical re-appraisal of certain existing notions relating to "simple" tension.

Many of the concepts to be examined below are basically physical rather than mathematical and are therefore deliberately described in such terms, any abstract axiomatic symbolism being kept to a minimum. This latter point was of special concern to the writer in view of the potentially controversial philosophical implications of the model which touch on areas well removed from those normally associated with concrete technology but which are, nevertheless, of fundamental importance to material science; it was felt that any inherent conceptual divergence should be identified and discussed (the concern with philosophy will become more understandable when the actual implications are made known) rather than remain as an unlisted model "characteristic", obscure within a mathematical disguise. Although the arguments to be advanced, both through generalisation and particular case, will often seem to venture far beyond the limited confines of concrete's mechanical properties - the stated object of this present study - experimental results obtained from that most tested of all "brittle" materials will also be seen to play an extremely critical role.

4.2 THE MODEL - A BEGINNING

4.2.1 Fundamentals

4.2.1.1 Foreword

A symposium of background concepts having relevance to the "new" model is presented below. The majority of those concepts, and of the subsequent developments of the model itself, follow along similar lines to the notional train as first expressed in the paper of Grimer and Hewitt[110] and/or as later enlarged upon by Clayton and Grimer[256]. Accordingly, the writer can make no claims as to conceptual originality in a total sense. However, this work attempts to assess/probe/argue the underlying rationale more deeply; in consequence, differences prevail with regard to aspects of form, emphasis, order and extent of coverage.

4.2.1.2 A Question of Origin

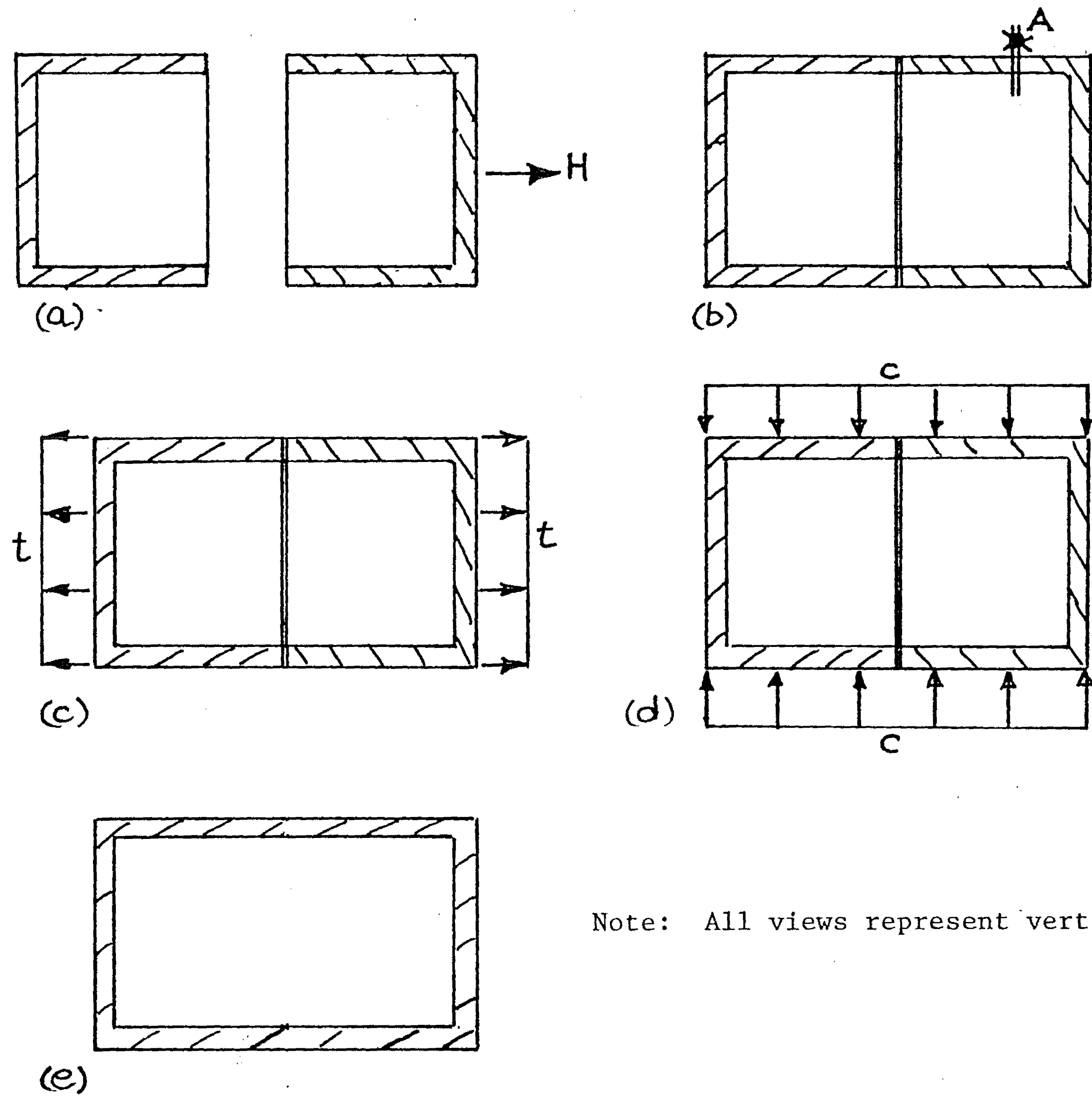
In the "pure" mathematical fields of abstract algebra and co-ordinate geometry, the importance of reference points is not great - "origin shifts" can be accommodated at will. However, in the application of mathematics to physical concepts certain preferential origins of consideration have emerged or have been defined as being more "suitable" than others from the viewpoint of either implied direct relevance, simplicity, or consistent interpretational significance. It is pertinent to note that the Copernican view of the solar system, the cause of perhaps the greatest upheaval in the history of rational scientific thinking, involved a simple shift of spatial origin from the previously accepted Ptolomaic position. The ultimate justification of taking the sun-centre as a relatively "stationary" co-ordinate datum hinges not on reality, as this is strictly indeterminate, but on geometric simplicity and philosophical convenience; the alternative celestial schemes, while mathematically equivalent, demand complex epicyclic planetary orbits apparently inconsistent with the basic axiomatic and consequential laws of earthbound physics. Thus, to maintain consistency, which is of course the principal aim of all rational scientific description, Newton's inverse square law of gravitation - a very simple, effective, and verifiable (Cavendish balance) link between terrestial mechanics and Kepler's first and third* empirical laws - would have to be replaced by

---

* Kepler's second law is guaranteed by any "central" force function.

an extremely complicated interactive force function relating only to the "mutual attraction" of the heavenly bodies. In a similar manner, although the concept of temperature has achieved philosophical standing through the theories of thermodynamics and beyond, both its inherent significance as a phenomenological parameter and the nature of quantitative relationships between the absolute temperature scale (having, for the most part [257], no physically "meaningful" sub-zero values) and other more arbitrary measures, based on specific material transition points, were initially founded on the mathematical simplicity of Gay-Lussac's experimental gas law (commonly, but quite wrongly, attributed to Charles or Amontons).

The "understanding" of interactions within specific systems is necessarily influenced either directly or indirectly by the chosen origins of consideration and by the precise manner in which the systems themselves are conceived. As a result, some degree of origin flexibility is frequently justifiable. For example, in a sociological context, different "zeroes" for measuring time interval are often employed in different situations; an event in an individual person's lifetime can be designated via an external datum, such as by using a calendar date, or in terms of singularly internal frames of reference such as birth and death, the latter having the notable characteristic of reversing the normal numerical significance of the notionally dominant past-future time sequence. Depending on the particular situation involved, certain origins may prove more "suitable" than others in terms of possible interpretational inference leading to consistent "explanation". An origin will necessarily always have some "internal" significance with respect to the system for which it is defined: e.g., 0^0C. Its inherent relevance as regards associative "understanding" of a particular situation will, however, depend on the "suitability" of the system envisaged to describe the circumstances under examination. Unfortunately, in that area of material science dealing with the behaviour of materials under applied load, the systematic physical relevance of reference origins has largely been ignored. Undoubtedly, the distinct penchant among successive generations of mathematically-inclined analysts towards model systems having continuous linear characteristics (and hence no one datum worthy of any particular consideration) has contributed towards the general, but typically unreasoned, acceptance of the nominally "unloaded" stress state as an appropriate single origin of consideration for distributed force. While the practical convenience of the origin is



Note: All views represent vertical section

FIGURES 4.3: The Thin-Walled Box Analogy

(a) No interaction between components

(b) Securing an interaction

(c) Interactive assembly subject to horizontal "tension", t

(d) Interactive assembly subject to vertical compression, c

(e) Single closed box

undeniable, so too is the completely arbitrary nature of the "unloaded" assumption: that the use of the conventional stress datum has led to little consistent understanding of material behaviour should not therefore come as any great surprise.

Consider now the two singly open-ended, thin-walled, elastic box structures shown in vertical section in Figure 4.3(a). Apart from the almost negligible influence of mutual gravitational "attraction" there is nothing to prevent the relative separation of the bodies if a horizontal force, H, is applied specifically for this purpose. Overall structural continuity requires some degree of physical interaction between the boxes. If, for instance, these are brought together and the internal pressure is reduced by removing some of the entrapped air, via a vacuum pump and valve arrangement at A (Figure 4.3(b)), the resulting system will possess horizontal "tensile" strength against separation, provided the perimeter junction acts as an efficient seal*. An identical final situation could be produced by compressing the boxes horizontally with the valve at A open, closing the valve, and then releasing the applied external forces. The basic integrity of such a configuration has obvious similarities with many other prestressed systems which incorporate a seemingly "internal" tension-compression balance, but an examination of the elementary physics of Figure 4.3(b) reveals that, in this case, the balance is not internal and that tension plays no part whatsoever in the between-box interaction; the individual bodies are held in compression and hence in mutual contact by the external/internal pressure difference, each wall of both boxes being subject to an essentially biaxial compressive stress state. Despite the original reduction in internal pressure, it would seem that this must remain compressive by nature since the concept of a near-ideal gas providing (or sustaining) apparent isotropic tension is both practically and philosophically untenable in the light of current views. It is possible to visualise a mathematically-equivalent, purely internal force balance by adopting the external pressure as a "convenient" zero reference datum; however, this type of approach imparts unrealistic physical properties to the contained air and is therefore as intellectually unwarranted as the popular misconception of suction phenomena in terms of internal "pull" rather than of external "push".

---

* The function of this example is to illustrate elementary principles only. Factors such as plate bending will not be encompassed.

If increasing uniaxial horizontal tension, t, is now superimposed on the prevailing stress system (Figure 4.3(c)), the external/internal pressure differential must alter continuously if static equilibrium is to be maintained. The "applied tension" can best be viewed as a lessening of external compression in one direction only, this being quantitatively greater than the associated drop in internal pressure since the horizontal compression, parallel with t, in the box walls must also decrease, resulting in relative axial elongation. When this particular wall compression is finally reduced to zero the composite system is no longer capable of sustaining further "tension"; any subsequent increase in the applied load produces a dynamic condition and the individual box sections will separate. Up to this point, the pressure differential between the ambient external environment and the internal volume has increased steadily and, therefore, progressive lateral contraction or a structural Poisson's ratio effect becomes a logical consequence of the particular system envisaged; failure, the instant of separation, will necessarily be accompanied by a sudden equalisation of external and internal pressure as the external atmosphere gains internal access.

An application of slowly increasing uniaxial vertical compression, c, to the box combination (Figure 4.3(d)) leads, from similar considerations, to increasing internal pressure, axial contraction, lateral expansion, and the eventual separation (finite but not extensive) of the constitutive elements when the mutual physical interaction is effectively eliminated. However, compared to the uniaxial horizontal tension case, there is a very much smaller pressure differential (and of a different sign) between the internal and ambient external environments at the point of failure, the precise value depending on the linear dimensions of the boxes considered and the magnitude of the prevailing external pressure.

The particular structural system outlined above derived its "strength" in both horizontal tension and vertical compression from an air pressure differential. The stipulation of air as a specific medium was, however, strictly unnecessary. Composite integrity would have been guaranteed by any fluid, provided a finite external/internal pressure differential existed; indeed, the external and internal fluids need not have been of a common composition, although it is worth noting that the behavioural traits described were implicitly dependent on the existence of a "real" internal fluid - the structural consequences of an internal "vacuum" would have been quite different. Of course, the boxes could have been joined by much more conventional means and perhaps even constructed as a single assemblage (Figure 4.3(e)). This being the case, at the previous level of discrimination a pressure difference no longer appears operative as a strengthening mechanism and the load-carrying behaviour of the structure would now seem to be governed primarily by that of the constituent material: however, if the material is considered as a structure in its own right then a source of strength conceptually similar to that already examined can be contemplated. It is not in any way suggested that real materials are somehow composed of mysterious sub-microscopic systems of interacting boxes, but rather that basic material properties such as structural integrity (all too often taken for granted) and important phenomena such as the various possible load-induced failure characteristics (recognised but imperfectly understood) can be simply visualised in terms of effective external/internal pressure differentials, these being sensitive to the magnitude and nature of any applied loads. As will probably be apparent, even at this introductory stage, the use of the term "external" with regard to pressure in a generalised material context immediately implies that some conceptual and/or quantitative shift in the reference datum for this physical quantity from its currently accepted position may well be necessary to accommodate such a visualisation.

In the boxes and air pressure example a mathematically equivalent tension-compression system was rejected as physically unrealistic, the existence of an absolute pressure datum for an ideal or near-ideal gas being a fundamental tenet of the kinetic theory for such substances. If, however, the idea of regarding tension as merely a reduction of prevailing compression is to be extended to the more arbitrary structural systems of which, it is suggested, materials are composed, it would seem that the conventional datum level for "total" external pressure* must prove to be quite inadequate for general descriptive purposes since, in many situations, distinct forms of "apparent" tension do prevail. To overcome this deficiency, the additional presence of an "unrecognised" background pressure will be postulated. The relative magnitude of this pressure will be unspecified but will be taken as being sufficiently

* Apart from the theories of gaseous phenomena, the accepted zero stress or pressure level for an "unloaded" material is rarely justified in anything more than the superficial manner typically reserved for "the self-evident".

large to negate any possibility of net tensile action in an absolute sense. As a result, the nominal vacuum condition as typically understood will no longer be capable of sustaining its privileged position as the "obvious" origin from which to measure absolute pressure: indeed, even its potential phenomenological (as distinct from practical) relevance as a secondary or intermediate origin (cf. 0^0C) from which to measure pressure intervals will be scrutinised and found to be only marginal. More "suitable" origins for this latter purpose will be proposed; however, in relation to conventional scales of pressure these will not be fixed points but will be both system- and condition-dependent.

4.2.1.3 Solids, Fluids, and Hierarchical Structuring

It is often deemed convenient to classify substances as either solid or fluid, the discrimination between these terms being primarily a function of different observed behavioural traits under sustained shearing action. Thus, Chambers[258] defines a fluid as *"a substance which flows - it differs from a solid in that it can offer no permanent resistance to change of shape"*. Such a definition implies, therefore, that a solid has this latter capability. However, materials commonly classified as solids invariably show some susceptibility to creep under sustained loading and hence exhibit what might be termed as "quasi-fluid" characteristics. It is therefore apparent that the observation time involved in making an implicit judgement regarding permanent resistance is extremely important. Definitions of a solid generally involve references to the restricted mobility of constituents, these having *"fixed mean positions"*, to *"definite shape"*, and to *"resistance to deforming force"* (the quotations are once more from Chambers[258]). In the application of these concepts, the observation time is again crucial. For example, a high-board diver who misjudges his angle of entry could well classify water as a "quasi-solid" (and have marks to "prove" the resisting force).

The above illustrates that all real substances can combine the attributes of both the defined solid and the defined fluid to varying degrees depending on the particular behavioural situation envisaged, and that therefore a complete and strict distinction, although potentially useful in certain sets of circumstances, is in general philosophically unwarranted; i.e. the mutually exclusive aspect of solid-fluid conceptualisation is merely an abstraction applicable only to defined ideal models. Failure to recognise the obvious absence of a distinct "dividing line" leads inevitably to the proposal of somewhat artificial notional discontinuities; the latter frequently manifest themselves within the suspect pseudo-scientific premise of conceptual dualism in one of its many forms. Thus, for example, physicists today tend to present two "pictures" of a liquid - either that of a condensed gas (in relation to its behaviour in the so-called "critical zone") or that of a disordered solid (in relation to its properties at temperatures and pressures outwith that particular region).

The difficulties associated with any attempted classification of materials in a total sense are brought about by the apparently discrete and positively hierarchical nature of matter itself. Consider, for instance, a sample of a nominal solid such as concrete. At the first level of discrimination, or hierarchy, below its actual bulk presence the material may appear as a distributed solid phase (aggregate) within a solid matrix of hydrated or partly hydrated cement paste. However, further discrimination would reveal both constituent "solids" as inherently porous substances containing fluids in either gaseous, liquid, or vapour forms. Continuing this branching process with regard to the pore "fluids", it may be inferred that each of these will possess a distributed solid component as a direct consequence of their accepted multi-molecular nature.

Before proceeding, it is most pertinent to pose the question as to how far this hierarchical differentiation into quasi-solid and quasi-fluid fractions* can (or should) be taken, since successive reapplications of the process very quickly bring the level of discrimination towards the limits of present knowledge and/or understanding regarding the fundamental aspects of matter. Thus, every molecule could be said to have a quasi-solid nucleus (capable of further hierarchical structuring) and a quasi-fluid electron space, this latter quantity in turn having a quasi-solid component because of the electrons themselves. At and beyond this level a potential problem of interpretation arises as to the physical nature of the complement (quasi-fluid) to any postulated quasi-solid, if current views are to be maintained. An open-ended hierarchical visualisation suggests that <u>any</u> quasi-fluid or quasi-solid is capable of an unlimited degree of sub-component

---

* Hierarchical differentiation does <u>not</u> conform to the strict "either-or" condition of the dualist thesis. This important distinction, although perhaps subtle, is nonetheless real.

differentiation, and in this context neither an "empty" space nor a fundamental solid particle has any relevance unless trivial arguments are introduced. Therefore, it would seem that either a totally open-ended concept has no justification or that presently accepted viewpoints are unsatisfactory. Unfortunately, there are at the moment very few grounds on which the alternative having most merit could be positively established; indeed, both could be satisfied simultaneously by the existence of a fundamental level which is lower than that imagined at this time. However, it is perhaps enlightening to note that the history of science reveals the continued thwarting of man's apparent wish to establish the credibility of ultimate and indivisible particles of matter, brought about by the subsequent discovery of even more elementary quantities.

The problem of open-endedness is not limited to the lower hierarchies of discrimination. Returning to the quasi-solid state of a concrete specimen at the bulk or immediate visual level, there should exist a complementary quasi-fluid at this same level, if the ideas of hierarchical structuring are to have any extended generality; in addition this quasi-fluid should be capable of sub-component differentiation. Under "normal" conditions there would be an obvious fluid in the surrounding air but the specimen could supposedly also exist in a "perfect vacuum" environment, devoid of known fluids. Thus, once again the general concept of quasi-solid/quasi-fluid differentiation and the accepted visualisation of "empty" or "free" space appear to be inconsistent. (From an extended hierarchical point of view, the traditional analogy drawn between the motion of a planetary system and the structure of an atom takes on a much deeper significance than is usually inferred.) The idea of an external but apparently undetectable quasi-fluid may initially seem so absurd and "unrealistic" as to merit little genuine consideration. However, it is well worth reflecting on whether such an interpretation is any more conceptually fanciful than the impressive list of properties, "constants", and characteristics bestowed on the so-called nothingness of "free" space by successive generations of physicists. (The writer would maintain not.) If all materials are, indeed, considered to manifest their existence within a fluid-like medium, and consistency with similar systems is to be maintained, then an effectively external environmental pressure becomes an expected "phenomenon" as a simple matter of course.

4.2.1.4 Questions of Measurement, Theories, and Change

Although the practical details of measurement are well documented, the philosophical aspects of this most positive of all links between science and mathematics has rarely been subject to the same degree of scrutiny as have some of the more "glamorous" fields securely positioned within either. The overt lack of glamour has been put forward[(259)] as both a cause and a consequence of this situation which has led in turn to a partial and somewhat unjustified sense of complacency regarding the implications of measurement, attained through an unconcious acceptance of certain precepts and indirectly "measurable" by the dearth and relative longevity of the recognised major works[(260, 261)] in the area.

Stevens[(262)] has described the process of measurement as *"the assignment of numerals to objects and events according to rule - any rule"*. Apart from the unfortunate rider to the statement which almost seems to invite immediate Realist criticism, such a description does conform readily (if not quite rigorously) with many inherited notions of measurement. The underlined section will thus be used as a basis for that which follows; strict accordance with Stevens' work ceases, however, at this point. It should be noted that, while no explicit mention has been made of the word "quantity", the statement as it stands is in no way concomitant to an unbridled acceptance (as suitable) of a purely Operationalist philosophy.

Completely general statements regarding measurement are difficult to make; measures have evolved and been developed in a variety of ways and for diverse purposes. This and similar aspects of limited generality do not compromise the validity of the phrase *"assignment .... according to rule"* but go some way to justifying its relative imprecision, which is such that it can accommodate any relevant details of procedural methodology, scales and the like. Indeed, Stevens' rider may well have been prompted by a wish to emphasise the width of possible interpretation rather than to highlight potentially trivial applications of the definition. The reference to *"objects and events"* is, to a certain extent, indicative of a view that many of the distinctions typically drawn between different types of measurement are often of a more arbitrary than "fundamental" nature. This view, which will be justified at various points in the subsequent text, is contrary to that sizeable body of "scientific" opinion which strives to maintain the maximum possible levels of superficial differentiation; Brillouin[(263)], for example, has criticised the common phenomenological

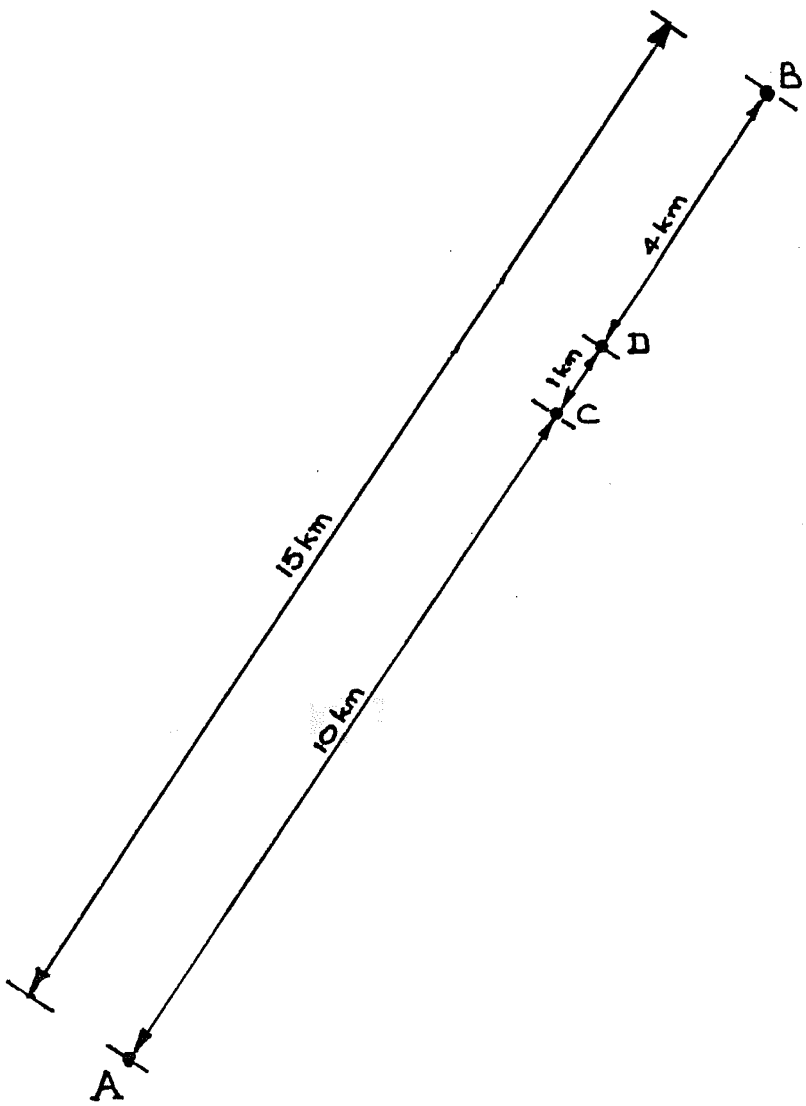
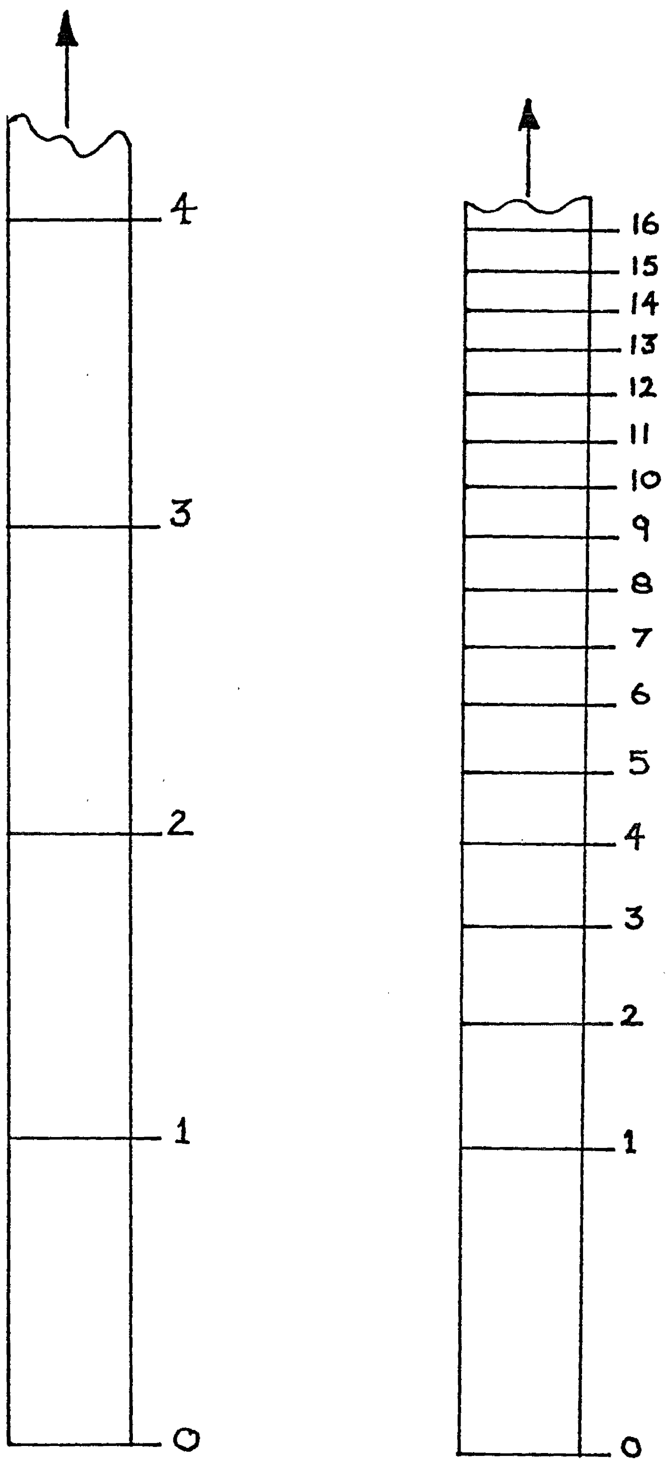


FIGURE 4.4: Travel from A to B via C and D

basis of the defined reference standards for time interval (spectral frequency) and length (wavelength) on the grounds that this does not afford an adequate degree of fundamental distinction.

Having briefly analysed the minor word structure of the measurement description, its most essential feature - "numerals" - can now be examined. The end result of measurement is a number. Although the actual value will obviously not be independent of the detailed contents of the "rule", its basic numeric character will remain unimpaired regardless of the actual prescriptions laid down. In order to reconcile this view with some typical misconceptions regarding measurement, and also to introduce some related ideas, consider the case of a traveller moving from points A to B via C and D as shown in Figure 4.4. Firstly it will be noted that the distance from A to B is stipulated as 15km - presumably the result of some length measuring process: this may seem initially to be something more than just the straightforward number which was forecast by the above, but such is not the case. The "km" postscript is a consequence of history - not logic. The actual measurement is strictly relative - the implicit ratio thereof, like all ratios, is a number. To attach any particular significance to the reference denominator of the ratio would be equivalent to stating that the number 5 should be treated specially as 30 sixths, or $1\frac{1}{4}$ two-squareds. Had one particular reference for length been chosen and accepted by all the need for any quotation of length "units" would have become obsolete; i.e. the customary unit would have been "understood" in the same way that human age is typically given as a number only (cf. relative density, etc.). Viewed rationally, the often over-stressed importance placed on all customary physical units can be seen in a similar light: i.e. a multiplicity of separately defined, but conceptually equivalent, systems has led to the practical necessity of assigning units of measurement to specify the system-dependent reference quantity involved in the denominator of the relative measure. The kilometre reference is basically external to the distance AB. The number produced by this "standard" only has meaning to the traveller in an associative sense in that he can compare it with other "lengths" which he has experienced. Thus, he uses the "km" as a common denominator with which to quantify a concept. It is equally possible, however, to quantify distances through internal references, peculiar to the system described. Numerous relative measures (R.M.) of the distance AC can therefore be listed:



Ruler X

Ruler Y $(Y = X^2)$

FIGURES 4.5: Two Rulers - X and Y

e.g. (i) R.M. = 10 .... external kilometre reference
(ii) R.M. = $^{2}/_{3}$ .... internal AB reference
(iii) R.M. = 2 .... internal CB reference
(iv) R.M. = 1 .... internal AC reference, etc.

Measures (ii), (iii) and (iv) may or may not be more relevant to the traveller than (i), although measures (ii) and (iii) do contain more information. Indeed, despite the nominally "internal" classification (ii) and (iii) are coloured by the same Platonic idealism which influences (i) - namely that a relative measure should somehow be proportional to the "quantity" under observation. The values given for (ii) and (iii) could only be obtained in practice by the use of an "external" fixed interval scale similar to that employed for (i); any other scale type used indirectly would, however, render (ii) and (iii) trivial if the implicit ratios involved were evaluated arithmetically or "simplified" in any way - the results would be relative magnitude numbers only, without any strict meaning in terms of relative length measure. Thus, for example, comparative length ratios such as $^{20}/_{10}$ $^{4}/_{2}$, and $^{16}/_{8}$, where each numerator and denominator was valued accordingly to the same non-linear scale, would not generally share the same physical significance, and the appreciation of difference would automatically be forfeited through expressing each as simply the number, 2.

It is not intended to venture too deeply into the philosophical quicksand which awaits any discussion as to the validity or otherwise of quantitative existence, but in view of the last statements above some comment is necessary. The two "rulers" shown in Figure 4.5 could each be used to "measure" a relative length; however, only one (scale X) would generally be deemed suitable, even although there is no *prima facie* principle on which to base such a decision - cf. a choice between two number systems of a different base. (Simplistic arguments are often advanced that customary length scales are "natural" in that length can be counted in equal units in a similar way to counting numbers of individual apples or the like; of course, the units of scale Y, in common with most apples, are not of uniform size but are none the less countable.) It is an important feature of all relative measure that, having established an "origin", any other primitive concept behind this is effectively "cancelled out" through the act of comparison[(264)]. The "object length" figures shown on the two rulers, for example, give the number of direct unit comparisons (a sum) counted from one end - the zero origin common to all length scales - to the other; sub-unit comparisons are also technically possible through scale expansion (e.g. 19.2 = 192 sub-unit comparisons, etc.). Thus although it is tempting to assert that, despite the procedural identity involved in using the two scales, ruler Y does not measure length, there is nothing about ruler X to indicate what exactly length is (or isn't?). Any particular "understanding" of length requires axiomatic definition. It is subsequent familiarity with the system "properties" so granted which then acts to establish scale preference. The conventional axiom that the relative length of a multi-part object is the sum of the relative lengths of its individual parts* (provided that certain procedural details are complied with) is enough to produce the whole class of similar fixed-interval length scales (metres, feet, yards, etc.) to which engineering science is accustomed. The theory, or system, which gives customary relative length both its functional credibility and its apparent significance is, of course, Euclidean geometry. The inverse of this statement, although equally valid, is rarely contemplated. However, the numerical "truth" of propositions such as Pythogaras' theorem, as generally stated, rests totally on an implicit assumption of (if not a metaphysical belief in) a fixed interval length reference. (The use of scale Y would lead to the hypotenuse length of a right-angled triangle being equated with the sum of the lengths of the other two sides.) Considering that Euclidean geometry has been for countless generations the basis of all "scientific" education, it is hardly surprising that the idea of an absolute unit length, which is not a necessary part of any nominal length measuring process, should seem "obvious" in the minds of so many. But, just as it is possible (despite Kant[265]) to conceive of other geometries, it is also possible (scale Y is an example) to conceive of non-trivial reference quantities for relative length other than of the fixed interval variety, while still retaining the primitive concept of length.

For any point between A and B, (Figure 4.4), the relative "distance from A" measures, corresponding to measures (i) - (iv) for the "distance" AC, could be ascertained. (It will be assumed, as it was initially but without reference, that a fixed interval scale is used directly for (i) and indirectly for (ii) and (iii); measure (iv) is,

---

* This is an additive property obeyed by numbers of apples, but not necessarily by numbers of molecules: i.e. apples do not interact!

of course, quite independent of any indirect scale.) Providing B remains "fixed", measures (i) and (ii) will always be in the same proportion to each other - a result employed through definition to classify similar scales (both implicitly of the fixed interval variety in this case). Measure (iii), on the other hand, is of the fixed interval type for any one distance (e.g. AC), but the actual interval used depends on which distance is to be measured*: it should be mentioned that this is quite different again from the "rule" associated with scale Y above - a sum of variable interval direct comparisons. Thus, although each measure (i) - (iv) is structured in the conceptual and procedural context of a length, complete equivalence in the fullest sense does not exist. The distinctions which have been drawn are not, however, those which might commonly be made in the hazy connotational light of the term "dimension"** - namely, that (i) could be given the dimension of length while (ii)-(iv) should each be regarded as "dimensionless". The nonsensical aspects of this latter attempted differentiation deserve no further comment, bearing in mind what has already been stated. Ellis(258) considers a dimension as simply a name for a class of similar scales. Thus, with that in view, he states: *"The same quantity may be measured on scales of many different dimensions"*. The important word is *"may"*. The fact that most quantities are treated as having iso-dimensional scales need not, therefore, act as a barrier to alternative developments. Accordingly, measures (i) and (ii) can be associated with one dimension of length, (iii) with another, and (iv) with yet another. Any number of dimensions are obviously possible: it is also worth noting that the dimension associated with (iv) is a class of one. The quoted statement, taken strictly at face value, is naturally quite incompatible with any reference to the (unqualified) dimension of say, length, and it might therefore be seen initially as if this somewhat general approach would automatically forfeit the benefits, such as dimensional analysis, afforded by a much narrower interpretation. However, this is not the case: in effect the general approach highlights both the potential scope and the inherent limitations of Bridgman's theory(260), which is based on particular (customary) dimensions of specific quantities and

* "Unit" variability with time, rather than with distance, has long been recognised in the field of economics.

** A statement by Ellis(258) is most relevant: *"It is difficult to say how dimensions are usually regarded. For no one seems to have any clear conception."*.

the interactive algebra relating to these. Despite frequent claims as to the global incontrovertibility of dimensional analysis, it undoubtedly operates in the field of strictly local (defined) truths. Also, although it deals for the most part with prescribed conceptual reference quantities and the theoretical (axiomatic) links between them, it is not entirely devoid of empirical content[258]; in Bridgman's own words, it is *"an analysis of an analysis"* and can therefore only offer aid in consequential reasoning where a logical foundation has been established a priori. From his contentions regarding the *"absolute significance of relative magnitude"**, it is obvious that Bridgman did not seriously consider the possibility of more than one dimension (either simple or complex) as having relevance to a particular physical concept or quantity. It is precisely this restrictive view (cf. Gauss's erroneous assertion regarding the universal scope of the "normal" distribution) and its subsequent implicit acceptance by others as a truism which has produced the term "dimensionless" complete with all its spurious special connotations. However, the lack of an obvious external and absolute standard does not require an interpretation that no reference exists: parameters such as strain and angle interval have, by definition of their measure, patently "internal" references. According to the more general approach, if a named quantity can be measured, then some form of scale must be present, and therefore a dimensional statement of some form may be made. Indeed, it can be shown[258] that the potential scope of dimensional analysis itself is actually capable of some extension by treating nominally unitless quantities such as angle interval as having a distinct dimension.

Now, consider that the traveller, having initially moved from A to C, moves on to point D; i.e. in the "interval" CD he has changed position with respect to the reference system (but not, incidentally, with respect to himself - a quite separate system). Before any relative measurement pertinent to this "shift" can be contemplated, one important question must be broached - namely, the precise manner in which change

---

* The potential distinction between relative magnitude and relative measure in the context of indirect quantitative comparison has already been drawn. Unlike indirect relative magnitude values, indirect relative measures are firmly bound within the framework of fixed interval scales. From a generalised standpoint on measurement, which encompasses the possibility of non-linear scales, indirect relative magnitude values need have no significance whatsoever!

is to be conceived. In this case two interpretations seem open; the change of position can be viewed either as a straightforward "length difference" or as a discrete "length" in its own right. The two are only synonomous if a fixed interval scale for length is adopted throughout; the nature and dominance of conventional scales explains why the question of a possible choice is rarely considered. Since the difference approach (like many relative magnitudes) can give numbers with no associative meaning (for instance, a quoted length difference of 4 units on scale Y above would have little significance*), the latter proposition will be adopted here; i.e. a "change of length" will be thought of as a "length of change".

An illustrative example of the way in which the previous reflections on relative length measures can be carried over into the realm of length change is afforded by the materials-oriented parameter of true-strain (after Hencky[175]). The elemental increment of true strain is defined as:

$$d\varepsilon = \frac{d\ell}{\ell}$$

Taking the customary view and interpreting ℓ as the "instantaneous" length and $d\ell$ as the incremental change in length (both referenced to an implicit absolute), $d\varepsilon$ is typically described as a "fractional" length change: indeed its "important dimensionless character" is often emphasised! However, since both the numerator and the denominator of the given ratio conform quite naturally to the concept of length - cf. measure (iii) above - $d\varepsilon$ can equally well be taken as a simple internal relative measure (indirect comparison) of elemental "change-length" per se (without any special fractional connotation) by recognising the logical possibility of reference variability - cf. a "unit" of length on scale Y above. Of course, had a number of elemental increments of strain been defined (e.g. $d\ell/2\ell$ $d\ell/0.54\ell$, etc.) the structural equivalence with the various similar customary length measures would have been more obvious. It should be noted that, although the customary

---

* If a non-linear scale length difference is not evaluated arithmetically, but is instead left in its algebraic (common origin) difference form, then it will have physically meaningful significance: e.g. (7-3). In the same context, it is worth emphasising that a difference such as (7-3) will not generally be equivalent to a difference such as (12-8).

relative length, ℓ, may appear variable to an external observer with a fixed reference, the relative length of this same "quantity" according to the latter internal scale - based on ℓ itself - is in effect fixed at unity (cf. measure (iv) above). Since the external observer would classify the reference producing this result as variable it may be seen that interpretations of fixity or otherwise are purely dependent on observational position; i.e. the term "fixed" should strictly always be prefixed by the qualifying adverb "relatively". The potential paradox that a fixed length and a finite strain should appear compatible only arises if this latter aspect is not fully appreciated (cf. the statement regarding the traveller not changing position with respect to himself). Adopting $d\varepsilon$ as an incremental "length of change" implies that the true strain, ε, is itself a relative length of sorts; the relationship between ε and any particular customary relative length, ℓ_c, can be obtained by integration (the act of summing - i.e. counting - the "sub-units"), the necessary constant thereof, k, having a value in accordance with the initial boundary conditions stipulated:

viz, $$\varepsilon = \log_e k.\ell_c$$

The ε scale is somewhat analogous to the Y length scale above in that it is composed of nominally unequal "units". As a result, the latter could not be used as the basis of another ε scale since it would be impossible to accommodate the implicit indirect comparison required at the elemental $d\varepsilon$ level. In addition, fixed interval scales are fundamental to the functional implementation of integral calculus. However, this fact does not invalidate Y as a possible (as distinct from a necessarily suitable) length scale. Indeed, a knowledge of the "conversion", f, between any length, ℓ_y, on the Y scale and the corresponding customary length, ℓ_c, would, in turn, establish a link between the Y ("length") and the ε ("change-length") scales,

i.e. $$\varepsilon = \log_e k.f(\ell_y)$$

Among the infinite variety of measures available to quantify the change CD (including the direct application of scales such as Y above), it is possible, although strictly out of context, to employ the true strain parameter ε. The value so determined will not, however, be unique unless a particular origin of consideration is stipulated; for example, taking this to be at A or B yields respectively,

$$\varepsilon_{CD} = \log_e 1.1$$

or
$$\varepsilon_{CD} = \log_e 0.8$$

The latter number has a negative prefix, as would be expected of any other consistent measure of length change CD as viewed from B. There is no suggestion of a negative "quantity". The sign of the change merely gives information on the direction - approach or departure - in relation to the specific origin chosen. Thus a "length of change" is a discrete length qualified by a sign indicative of an underlying reference position.

All physical systems may be characterised, through measurement, by a set of numbers. (The application of this principle - without any direct mention of measurement - to physical and mathematical models is of course the basis of the various field theories.) The procedural sources of the numbers, and the accepted minimum size of the set required for adequate specification, will generally be influenced by theories, concepts, and related experience. Language is not unimportant but, while unit and quantity names are useful carriers of information, the common comparative basis of all physical measurement should not be overlooked. Thus, for example, it is possible in practice (and in thought-experiments) to obtain customary measures of all "fundamental" and "derived" quantities such as mass, temperature, volume, pressure, and time interval, by methods within specifiable systems which each involve a single customary relative length determination*. This does not mean that the terms are equivalent to "length" as typically understood. The names serve as a logical alternative to the confusion which would obviously follow from a strictly procedural classification such as Type II length, etc. It is only through a combination of theories and concepts that any measured <u>number</u> can be associated with a particular named quantity. (While theories can give "meaning" to measurement, the concepts of a scientific theory can <u>not</u> be defined solely in terms of experimental operations(266); the assertion that they can is, of course, the fundamental flaw in the Operationalist thesis). With respect to quantitative differentiation, customary dimensional considerations can be less than helpful; e.g. strength and elastic modulus are <u>not</u>

* The reverse process whereby a time interval is measured in order to determine a length is very much a part of modern surveying practice.

equivalent measures, despite their sharing a common "unit". The confusion which can result from a mistaken appreciation of dimensions (see earlier comments) may be witnessed from those discussions in the technical press which followed the introduction of the S.I. system to countries such as the U.K., Australia, and New Zealand[266]. The "dimensional significance" of work and moment units suddenly became a favourite topic for question; apparently the degree of unit distinction offered by the engineering version of the Imperial System (ft. lbs., and lb. ft., or similar) had been quite adequate to dispel any previous doubts! Again it is worth stressing that all numerical "laws" - axiomatic, consequential, or empirical - are necessarily system-dependent by way of definition: the apparent simplicity or complexity of functional inter-relationships between different conceptual quantities, and, indeed, of the quantities themselves, is influenced to a great extent by the observational standpoint or origin chosen. Thus, for example, it is frequently the case in routine structural analysis that the modelled behaviour of an assemblage seems to take on a much more complex form than that of any individual member or component material; this is because the usual choice of "appropriate" descriptive parameters (an arbitrary decision) tends to impose a degree of simplicity on the latter from the outset. Unfortunately, the crucial feature of inherent definition which commonly underlies scientific thinking is often overlooked: there is a general lack of formal recognition as to the extreme fineness of the would-be distinction between analysis and synthesis, especially when the former involves constitutive premises. Unlike the "contempt" forecast by the old adage, "familiarity" in science appears to breed a degree of undeserved and alas largely unquestioned respect.

"Basic" length is perhaps the most primitive of all concepts - reflected in the English language by phrases such as "the length of time" - and mainly for this reason it has been concentrated upon here. Its roots are phenomenological and mingled inexorably with ideas of objects and their size (relative existence). Negative length (like negative volume and negative pressure) has no physical meaning; i.e. an object may exhibit varying degrees of existence but only one degree of non-existence. Similarly, since any phenomenological description obviously presupposes that there exists a phenomenon to describe, this must primarily involve concepts which are positive in relation to an underlying phenomenological origin. In the travel example, distance was the primary concept (no distance, no travel) and as such was positive whether measured from A or from B. The secondary concept,

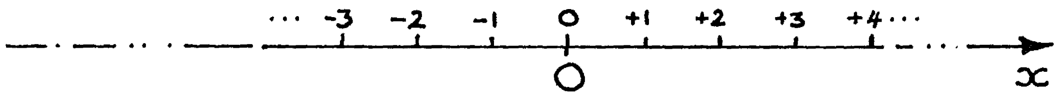


FIGURE 4.6: Typical Coordinate Axis

positional change, was derivative in that it contained information regarding the parent system origin deemed operable. Although the distance and the positional change were each measured in terms of the "fundamental" length concept, strict scale correspondence was not demanded. The origin implications of a primary concept are only absolute in terms of the system in which this is envisaged or to which this is applied. The nature of other systems may require that the same concept be viewed alternatively in a secondary (change) context. For instance, in the travel example, the "distance" AB could have been taken as a change (and hence signed) with respect to another reference system with a separate origin; similarly, the "distance" AC is a change in terms of a distance origin at B; conversely the "change" CD is capable of interpretation as a primary distance with respect to an origin at C: the obvious parallel between these ideas and those of hierarchical structuring alluded to previously is worthy of note. In general, therefore, it may be stated that the existence of any measured quantity within a system to which either a positive or negative sign must be prefixed to give "meaning" is indicative of an underlying primary origin. Several implications of this view will now be examined briefly.

Consider firstly the specification of position with respect to a typical coordinate axis as shown in Figure 4.6 (the system is open-ended, unlike that of the travel example). The origin of coordinates, O, is not a primary origin as described above; i.e. a quoted x coordinate may be treated essentially as a "change-length" measured from O, the primary origin in this case being implicitly at $x = -\infty$. Thus, in turn, a coordinate change can be taken in effect as a "change of a change" with reference to the primary origin: the directional information carried by the sign of any derivative change retains an identical significance regardless of the degree to which the parent (primary) system is subdivided. (While the alternative use of a polar coordinate system would quite definitely locate the primary distance origin at O, the interpretive position with regard to the primary angle origin would inherently depend on the precise manner in which "angle" was defined.)

The customary treatment of "balanced" uniform stress - tension/compression - is very similar to that of position on a coordinate axis: the same may indeed be said of "balanced" force itself, but the distributed aspect of "real" forces renders stress a more appropriate parameter for immediate concern. Although the conventions vary, different signs must be employed to give stress an associative meaning with reference to deformable bodies. Therefore, unless the dubious and unhelpful proposition that tension and compression are quite separate "positive" quantities* is accepted, stress must be taken as a change concept of some form; in such a context its primary origin can not be the conventional zero. There is, of course, nothing in the axioms of mechanics which demands that a body in equilibrium in its "free" state is not acted upon by balanced forces of a finite magnitude : even in dynamics absolute force has no particular significance since the nominal "source of action" is effectively net force. As the sign of stress is ostensibly derived from material behaviour it is evident that the primary origin relevant to this "measurable" change can only be ascertained through a conceptual understanding of material systems. Despite the obvious connection between stress and length change there is no a priori requirement that the primary origin for stress and that typically adopted for "original length" should be directly associable. By way of contrast, in the defined system of simple rigid body dynamics the sign of a net force is related to a primary positional origin.

Finally, the important physical parameter, electric charge, is worthy of a little investigation. Unlike the consistently "attractive" nature of gravitational (positive mass) effects, the additional forces of mutual interaction between two charged bodies may indicate the presence of either attractive or repulsive potential. As a result, quantitative charge measures must be given an appropriate algebraic sign to make due allowance for this in any consistent application of Coulomb's Law relating to electrostatic force (like signs repel, unlike signs attract). It is worth noting that the ultimate sign of the interactive force so produced - a simple matter of convention - is, of course, concerned solely with its relative spatial direction; i.e. the sign of an electrostatic force is quite independent of the actual algebraic notation (+ or -) adopted for the different senses of quantitative charge. Momentary reflection should be sufficient to also confirm that the sign of an individual charged particle is not affected in any way by either its relative or "absolute" position. Exactly what the sign of

---

* The distinction through name is an indication that this view has been held intuitively in the past. The extent to which it has been carried through to the present day is uncertain, but its influence is undoubtedly still felt; cf. heat and cold.

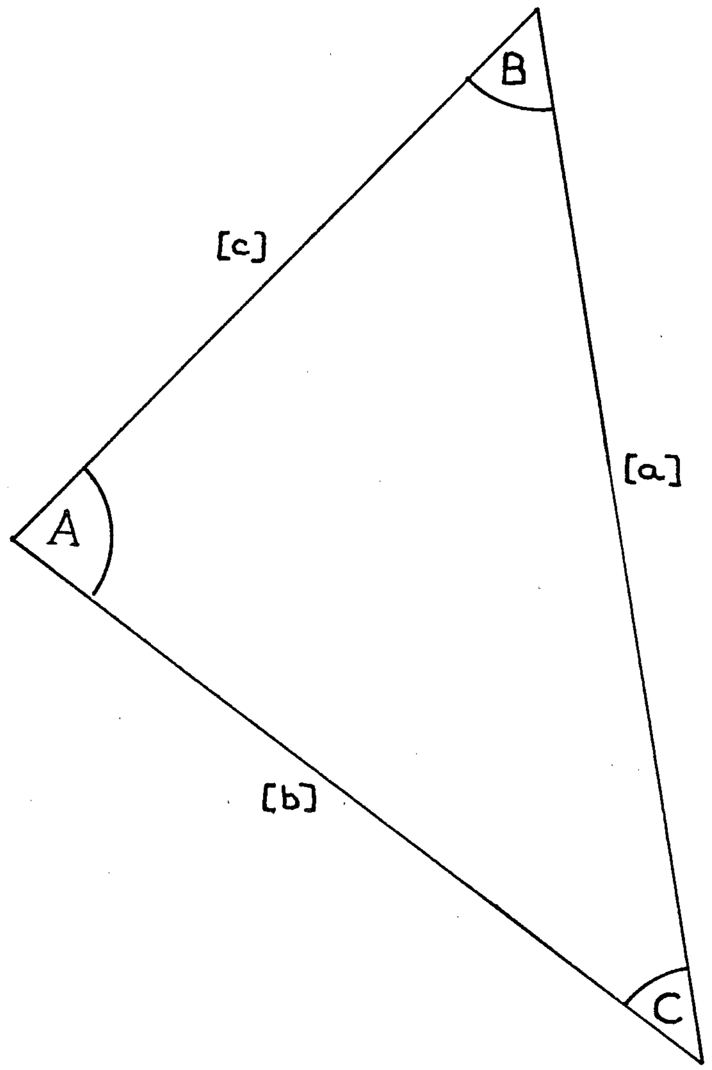


FIGURE 4.7: Triangular System

charge <u>does</u> refer to is far from clear in the "normal" view. Thus, in summary, the conventional origin of consideration for quantitative charge measurement, the so-called uncharged particle, is unquestionably of the secondary type, since it forms the basis of a system which yields both positive and negative values: the status of the relevant underlying primary origin is, however, rather vague as the "normal" view of charge is largely devoid of the associative physical "meaning" which typically characterises the primitive understanding of other quantitative properties such as length, mass and time. An alternative conceptual model which includes a meaningful primary charge origin awaits subsequent examination later in the text.

Mach[268] argued - and many have since reiterated his assertion - that care must always be taken with regard to the drawing of inferences from the properties of scale number systems: having given examples of perfectly viable non-standard length scales, the writer would certainly not seek to dispute this. Nevertheless, the idea of a primary origin is fundamental to the conception of system applicability and is therefore independent of any scale used. Thus, as an illustration, while a new temperature measure based on the logarithm of the present absolute scale would certainly reinforce the idea of an unobtainable lower limit for this "quantity" ($-\infty$ according to the new measure) it would not interfere with the basic concepts of thermodynamics, only their numerical implementation.

4.2.1.5 Interactive Change

The nature of some typical system interactions will now be examined, and the following notation will be used; a symbol within squared brackets will indicate a "named" quantity, while the same symbol without squared brackets will represent a measured value of this quantity according to some prescribed "rule".

Example 1. Consider the triangle shown in Figure 4.7. Using customary length measures for [a], [b], and [c], the cosine of the angle A can be described in terms of the triangle sides by the appropriate algebraic formula.

viz.,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

If it were wished to establish a relationship linking a and cosA only, the quantities [b] and [c] would have to be "absorbed" into the concept of [a] within the system. This is immediately possible since b and c control the maximum ($a_{max.}$) and minimum ($a_{min.}$) values* of a which are consistent with a triangular form,

$$\text{i.e.} \quad a_{max.} = b + c \quad , \text{ and } \quad a_{min.} = |b - c|$$

Unlike a ($a_{min.} \leqslant a \leqslant a_{max.}$) and A ($0 \leqslant A \leqslant A_{max.} = \pi$), cos A can be signed either positive or negative. The measure of the quantity [1 - cosA] is, however, always signed positive. It can be easily shown that, through rearrangement, the "cosine rule" is capable of alternative expression in the form,

$$\frac{1 - \cos A}{2} = \frac{a^2 - a^2_{min.}}{a^2_{max.} - a^2_{min.}}$$

, or

$$\frac{1 - \cos A}{(1 - \cos A)_{max.}} = \frac{a^2 - a^2_{min.}}{(a^2 - a^2_{min.})_{max.}} \qquad \ldots\ldots 4.1$$

Note: $(\cos A)_{max.} = \cos(A_{min.}) = 1$

Thus, it may be seen that, in this system, the customary measures of the positive quantities, [1 - cosA] and [$a^2 - a^2_{min.}$], interact in a very simple way and that the quantities themselves share a common "physical" origin. (One particularly notable feature in the light of previous comments is that the quantity [$a^2 - a^2_{min.}$] only has connotations of area difference in the context of customary length measurement: the values a^2 and $a^2_{min.}$ could equally well be envisaged as particular measures of the "length" quantity [a] according to the Y scale shown earlier, although the difference "quantity", if evaluated arithmetically, would not itself be a consistent "length" measure in such a case.)
Equation 4.1 can be expressed in a differential form either as,

$$d(1 - \cos A) = k.\, d(a^2 - a^2_{min.}) \qquad \ldots\ldots 4.2$$

---

* Similar subscripts will henceforth be used to denote maximum and minimum values of other quantities also.

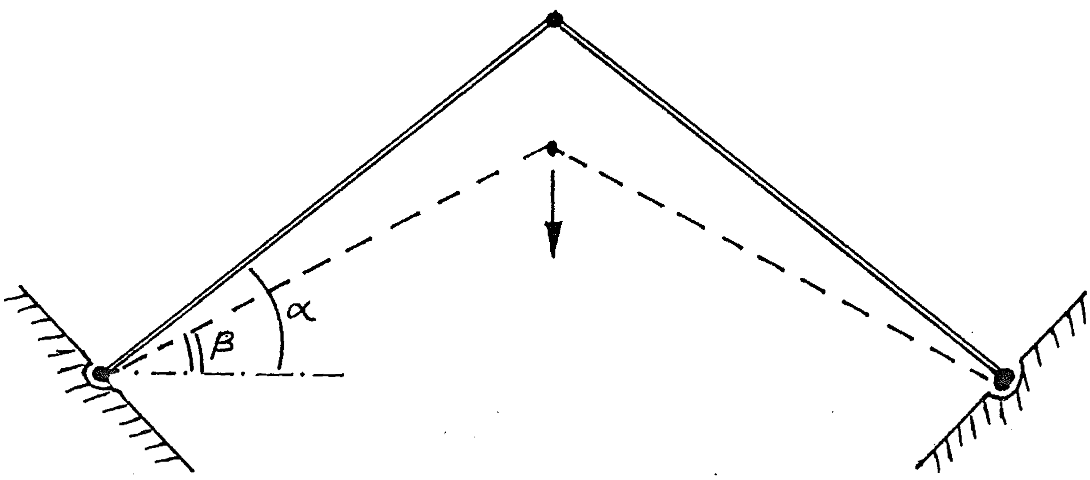


FIGURE 4.8: Snap-Through Example

, where $k = (1 - \cos A)_{max.} / (a^2 - a^2_{min.})_{max.} = 1/2bc$

, or as
$$\frac{d(1 - \cos A)}{(1 - \cos A)} = \frac{d(a^2 - a^2_{min.})}{(a^2 - a^2_{min.})} \qquad \ldots\ldots 4.2(a)$$

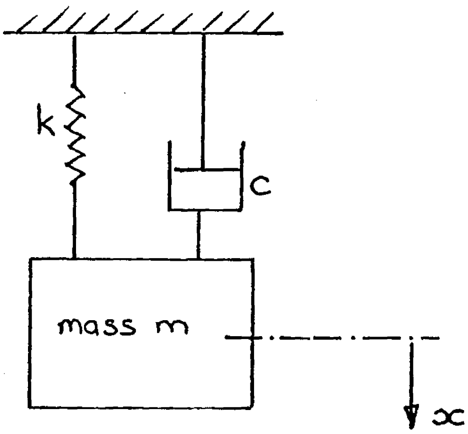
The first, 4.2, is similar to the defined basis of many modelled physical system parameters. It indicates the relationship between incremental changes in prescribed quantitative measurements. In this case the quantities, and hence the increments, are valued externally with respect to the triangle system considered. The alternative, 4.2(a), simply equates specific internal measures of incremental change.

Bearing in mind its extensive use for the purposes of empirical correlation, the arbitrary linear equation, $y = mx + c$, merits some brief comment in passing. The mere determination of the appropriate constants, m and c, from a set of "raw" x - y data, does not, of course, in itself generally provide any significant "understanding" of the actual system from which the "interdependent" quantitative measures of [x] and [y] so utilised are originally derived. Thus, although the equation may be expressed in a simple differential form like that of 4.2 ($dy = mdx$), any reference to internal measures of incremental change is quite meaningless unless a definite system origin and hence relevant positive quantities can be (or have been) established. If this is not done the equivalent to 4.2(a) is strictly devoid of any implicit systematic information, and an infinite variety of alternative expressions, consistent with an indeterminate origin of consideration, is therefore possible: i.e. since $y = mx + c$ is equivalent to $y + a = m(x + \frac{c}{m} + \frac{a}{m})$, where a is any constant, positive or negative, the appropriate differential form thereof,

viz,
$$\frac{d(y + a)}{(y + a)} = \frac{d(x + b)}{(x + b)}, \text{ where } b = (a+c)/m$$

, is not unique.

Example 2. A simple model structure consisting of ideal linearly elastic rods and ideal hinges is shown in Figure 4.8. It will be assumed that "failure" occurs by way of a snap-through phenomenon under an applied vertical load (possible member buckling and/or material breakdown is disregarded). The full lines represent the unloaded configuration and the dotted lines that at the onset of instability. An



$$\omega = \sqrt{\frac{k}{m}} \qquad \lambda = \frac{c}{\sqrt{2m\omega}}$$

whence $$B = \frac{V_0 + \lambda\omega x_0}{\omega\sqrt{1-\lambda^2}}$$

FIGURE 4.9: Linear Underdamped Mass/Spring System

Note: (i) x is measured from position of static equilibrium

(ii) at time $t = 0$, $x = x_o$ and $v = dx/dt = v_o$

(iii) k = spring stiffness, c = damping coefficient analysis of the structure (geometrically non-linear) would reveal that $\cos\alpha$ and $\cos\beta$ (both positive quantities and of a common origin as a result of system specification) are inter-related in a very definite manner:

viz, $$\cos\alpha = (\cos\beta)^3$$

In this case, the appropriate "internal" measures of corresponding incremental change are not equal, as before, but are linked by a constant.

i.e. $$\frac{d(\cos\alpha)}{(\cos\alpha)} = 3.\frac{d(\cos\beta)}{(\cos\beta)}$$

Of course, the previous equality (4.2(a)) can be viewed as a special case of the same form with a constant of unity.

In both the triangle and snap-through examples, the "linear" relationships between different specific internal measures of incremental change were established consequentially through the use of more fundamental propositions belonging to more general systems: these "external" influences had, in turn, their own defined (internal) change interactions and implicit origins. Many other examples could be cited to show the frequent recurrence of "similar scales" of related internal incremental change within "secondary" systems; it should be noted, however, that the appropriate system reference which effectively renders an incremental change "internal" need not always be the equivalent externally valued parent quantity, as was the case in both examples cited above. This latter point will be illustrated in some detail later in the text. Indeed, in certain circumstances, particular incremental changes may not require any special systematic or conceptual restructuring: thus, for instance, the fundamental (underlying) amplitude, A, of a single degree of freedom, linear underdamped mass/spring system (see Figure 4.9) with an initial displacement, x_o, and initial velocity, v_o, can be expressed as an exponential function of the subsequent time interval, t.

i.e. $$A = A_o e^{-rt}, \text{ where } A_o = (x_o^2 + B^2)^{1/2} \text{ and } r = \lambda\omega \quad \left.\right\} \text{ each being a constant for any one system}$$

or, in a differential form, $\frac{dA}{A} = -rdt$

In this particular case the incremental change relationship exhibits "external / internal" equivalence with respect to elemental time interval but not with respect to elemental amplitude.

For those real systems in which some basic interaction is undefined (having no effective physical/mathematical analogue) but is nonetheless consistently measurable, an empirical approach may be adopted with a view to establishing a "fundamental" description"*. The most useful empirical relationships are those linking positive quantities, especially when these can be expressed simply in terms of the measurable parameters normally considered relevant to the system. In any initial attempt to establish a "suitable" relationship, some clear conception of the pertinent system origins would obviously be extremely helpful since this would at least give an indication of what the positive quantities might be.

Empirical "laws" take many forms which vary much in the degree of apparent complexity exhibited. Some can undoubtedly be viewed as little more than the result of arbitrary "curve-fitting" procedures (e.g. expanded polynomial regression techniques). However, two simple forms commonly employed to associate mutually inter-dependent quantities ([Y] and [X], say) may be seen to immediately embody the same underlying ideas of similar incremental change "scales" as were alluded to above:

viz, $$Y = kX^n \qquad \text{..... 4.3}$$

i.e. $$\frac{dY}{Y} = n\frac{dX}{X} \qquad \text{..... 4.3(a)}$$

and, $$Y = ae^{mX}$$

i.e. $$\frac{dY}{Y} = m\,dX \qquad \text{..... 4.4(a)}$$

where, k, n, a, and m are constants.

Although power formulae have been used extensively in many fields of material science to correlate experimental data, there is a tendency

---

* An alternative is to define further parameters, prescribe their interaction with others, and hence measure their values in relation to the systems under examination. The use of the state variable entropy in thermodynamics is one example of this approach.

for these to be considered, in the main, as "purely" empirical expressions, having no real merit as possible theoretical foundations. Indeed, their indiscriminate use has often been subject to quite justified criticism. Among others, Reiner[175] has drawn attention to the glaring inadequacies of certain proposed power "laws" which do not conform to known or reasonable boundary conditions (origin inconsistencies are a common feature). In all instances of this kind the power expressions of the would-be "laws" merely serve the practical role of convenient interpolation formulae, having strictly limited numerical validity and hence no particular significance; as such, these are fully deserving of the "purely empirical" label. The potential relevance of many other power formulae can not, however, be dismissed so easily. Considerable emphasis is frequently placed by theoreticians on the material-independent nature of the historically famous power laws of Newton (gravitation), Coulomb (electrostatic attraction/repulsion), and Stefan-Boltzmann (black body emission), which each have a <u>fixed</u> integer or simple fraction power index, n, for the particular phenomenon described, and therefore a "dimensionally invariant" value of k. If, conversely, n is both phenomenon and material dependent, then the dimensions of k for any one phenomenological description will necessarily vary and so this quantity can not be interpreted as a "universal" fundamental constant in the way that it generally is with regard to those classical power laws mentioned above. This qualitative variability in the significance of k has been termed as the "dimension objection" by Reiner. However, as has already been stressed, all measures are strictly relative and consequently the actual <u>numbers</u> involved will be inherently dependent on some system or other: therefore, the rejection of a power formula on the sole basis of what is essentially an artificial restriction - that k need be <u>universally</u> fundamental - hardly seems capable of rational justification. Furthermore, the quantity k does <u>not</u> appear in 4.3(a), the basic equation of internal incremental change interaction, which is in fact "dimensionless" by customary reasoning; of course, the would-be problematic aspects of k can be effectively circumscribed by adopting any suitable base-reference values for Y and X, Y_r and X_r respectively.

i.e. $$Y = kX^n \rightarrow \frac{Y}{Y_r} = \left(\frac{X}{X_r}\right)^n$$

A number of workers, including Nadai[268], have used this latter approach to avoid the "dimension objection", but have tended to choose particular values of Y_r and X_r, labelling these as "material constants", n being usually taken as a combined material/phenomenon parameter. If specific ranges of conceptual applicability, or even of direct interest, exist for Y and X then a choice of these for Y_r and X_r automatically normalises the power relationship (cf. equation 4.1 - a "derived" power law with $n = 1$).

From any implicational analysis of theoreticians' comments regarding the relevance and usefulness of empirical power laws, a definite impression might be gained that, without an apparent reason being ever specified, power indices which are neither integers nor simple fractions often seem liable to be treated with more immediate suspicion than others which comply with this "convenient" but quite arbitrary condition. Viewing n as a scale factor of sorts (linking equivalent measures of incremental change) the totally groundless nature of such an attempted differentiation becomes abundantly clear; thus, for example by association, the scale factor linking the S.I. metre and the Imperial foot is no more or less relevant than that linking the S.I. metre and the S.I. millimetre.

Despite the obvious similarities of structural form displayed by equations 4.3(a) and 4.4(a), the conceptually restrictive aspects of customary dimensional considerations require that an artificial distinction - based on the assumed "significance" of distinctly arbitrary precepts - be drawn between the "dimensionless" nature of n as compared to that of m (also to that of k and a). Of course, by giving m a "relevant" customary dimension, the interactive unit algebra of an exponential law is immediately and uniquely satisfied through automatic (defined) self-consistency; numerical values of m which are nominally material- and/or system-dependent for the same apparent phenomenon or physical process are thus freed from the "fundamentalist" criticism which unjustifiably plague any similarly variable power exponent, n.

In terms of what has already been stated, any formal interpretation of dX in equation 4.4(a) as an appropriate "internal" measure of incremental change necessarily implies that there must exist complete external/internal system equivalence with regard to that particular quantity; i.e. that the "external" measure has immediate system relevance. It is perhaps significant that the two parameters which appear most frequently in empirical exponential expressions of phenomenological description, time interval and temperature, each has a physical phenomenon as the basis of its standardised measure. However, although descriptive relationships of the exponential type in either time interval or temperature are fairly common (as might almost have been "expected"), they are by no means general. The differential equation 4.4(a) has, of course, two sides; both must be conformed with in order to produce an exponential relationship at the integrated level. In addition, the quantitative nature of certain "known" interactions indicates that the standard increments of time interval and temperature are not necessarily appropriate measures of internal change for all physical systems involving either of these parameters; thus, for example, the well-established power law of Stefan-Boltzmann, mentioned earlier, would obviously suggest that, in the context of black body radiation and its total intensity, the appropriate "internal" temperature increment be taken as dT/T (rather than simply dT), where T is the absolute temperature of the source. From the above, an overriding principle of non-generality may be inferred - namely that the "internal" reference denominator pertinent to any incremental relative change need not be unique to all physical system interactions which embrace the same "external" measure of an apparently common "parent" quantity. The underlying logic of this last statement might perhaps be best illustrated by the somewhat variable appreciation of elapsed time displayed by the human species. (From a philosophical standpoint the primitive notions of time and change are, of course, fundamentally inseparable.) In terms of its practical applications and, indeed, of its more formal developments, the "science" of time[270] relies for the most part on the "fixed" concept of a standard time interval. The "universal" time scale so produced is, however, almost completely external to those many factors which can, both directly and indirectly, influence human appreciation as regards temporal matters. Thus, in some situations a person's "internal" clock may indicate that external time seems to "drag", in others that it seems to "fly". The generally recognised human phenomenon of a diminishing appreciation of elapsed external time with increasing age is also consistent with a "changing" internal reference value which, in this particular case at least, appears to be based on some accumulated personal "measure" of total conscious experience.

If, within a certain system, two quantities [Y] and [X] are totally inter-dependent then, by the very definition of mutual inter-dependence, the nominal values of either may be used as a consistent systematic measure of the other, even although the quantities themselves might have quite different "external" conceptual foundations.

In general, the customary scales of [Y] and [X] may or may not be similar. Nevertheless, since changes in the value of [Y] will necessarily be reflected by changes in the value of [X], and vice versa, it should always be possible (at least in principle) to set up associated similar scales of specific incremental change. As was stressed earlier, and was reinforced immediately above, the same "quantity" - in this case incremental change - can have any number of "dimensions" in the broadest sense.

The obvious need to recognise potential reference diversity does not, however, seriously hinder the immediate formulation of a quite unrestricted expression indicative of <u>scaled</u> incremental change. Thus, for the somewhat elementary case of two mutually interdependent quantities [Y] and [X], the following interactive equation, of which both 4.3(a) and 4.4(a) may be considered as rather special (and extremely simple) applications, is perfectly general:

viz,
$$\frac{dY}{R_Y} = n \cdot \frac{dX}{R_X} \qquad \text{..... 4.5}$$

The respective denominator terms R_Y and R_X are systematic (internal) reference quantities for dY and dX, appropriate to the particular interaction (and system) involved: n is a constant which can be interpreted as an intrinsic "scale factor" linking corresponding "measures" of incremental change. Of course, since R_Y and R_X may well be functions of the "external" measurements, Y and X, this general form is capable of producing <u>any</u> mathematical relationship between the two parent variables at the integrated level: i.e. the presented ideas on "internal" incremental change do not place any significant emphasis, other than in terms of simplicity, on undefined or non-consequential numerical "laws" of either the power or exponential type.

By their very nature, empirical laws are strictly beyond the realms of a priori justification; this fundamental truism in no way detracts, however, from their potential usefulness in a practical and/or theoretical sense. They may, for example, provide valuable insight (a posteriori), leading to an inferred "understanding" of the systems to which they apply, by establishing "internal" conceptual foundations, identical in character with those of derived relationships arising from the mathematical consequences of defined systematic interactions. The "semi-empirical" approach, although frequently maligned, need not be without merit as an intellectual "prop": the very existence of the absolute temperature scale bears excellent testimony to this.

The above arguments involving only two quantities, each depending solely on the other, may easily be extended to cover the much more common practical case - that where the prevailing value of one particular quantity [Y] exhibits measurable dependence on the corresponding values of several other associated quantities [X_1], [X_2], ... etc., within some interactive system. Thus, the appropriately expanded version of equation 4.5,

$$\text{i.e.} \quad \frac{dY}{R_Y} = n_1 \frac{dX_1}{R_{X_1}} + n_2 \frac{dX_2}{R_{X_2}} + \ldots + n_n \frac{dX_n}{R_{X_n}} \qquad \ldots\ldots 4.6$$

, again has complete generality. Of course, despite its deceptively uncomplicated appearance, equation 4.6 embodies an infinite potential for both mathematical variety and complexity. As before, the precise integral form pertinent to this "linear" equation of associated incremental change will be sensitive to the prevailing mathematical character of the crucial reference denominator quantities, R. One obvious possibility is $R_Y = Y$, $R_{X_1} = X_1$, $R_{X_2} = X_2$ etc, which yields the extended power relationship,

$$Y = k(X_1)^{n_1} \cdot (X_2)^{n_2} \ \ldots . \cdot (X_n)^{n_n}$$

Other of the simpler integral manifestations of equation 4.6 include,

$$Y = a\, e^{n_1 X_1} \cdot e^{n_2 X_2} \ \ldots . \ e^{n_n X_n}$$

$$(R_Y = Y,\ R_{X_1} = R_{X_2} = \ \ldots . \ = R_{X_n} = 1)$$

and, $$Y = n_1X_1 + n_2X_2 + \; .... \; n_nX_n$$

$$(R_Y = R_{X_1} = R_{X_2} = \; .... \; R_{X_n} = 1)$$

The more complex are the reference denominator terms, then the more complex is the final integral form. In the vocabulary of the overall systematic approach so far adopted, any need for a complex reference quantity, R, may be interpreted as meaning that the corresponding "external" measure of incremental change has little immediate "internal" significance to the particular interaction concerned.

Complexity, like its converse, is a strictly relative notion: no absolute basis for either can survive the rigours of close logical scrutiny. As was stressed earlier, the degree of apparent complexity exhibited by the behaviour of an interactive system is nothing more than an indirect reflection of the exact manner in which the underlying basis for its formal description was originally defined or conceived. (Apparent complexity is frequently imposed by a prior assertion of preconceived simplicity.) Thus, bearing in mind the typically "external" aspect of most defined quantitative measures, it is hardly surprising that "standard" physical data derived for the experimental study of specific systematic interactions often seems to defy any form of simple correlation. Physical data can, of course, like the very concepts which lie behind the information it represents, be restructured; for example, origin shifts can be implemented, quantities combined and/or rearranged etc., (cf. the simple form of the rearranged cosine rule, equation 4.1). If such a restructuring is performed on a trial and error basis, with the sole aim of producing quantitative relationships which display an element of positive mathematical simplicity, any success this might achieve could probably be dismissed as being merely "fortuitous", although the nature of the "successful" changes would obviously merit at least some measure of consideration. If, however, in contrast to this "purely" empirical approach, the restructuring is in itself based upon some implied prior "understanding" of the particular system involved, then the emergence of simple quantitative relationships as an indirect consequence thereof effectively reinforces that initial understanding, almost to the point of would-be confirmation. A degree of confidence in the "internal" conceptual foundations can thus be established and any potential for further understanding developed accordingly.

4.2.2 Foundational Premises - Enunciation

The essential physical basis for the material/behaviour model to follow is three-fold and consists of:

(i) the implicit acceptance as "suitable" of the generalised hierarchical concept alluded to earlier, whereby the principal of complimentary (quasi-solid/quasi-fluid) differentiation is applicable at any appropriate level of discrimination;

(ii) the recognised existence of an external quasi-fluid (itself capable of further differentiation) in nominally "free" or "empty" space, which, like all fluids, exerts an influence (or, more specifically, a pressure) on anything within its "volume"; and

(iii) the notion that materials and components thereof are, in effect, held together from without, not within.

4.2.3 Foundational Premises - Justification

4.2.3.1 Foreword

As reasoned earlier, the first two features listed above are not totally independent; indeed, the latter presupposes the former: similarly, the second and third features are closely linked. Since the most potentially contentious aspect of the proposed foundations is likely to be that relating to the omnipresence of an external "fluid", some discussion of this particular notion seems appropriate as a starting point in terms of justification.

4.2.3.2 The Case for an "Aether"

The concept of an external fluid-like "environment" is not new; it can be traced back to Aristotle(271), the Stoic philosophers(272), and beyond. Although it has never played a particularly significant role in the strictly mathematical formulations of modelled physical behaviour, the idea of an external fluid has been alluded to widely in the past; it finds a regular exposition in the major physical/mathematical treatises of mechanics from the dawn of the "Scientific Age" (or the "Age of Reason") in the 16th/17th centuries to the beginning of this present century; e.g. the works of Descartes, Newton, Hooke, Leibniz, Euler, the Bernoullis, Young, Faraday, Kelvin, Maxwell, etc., - to name but a few! The properties bestowed upon this "aether" or "subtle fluid" were many and varied, frequently complex, and often hotly disputed. Indeed, the hypothesis of coexistent aethers, each "designed" to explain different classes of phenomena, was not uncommon. Of course, the most reasonable conceptual function of the external fluid was as a medium to support light propagation. ("Waves in nothing" was, and still is, philosophically repugnant to the physical thinker.)

Modern physics texts often summarily cite the famous Michelson/ Morley experiment of 1887 as sounding the death knell on the aether concept. It is, however, well worth reflecting on whether such was in fact the case. Whilst there can be little doubt that the null-result of Michelson and Morley quashed the essentially "Newtonian" notions relating to simultaneity and the absolute nature of length, space, and time (the "Galilean" transformation) which then prevailed, the subsequent theoretical developments stemming from this, and the full implications thereof, need careful examination before any rational judgement on actual aether rejection or otherwise can be made with some conviction, much less certainty. It is true that modern physics, in its seemingly unrelenting attempt to reduce all of mechanics to Geometry, makes no specific mention of an aether, per se, but it could reasonably be argued that the former "subtle fluid" has merely undergone a convenient change in nomenclature to become "the field" in one of its many forms or even perhaps the relativistic abstraction, "space-time"*. Sir Harrie Massey[274], for example, has indicated some accordance with this view, principally in relation to Dirac's "picture"[275] of empty space as a condition in which all the "negative mass" states - a theoretical consequence of relativistic invariance when applied to quantum theory - are occupied:

> *We see that, after having disposed of the luminiferous aether, relativity taken together with quantum theory has replaced it by a new and far more complex one.*

Of course, those theories categorised by the concept of energy in the field were in existence well prior to 1887. Faraday, Maxwell, and the other field theorists of the earlier periods of the 19th century had followed on from the notional groundwork laid by such workers as Boscovich[276] almost a century before. However, at the time of their introduction, the particular field theories of gravitation and electromagnetism in no way threatened the philosophical standing of the aether.

---

* In his thoroughly unconventional treatise, "Physics without Einstein", Aspden(273) goes so far as to quantify the density of space-time in gm/cc!

Indeed, they were often seen - especially by the physically-minded Faraday - to complement one another; i.e. the relevant properties of "the field", including its energy, were considered as being implicitly representative of the physical state of the aether itself. Even Maxwell, far more the mathematician by inclination (and ability) than Faraday, was prepared to give his electromagnetic theory[277] a physical background; an appropriate passage from that famous work is quoted below:

> *We have therefore some reason to believe from the phenomena of light and heat that there is an aetherial medium filling space and permeating bodies, capable of being set in motion and of transmitting motion from one part to another, and of communicating that motion to gross matter, so as to heat it and affect it in various ways.*

Perhaps the most significant aspect of Maxwell's statement is that he felt so inclined as to make it - this despite his own and others' realisation that the exact nature of the "elastic" aether postulated did not influence the ultimate theoretical predictions of his predominantly mathematical treatise.

In the "new" realms of natural philosophy it would be an understatement to say that physical models (as distinct from the "purely" mathematical variety) are no longer popular. Thus, for example, when dealing with the consequences of the Lorentz transformation, Christy and Pytte[278] make the following remarks regarding physical arguments and interpretation:

> *They are not really helpful, however, and they can lead to confusion. Therefore we prefer to avoid such explanations, and re-emphasise the content of equations .... and .... .*

The waning of interest in over-riding physical concepts, such as those of Maxwell given above, coincided to a large extent with the rise to scientific pre-eminence of Einstein, in the footsteps of Minkowski, rather than with the initial period of confusion and hurried rethinking which necessarily followed the publication of the Michelson/Morley null-result. It should be noted, however, that the persistence of confusion undoubtedly eased the task of gaining recognition for the "revolutionary" thesis of Special Relativity[279]: thus, it is extremely unlikely that Einstein's ideas would have escaped so much question and attracted such a high degree of immediate general acceptance, had these been presented fifty years earlier.

To understand why the Einstein approach with its repeated references to "observers" and "measurements" - usually the very basis of physical thought and inference - should actually discourage physical interpretation requires a coming to terms with the "meaning" of measurement, especially in regard to the important aspect of references (see previous comments). Consider, as an illustration, the results of an experiment designed to determine the effect of temperature changes on the "length" of a steel rod. Given the normal connotations of temperature, it would generally be stated (found?) that the length of the rod increased with increasing temperature; nevertheless, such an "expected" conclusion is firmly based on particular ideas of the length concept in relation to the rod. Alternative conclusions are equally viable. Thus, for example, if the "length" were simply measured by "reading off" numbers on a steel ruler, located in the same thermal environment as the rod, no change might be recorded - a null-result! (The consequences of "length" measurement using a ruler of a material having a "higher coefficient of thermal expansion" than steel might prove to be even more contentious.) As was reasoned in an earlier section, change is a strictly relative notion*, the interpretation of which in physical arguments necessarily relies upon a prior stipulation of those conceptual references deemed "suitable"; i.e. the latter serve the function of local, rather than universal, absolutes. The Einstein approach, however, makes no specific mention of conceptual references: its "measures" are largely unqualified and hence outwith the realms of physical argument. It is therefore interesting to compare the famous Second Postulate of Special Relativity - that the "measured" speed of light is constant in all inertial frames - with the potential measured constancy of length in the different "thermal frames" of the rod example. While the former has been widely taken to mean that any "understanding" of the speed of light in terms of a local conceptual absolute is redundant, such an interpretation is quite incompatible with the fundamental "realities" of physical measurement.

---

* The difficulties experienced by the Greek philosophers of old - especially Parmedes - in coping with the idea of change in a consistently rational manner can be attributed to their general failure to fully appreciate its relative nature. Conversely, it would also seem apposite to remark that the degree of apparent simplicity embodied within more modern treatments of the topic is often achieved through somewhat limited consideration, whereby the inherent complexities of notional change in the absence of "fixed" bases for comparison is deliberately avoided.

Several plausible attempts to effectively reconcile the Michelson/Morley null-result within the logical framework of a physical rationale pre-date the Special Theory of Relativity (1905). The significant contributions of Fitzgerald and Lorentz in this sphere[280] are especially noteworthy. Based upon a fairly reasonable premise of matter/aether interaction due to relative motion, it was suggested that all material bodies (including rulers) moving with respect to the aether - a local conceptual absolute - undergo an associated relative length change according to a notionally "fixed" external reference. This Lorentz-Fitzgerald contraction, as it came to be known, is naturally quite different in concept from the observational length differences of the Special Theory (despite the misleading assimilation of terms to be found in many modern physics texts) as may be judged by another quote from Christy and Pytte[278], this time regarding Einstein observers:

> *Each observer is using good instruments ......*
> *they arrive at different answers simply beacuse*
> *of the curious nature of space and time.*

Although, in 1904, Lorentz was responsible for deriving the important relativistic transformation (which still bears his name) securing the form invariance of Maxwell's equations of the electromagnetic field in all inertial frames, he and his followers (including Poincaré who was later to abandon the aether in favour of Einstein's geometry) were unable to link this with their aether assumptions. It has since been shown[281, 282], however, that a rational aether (or sub-stratum*) model, incorporating the conventions of Einstein but still based on local absolute concepts, is a totally viable proposition, offering a physical "explanation" of relativistic effects consistent with experimental observations, and solutions to the paradoxes (e.g. the travelling twins) of Special Relativity which have led some[283] to seriously dispute its validity.

To those for whom mathematics and physics are synonomous the fact that the mathematical expressions of "Neo-Lorentzian" Relativity are identical with those of the Special Theory might appear to render the former superfluous since it would seem to be based on "unnecessary" metaphysical premises. From the time of Descartes, scientists have been particularly sensitive (although generally in an ambivalent sense)

---

* The more modern term "sub-stratum" is used in an attempt to rid the basic aether concept of the unfortunate connotations relating to the many spurious mechanical features bestowed upon it during the 19th century.

about extraneous assumptions and metaphysical assertions. Although Descartes' aim[284] of establishing unquestionable "self-evident" scientific principles was doomed to both philosophical and practical failure, its repercussions in the light of continued mathematical development were many, especially when combined with Occams' famous "razor" (the "First Commandment" of the scientific method) that entities must not be multiplied beyond necessity. Pure mathematics in itself has, of course, no need for philosophy; its foundational axioms are defined within specific man-made systems and automatic self-consistency* is assured. Its relevance to the "real" world is, however, a matter for either philosophy or experience to judge upon; it certainly does not, as is often wrongly implied, rid science of metaphysics; indeed, as soon as it crosses the "boundary of reality", it tends to create its own. Thus, for example, as Landé[286] has pointed out, an assertion of determinism is no more or less metaphysical than an equivalent statement of denial. The same may be said of an aether. This concept, although perhaps redundant to mathematics, has strong claims for philosophical "need"; i.e. if the electromagnetic field is not a medium for light propagation but merely a set of equations expressing quantitative interactions, should it then be stated that light travels in mathematics?; similarly, if the idea of an aether is to be regarded derogatively as metaphysical, what standing has the concept of a wave in nothing? Metaphysics, it would seem, is therefore frequently like beauty, being essentially in the eye (or mind) of the beholder. (Incidentally, as Prokhovnik[282] has remarked, the orthodox version of Special Relativity is not without its own arbitrary and "extra-physical" assumptions, although they are rarely mentioned explicitly: it is hardly surprising therefore that Massey[274] refers to the *"Wonderland of Relativity"* - cf. the Christy and Pytte quote given above, regarding the *"curious nature of space and time"*.) To deny any "need" for overall notional consistency is tantamount to an intellectual surrender to the more idealistic abstractions of Heglian dialectics, according to which a mathematical concept does not exist but is real whereas an individual thing exists but is not real! The simplistic view that, to be classed as "scientific", a principle must be refutable only scratches the surface of an extremely complex issue. If it were applied rigorously, much of what is termed "science" today would

---

* Professor H.J. Hopkins[285] has made the pertinent observation that mathematicians frequently appear to hold the whiphand, *"for one can only reason with them in their own language"*.

need urgent reclassification. Consider briefly, for example, the principle of the conservation of energy, without doubt one of the major cornerstones of current scientific thought: to quote Theobald[287],

> *Ingenuity in fact knows no bounds when it comes to saving the principle of the conservation of energy. The neutrino, as is well known, had originally just this function.*

By definition it is strictly impossible to refute a fundamental metaphysical tenet: i.e. absence of proof is not equivalent to proof of absence. The energy conservation principle, which finds perhaps its most popular expression in the "book-keeping" First Law of Thermodynamics, is certainly not without distinct metaphysical* overtones. The usual interpretation of Bridgman's principle which asserts the "absolute significance of relative magnitude"[260] - see earlier comments - is another obvious case in point; according to Ellis[285], this is *"plainly metaphysical"*.

4.2.3.3 Matter as a Hierarchical Structure

When compared to the unconventional (strictly in the context of "modern" norms) proposal of an external quasi-fluid, the suggested view of matter as a hierarchical structure should seem eminently reasonable, since it conforms to both the particulate and continuum "pictures" usually offered as notional alternatives.

The divergence of opinion (or emphasis) as to the discrete or continuous nature of matter has probably occupied the scientific mind since man first tried to rationalise nature: within this field the continuing development of mathematics has played a significant role. Thus, in the Greek schools of thought, the replacement of the Pythagorean view of discrete physical points with that of the continuum - so ably developed by the Stoics[272] - can be traced to the discovery of irrational numbers. (The denial of an absolute void enunciated by Aristotle[271] is very much part of the generalised hierarchical approach now under examination.) Again, in much later times, the

---

* In view of unfortunate historical precedents, it is important to distinguish between "mild" metaphysics (limited hypothesis) and the more extravagant forms of conceptual imaginings.

discrete particle model of Newtonian mechanics dominated all alternative formulations from the period of its inception until the "new" mathematical methods of the late 18th century[288] created a basis for a satisfactory continuum physics fit to challenge the then status quo.

Of course, when viewed in their proper perspective, mathematical models represent no more than idealised abstractions of the "real" world, as observed either directly or indirectly. Thus, the fact that the descriptive mathematics relating to the ideal versions of discrete and continuous systems may be quite different does not legislate against the mutual inclusiveness of the discrete and continuous aspects of matter in terms of concept: for example, there is no fundamental conflict implied when a materials scientist refers to a "solid" object, such as a piece of metal, as an "open", spatially ordered, system of "particles" (atoms). The basic question is one of scale, observational position (level of discrimination), and the degree of resolution employed. It may be recalled that the difference between a "solid" and a "fluid" - both conceptual entities being capable of sustaining a nominal continuum interpretation - is generally argued in terms of varying degrees of discrete "constituent" mobility. Also worthy of note is the background of stress concept[209, 251] within the Theory of Elasticity. History shows that the original rise to prominence of the abstract mathematical visualisation of a material "body" as simply a geometric space within which various actions and/or properties are continuously distributed (i.e. the continuum mechanics approach formally initiated by Cauchy in 1822) was, to some extent, merely an intellectual follow-up to an earlier discrete "molecular" model due to Navier; although this precursor of the prevalent modern view of materials for most analytical purposes was an extremely crude physical conception, it incorporated the crucial feature of a continuous spatial distribution of inter-molecular forces, thus allowing the implementation of integral calculus to "sum" net local actions in terms of areas within volumes. Despite its many successes in application to the solution of real problems, and the high degree of refinement and mathematical development it has undergone (starting in the last century with the works of those such as Green), continuum mechanics is, nevertheless, inherently bound by the fundamental limitations which stem from the physical rationale of its rather inelegant origins. Bearing in mind the essential inhomogenuity of those forms of internal structure now known to exist beneath the immediate bulk presence of any real material at the nominally "visible" level, the high degree of idealisation and limited consideration involved in the common continuum-oriented expression relating to stresses at a "point" is undeniable: the exact relevance of a "point" in terms of a heterogeneous composite such as concrete, complete with fluid-filled voids and micro-cracks, is particularly dubious. Of course, what might be classified as the "point syndrome" goes no small part of the way to explaining the general failure of continuum mechanics as regards the formulation of rational strength criteria.

In one area of scientific endeavour at least, that of quantum physics, the need to reconcile the important interplay between the notions of continuity and discontinuity has been positively recognised*. Unfortunately, however, because the discrete aspects of the quantum approach are so often over-emphasised, the whole field tends to be commonly, but quite wrongly, identified with concepts of discontinuity. Thus, although the early history of quantum theory[289] is strictly beyond the scope of this present study, the frequently overlooked feature that Plank's original work (1900) on material radiation oscillators and the later postulation of a fundamental radiation energy "packet" by Einstein (1905) were each directed towards the specific formulation of a rational theory to account for an experimental continuity of effect** not forecast by the classical models is nevertheless most relevant to the current argument and therefore worthy of mention. The developments and applications of quantum mechanics have been many and varied: numerous continuity/discontinuity conflicts, such as the implied entropy discontinuity of Gibbs Paradox (a "real" problem which is essentially disputed rather than solved in classical thermodynamics), have fallen before its influence[286]. Quantum physics is not, however, without its philosophical problems, although by far the greatest of these - conceptual dualism - is largely of its own, or rather of a sizeable proportion of its proponents, making. Among the notable quantum physicists not

---

* For example, while the assumed fundamental continuity of energy radiation embodied within the classical models leads to the prediction of an unobserved higher level discontinuity, the alternative stipulation of a fundamental discontinuity yields a satisfactory "understanding" of that appropriate continuity of effect measured in practice at the phenomenological level.

** The case for "expecting" a general continuity of effect at the phenomenological level was, in fact, argued by Leibniz (290) some 200 years earlier, when countering a speculation of Descartes regarding collision.

subscribing to the "need" for dualism, Landé[286] has argued forcibly and well against what he considers a violation of one of the accepted rules or orderly thinking; viz, do not indulge in false opposites. Coming, as this does, from one who has actually succeeded in deducing the quantum laws (a task once thought impossible!) from plausible general postulates[291] of statistical invariance and symmetry, it might have been expected that this would have merited serious and widespread attention, but somehow the almost magic "strangeness of the quantum" still persists unshaken in the minds and teachings of the dualists. The so-called quantum paradoxes of wave/particle (either/or) coexistence at the same nominal level of observation (the exact manifestation "suitable" to any particular situation being at the sole discretion of the two-picture theorist) are not an inherent feature of quantum physics per se, but rather a simple consequence of a deliberate choice that such a confusing "explanation" should remain as acceptable. Thus, although the "wavelike" properties of matter "particles" (e.g. electrons) are not inconsistent with a sensible unified particulate view, the existence of Duane's[292] Third* Quantum Rule (1923) which makes such a view possible often appears to be quite deliberately overlooked and hence is generally unrecognised. Indeed, many introductory texts even prefer to cite, in passing, a mathematical (but hardly a consistently rational) alternative, known as the Second Quantitisation (1928) in which an atom (or an electron) finds description as a continuous electric "fluid".

However, just as concepts may not be mutually exclusive simply because the mathematics thereof are different, the equivalence of the appropriate mathematics is not, of course, a valid justification for conceptual dualism. As Landé has pointed out, if this were the case the planetary models of Ptolomy, Tycho Brahe, and Copernicus should all be held in equal standing: the "scientific" rejection of planetary triality is a particularly pertinent analogy since the Second Quantitisation of Klein, Jordan and Wigner involves an extremely complicated non-linear differential equation. Fortunately, mathematical elegance is not the sole criterion for notional acceptability. In some ways, the survival of dualism may be seen as the condescending mathematician's revenge on a scientific society to which he was unwillingly obliged to provide a

---

* The First and Second Quantum Rules to which this forms a logical complement are the Plank Rule (energy) and the Summerfield-Wilson Rule (angular momentum), respectively.

"picture". For example, Schrodinger's unitary wave theory - now almost always interpreted in a statistical context - originally floated the idea that a particle was "in reality" simply the high crest of a wave. It would therefore seem that quantum mathematicians do not feel so inherently secure and safe from Realist criticism as do their counterparts in the field of Relativity who generally tend to summarily dismiss any need for a sensible physical picture ("thinking in objects") as naively primitive.

The essential feature of the hierarchical approach which removes any aspect of dualism, which might wrongly be inferred from it, is the emphasis placed on different levels of discrimination. The model recognises both continuity and discontinuity on a local basis and thus accentuates *a priori* the importance of systematic structure within a material. Consider, for instance, a sample of a confined gas. According to the hierarchical view this can be treated as a continuous entity at the bulk volumetric level; however, in dropping one level of discrimination the gas is then visualised as a dynamic collection of discrete quasi-solid "particles" (molecules) within a continuous quasi-fluid space, this latter complementary "quantity" being as much a part of the overall structure in that particular hierarchy as are the molecules themselves. This space may be considered either as an "internal" environment with respect to the bulk volume or as an "external" environment with respect to the molecules. It is important to note that the subdivisibility of the molecules into further discrete and continuous entities at lower levels of discrimination is in no way sacrificed by treating each molecule as a quasi-solid at this relatively higher level. Of course, in the context of nominal fluids, and apart from certain implications regarding the nature of the complementary space (non-molecular volume), these ideas are far from being contentious. Fluid pressure, for example, has long been conceived of in physical terms as the combined (time-averaged) result of many discrete dynamic sub-solid interactions, although it was not until Einstein's reflections on Brownian motion that the inherent "realism" of the kinetic theories gained general credence[292]. Any visualisation of fluid pressure "acting" on a surface (observational position) which involves ideas of sub-solid bombardment is quite obviously positive; negative bombardment is meaningless - no bombardment, no pressure; i.e. fluid pressure may simply be taken as an implicit "measure" of the relative dynamic existence manifested by the inter-colliding sub-solid system with respect to the observational standpoint chosen. The only sub-solids normally considered pertinent to a "real" fluid are, of course, the constituent molecules, the "free" volume being treated typically as a dynamic but nonetheless "empty" space. The molecules need not, however, always be of a common type: miscible fluid systems may well contain a variety of molecular forms. The generalised hierarchical view of a fluid merely extends the principle of this latter characteristic indefinitely by imparting an effectively fluid-like physical structure to the apparent nothingness of the constituent "free" space. Ultimate justification for this approach will be by way of the degree of consistent "understanding" it provides.

Invariably, the mechanical testing of a material involves an initial incorporation of a "representative" sample thereof within some physical/structural system or other, and then the subsequent observation (through defined systematic measurement) of the behavioural traits exhibited by this sample as certain overall characteristics of the system are deliberately altered (controlled) or are allowed to change in an effectively self-regulating fashion. Thus, the performance of a material under test is basically a local systematic response to a changing "external" environment: the associated concepts of a material and its environment can not therefore be divorced in strictly rational terms. This has long been recognised in the field of thermodynamics in relation to the mechanical "performance" of fluids, especially gases. However, despite a vast amount of experimental evidence also confirming this latter principle with respect to the testing of nominal solids and, in effect, highlighting the potential quantitative and phenomenological importance of the testing system (environment) actually employed in any form of strength determination, the abundance of available intellectual information has largely been ignored in most theoretical approaches to "solid" material breakdown. Phrases such as "intrinsic strength"* - a concept of the absolute (undifferentiated) continuum - are indicative of the typically inversionist thinking which tends to pervade the fundamental dialogue of material science in this context, and which attempts to "explain" any inconsistencies between ideal models and observed reality

* If strength was indeed a truly intrinsic material property which could be simply tied to the appropriate material name or chemical symbol, as distinct from an interactive system parameter, it would not be influenced by the procedural details of its determination.

by conveniently finding fault, not with the models, but with reality itself. Thus, in the established "scientific" tradition of a reductionist philosophy, the idea that experimental results, being liable to "secondary test effects", are inherently more suspect than "reasoned" theoretical predictions generally prevails. It is not uncommon to read in elementary texts (often the only reliable sources from which a true impression of the assumptions and implicit fundamental tenets of more advanced treatises can be gained) that the tensile strength of "brittle" materials such as glass is "really" much greater than that determined by experiment, but that the "unfortunate" presence of flaws has a "weakening effect". This plainly metaphysical insinuation of an almost malevolant action on the part of the flaws exemplifies perhaps the worst feature of reductionism - an inbuilt capacity for endless tautological amendment. Viewed in a cynical light, the approach invariably starts with one set of ideal concepts, develops the pertinent theory thereof to a point where predictions can be tested, and then introduces as many qualifying concepts as are necessary to account for discrepancies, should these arise. By thus removing the empirical vulnerability of the ideal model, its fundamental relevance and suitability need never even be considered, much less questioned. The associated aspects of inversionist thinking can best be illustrated by the somewhat misleading title commonly given to the qualifying process; viz, saving the phenomenon (not the model). From a hierarchical standpoint, the flaws in any particular sample of a nominally brittle solid material are as much a part of the representative material system as are the nucleii of the molecular constituents; accordingly, the "real" strength of a glass specimen is seen as no more or less than the ultimate capability of the <u>whole</u> sample system (including flaws) to adapt to a changing external environment without the unstable bulk disintegration of its solid substructures taking place. Two glass specimens, one with and one without flaws are thus viewed as basically <u>different</u> systems, even although the chemical classifications for each may be totally identical: any process of flaw removal is seen as involving an effective change of local environment at some level or other. As in all systems approaches, the hierarchical description of materials (applicable at <u>any</u> particular level of discrimination) recognises three important interactive concepts - firstly, the input (control variables), secondly the system itself (transformation functions) and thirdly the output (state variables). Within these terms of reference, phrases relating to material behaviour (output) such as "the true stress-strain curve" or the compressive strength cease to have any real descriptive validity.

Although it might appear so at first sight, there is in fact no basic conflict between the ideas of "the quantum" or fundamental unit and the indefinitely extended principles of hierarchical differentiation, since the quantum language is essentially designed to cater for the description of processes at a particular level. Consider as a general illustration, the common exercise of puting a number to a gathering of people. The quantum of attendance, as it might be called in such a case, is frequently taken as a "head". This does not mean to imply that the human head is not capable of notional or even physical subdivision, but rather that it serves as an extremely useful grouped unit having distinct relevance, in an associative sense, to the actual situation "seen" to be involved; of course, from a practical standpoint alone, the merits of possible alternative "units" of presence based on a lower level of discrimination than that of a head count (e.g. a tooth count, hair count, brain-cell count, etc.) would be quite dubious!

There are, however, certain distinct conflicts between the extended hierarchical approach and the currently accepted view of matter. Neither the concept of an absolutely empty space nor that of indivisible fundamental matter "particles" has any place within the former. (It may be recalled that the number of claimed "ultimately fundamental" particles has increased steadily since the atom was first postulated to fill that role over 150 years ago. The so-called building blocks of matter would therefore seem to be no less immune to subdivision - or, indeed, to temporal disintegration - than are those of masonry construction; bearing in mind the final demise of the fundamental atom, which was, of course, split notionally long before its physical division was ever achieved, and the progressive trend thereby established, the long-term future of the "elementary" quark can hardly be considered as secure.) It must be emphasised that the concept of extended hierarchical differentiation being applicable ad infinitum is not equivalent to the idealistic (single level) continuum approach with its typical descriptions of finite material volumes as containing infinite numbers of points of no physical extent. The formal mathematics of infinity is such that it is as feasible to divide a finite quantity into an infinite number of finite parts (the hierarchical approach) as it is to divide the same quantity into an infinite number of infinitesimal parts (the continuum approach): had the former not been explicitly recognised, the famous paradoxes of Zeno would still be unresolved!

Having been taught to count from an early age in numbers of objects, most students of science have far greater initial difficulty in coming to terms with the concept of infinity than with that of its inverse, i.e. zero. As a result, the development of an "understanding" of those "in-limit" aspects of the notion of infinity often tends to occur in the absence of a corresponding reassessment with regard to the full implications of the zero concept. Such imbalance is unfortunate because the latter should also be seen to embody in-limit connotations. This line of thought is naturally as pertinent to the empty space conflict as it is to the recognition (as potentially "suitable") of the ideas relating to the indefinitely continued <u>finite</u> divisibility of "solid" matter. Thus, while local zeroes arising from the absence of <u>defined</u> existences are quite unobjectionable, any <u>absolute</u> zero of existence which might be proposed can have no defined basis for rational justification and must therefore be quite arbitrary: i.e. there is a world of philosophical difference between a statement that a box contains no apples, for example, and one that a box contains <u>absolutely</u> nothing!

4.2.3.4 <u>Environmental Variation as a Source of Action</u>

In terms of application, the last feature of the proposed model is perhaps its most important. The notion of materials being held together from without rather than from within, simply involves a shift of conceptual origin with respect to "source of action" from that which generally prevails; i.e. the existence of so-called "bonding" effects will be seen as a natural <u>consequence</u> of specific interactions at various levels within an overall system, rather than as a <u>defined</u> (self-consistent) conceptual axiom of strictly limited development potential. (Such is the frequency of applied tautology masquerading as would-be logic in this context - see earlier comments on "strength" - that explicit recognition of "internal bonding" as a somewhat arbitrary primary axiom is seldom given formal expression; it is hardly surprising therefore that the suitability of the axiom itself to the situation it claims to describe is rarely the subject of close scrutiny.) The greatest immediate benefit of such a datum shift is that it offers a welcome opportunity to rid materials science (and indeed, physics in general) of its most "magic" element of inherited dogma - action at a distance.

Although this troublesome concept is undoubtedly "Newtonian" in origin, it would be quite wrong to ascribe it totally to Newton himself, since it owes much of its perpetuation in the form now "understood" to other influences, including the attitudes of both his followers and his critics. Living when he did, Newton was only too aware of the Cartesian criteria for scientific acceptability* (see earlier comments on Descartes) and he trod a narrow path between his philosophy on the one hand and his mathematics on the other. Thus, while his personal philosophy admitted the necessity for an aether (or "subtle spirit") as a source of action[294], the formal mathematics of his mechanics did not. Newton maintained that the "subtle spirit" was omnipresent, but that its density varied, being greatest in the "free" interplanetary spaces and least within the "pores" of solid bodies. (This feature of "environmental" density variation will, in fact, turn out to be an integral facet of the "new" model.) Unfortunately, some of those who have chosen to write the history of science have often created a misleading facsimile of the exact manner in which notional developments actually occurred; the common implication to be found in (patriotic?) British texts (largely due to the influence of Tait[295]) that the concept of energy derives its existence from Newtonian physics, while in reality it undoubtedly grew from the alternative mechanics of Huygens and Leibnitz, is only one example of this type of distortion. To avoid interpretive misunderstandings (deliberate or otherwise), Newton's own views[293] on action at a distance must be consulted:

> *And now we might add something concerning that most subtle spirit which pervades and lies hid in all gross bodies; and by the force and action of which spirit the particles of bodies attract one another at near distances and cohere if contiguous; .....*

It may be seen, from this quote at least, that Newton considered the mutual "attraction" of bodies, and hence the mathematical expression thereof, in an "as-if" context. As Koyré[296] has pointed out, Newton's fundamental belief in an aether was such that he included in the Principia details of an experiment actually purporting to "verify" its existence. However, because he was not (and indeed could not) be as

---

* A quote from the Principia Mathematica[293] illustrates this point most forcibly: *We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearance.*

authoritative in his philosophical approach as he was in his mathematics*, others were prepared to read into his work on the latter an alternative philosophy based on a "realistic" acceptance of action at a distance. To a large extent, it is this implied philosophy which is accepted today as "Newtonian" and which caused most of the argument between Newton and his doubting contemporaries, especially Leibniz. Although Leibniz held the now prevalent mathematical idea that space is nothing more than a system of relations[(298)], it is interesting to note that in his correspondence with Clark[(299)] regarding the impossibility of action at a distance, the mutually agreed postulate of an "undiscovered" contact mechanism, not unlike a "kinetic theory" version of Newton's earlier aether, finds formal expression. (Certain consequences of this postulate will shortly be developed within the framework of the "new" model.)

By the 19th century and in the wake of the partial success enjoyed by the elastic fluid theories of heat, magnetism and electricity, physicists were prepared to recognise three sources of physical action - viz, action by contact, action at a distance, and action in "the field". The predominantly mathematical attitude of present day field theorists to this trialistic view is most illuminating: thus Theobald[(287)] states,

> *It is clear that ..... (the first) ..... conceals a conceptually complex situation which may in fact be able to be analysed away in terms of .... (the second) ..... and certainly .....(the third).*

Of course, from a hierarchical standpoint there is nothing so especially complex about collision (perhaps the simplest of all physical interactions) that it needs to be "analysed away" in terms of an unnecessary metaphysical concept. Field theorists might well claim to the contrary that the postulation of a medium (other than mathematics?) for the propagation of action is in itself somewhat gratuitous, since the use of a medium solely for explanatory purposes must lead eventually to the logical necessity for yet another medium (or sub-medium) to "explain" the properties of the first, and so on. However, these aspects of possible indefinite differentiation, continuity/discontinuity, etc.,

---

* In common with most scientists of his time, Newton was indefinitive and far from precise, or even consistent, in his use of language (the strict "scientific" meaning of many terms was not to be agreed upon until much later) and this certainly facilitated misinterpretation. For example, in his other major treatise, Optiks (297), several conflicting hypotheses on heat are proposed, without any apparent statement of notional preference being offered by way of discussed comparison.

being naturally at the very foundation of the extended hierarchical approach, need pose no particular philosophical difficulties; i.e. the choice of a "fundamental" level is a quite arbitrary decision. Conversely, despite the would-be persuasiveness of their quasi-physical rhetoric which includes inter alia the mnemonic description of gravity as a "local distortion of space-time", the proponents of the purely mathematical field must adopt a rather ambiguous philosophical stance in order to justify their views. Thus, they can but attempt to side-step the basic dilemma of implied action at a distance to which they have become almost inextricably bound (unwilling victims of a deliberate self-made choice); this element of evasion may be judged by another quote from Theobald(287):

> *Field theories are essentially process theories, theories in which object language is at a discount, and in which therefore action at a distance becomes more intelligible.*

Although perhaps also appropriate at the present point, further possible comment on the unquestionable ambivalence embodied within the typical field theorist's view of the whole action at a distance topic will be reserved for the moment, until the "physical" alternative offered by the hierarchical approach has undergone some degree of development.

4.2.4 Towards a Physical View of Gravity

That the presence of physical variations wihin a fluid environment can account for apparent action at a distance may be argued as follows. Consider firstly the emotional reaction of a person who unwittingly walks in front of the air intake of a large gas turbine. While he might feel as if the throat of the turbine is attracting him ("sucking" him in) at a distance, the "real" source of action is the net "push" of the air molecules from the other side. (Of course, those who would maintain that conceptual dualism is strictly rational would supposedly disagree: thankfully, however, the areas in which such minds operate and have influence are fairly limited, and Otto von Guericke's famous Magdeburg spheres experiment(300) has never suffered from "two possible interpretations".)

Consider now a single "solid" sphere located within an infinite volume of a near-ideal gas. (A near-ideal gas is essentially the same as the ideal gas of elementary thermodynamics, but the molecules are

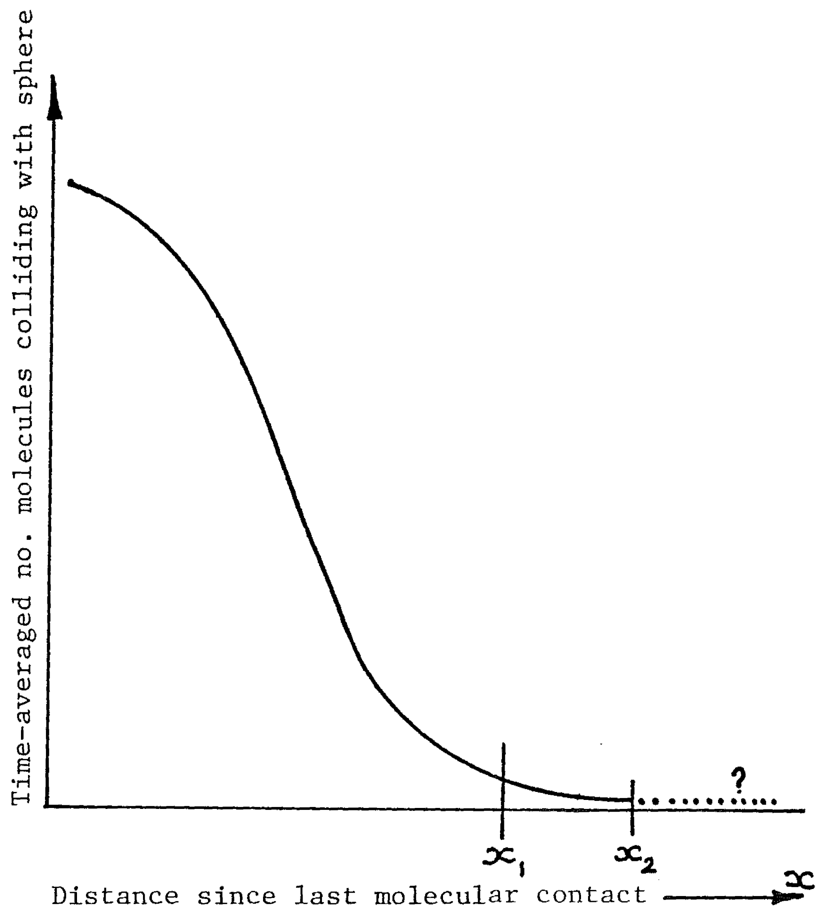


FIGURE 4.10: A Molecular Distribution

considered to be of a finite size*, thus allowing for the possibility of inter-molecular collisions.) If a kinetic view of the hypothetical fluid is accepted as reasonable (i.e. if a sub-level of random dynamic existence below its immediate bulk presence is recognised), the "continuous" pressure on the external surface of the spherical body, exerted by the gas, may be associated conceptually with a time-averaged sum of the many local dynamic actions arising from all-round molecular bombardment (discrete collisions, momentum transfers, etc.). Through the implementation of a notional time scale which yields apparent continuity, the appropriate average pressure may "safely" be assumed constant; according to such a scale no net time-averaged pressure force would be experienced by the body. However, taking into account the important physical phenomenon of Brownian motion, observed in connection with real fluids, it should be clear that any blanket generalisation regarding the "absolute" constancy of hydrostatic type pressure is philosophically unjustified - although obviously "convenient" on the grounds of simplicity. Ignoring the statistical nature of fluid pressure is not, of course, an especially critical omission in terms of most practical time scales; nevertheless, the general application of an unqualified mathematical model, which is based upon a tacit assumption of non-variability, necessarily precludes any further potential "understanding" of solid/fluid interaction.

The fluid particles will be involved in a "continuous" mutual interchange of both energy and momentum through their motion and successive impacts. Those molecules which actually impinge on the sphere will therefore vary as to the time or distance since their last "communication" with the overall fluid system. A distribution such as that shown in Figure 4.10 might reasonably be assumed, the illustrated distance axis being scaled in terms of some customary (fixed interval) length measure. (In order to avoid the frequent repetition of such qualifying dimensional statements, all quantitative measures henceforth referred to should be taken as being of the customary type unless otherwise specified.) It would not seem unrealistic to expect an identical distribution at each "point" or discrete sub-area on the surface of the sphere, provided that the observation time used for averaging the

* This molecular "size" will, however, be treated as being relatively small (almost to the point of near-insignificance) compared to the characteristic dimension of the sphere.

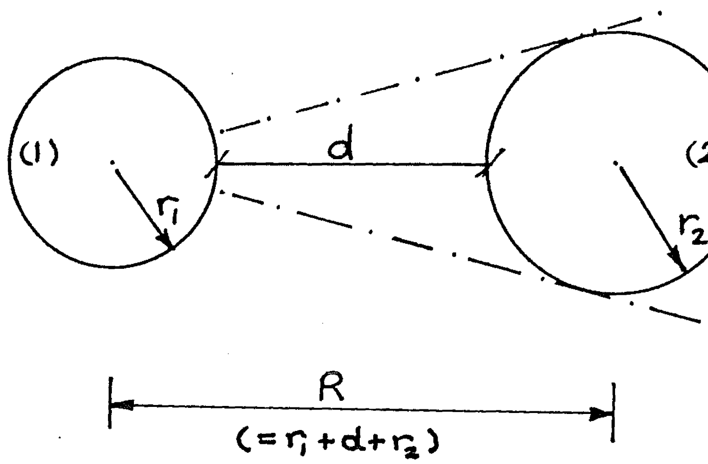


FIGURE 4.11: Two Solid Spheres Within A Shared Environment

collision numbers is sufficiently large. The "tail" of the skewed distribution may, of course, only account for a very small proportion of the time-averaged total number of molecules contributing to the hydrostatic type pressure but, nevertheless, it must be finite. Also, since the tail is infinitely long the fractional contribution of this towards the total pressure on any one elemental area will not be especially sensitive to changes in x beyond $x = x_1$; such "tail-area" insensitivity is a common feature of the statistics of extremes[301]. The statistical continuity of that part of the distribution shown depends implicitly on the choice of a "suitable" observation time for the purposes of averaging. However, no finite observation time can be adopted which would bestow a similar degree of continuity upon the extreme "regions" of the tail - hence the apparent vagueness of the illustrated distribution beyond $x = x_2$ in Figure 4.10. This "random" aspect of the extreme tail in no way prejudices the previous assertion that the tail itself makes a finite contribution to the suitably time-averaged total; rather it means that the effective "background" contribution from beyond $x = x_2$ is essentially the same as that from beyond $x = x_2 + i$, where i is any finite incremental distance quantity.

The relative simplicity (and symmetry) of the physical situation described above is, however, totally dependent on the stipulation of a single spherical body. Thus, the "additional" presence of a second spherical body (2) within the environmental fluid, as shown in Figure 4.11, automatically precludes the existence of a unique time-averaged number/distance distribution for each and every "point" on the surface of sphere (1) as was previously the case: i.e. the two spheres effectively interact within the fluid system, and some form of interactive force (applied by the fluid) becomes, in essence, an "expected" manifestation of this. Assuming that the "random" molecular motion of the environment can be taken as a source of radial surface pressure on each body, then sphere (2) could be said to shield sphere (1) - and vice versa - from a fraction of those long-range "free" molecules which would have been involved in impact with the first (second) sphere in the absence of the second (first): any "realistic" acceptance of net effective shielding occurring between the two solid "presences" also naturally implies that the time-averaged molecular density is no longer of a constant magnitude throughout the entire fluid system as before but, rather, varies according to the relative position of that locality actually considered*. This visualisation of a one-sided (density-related) lack of "tail pressure" for each sphere immediately yields an interactive force of apparent attraction between the two bodies. A simple net force analysis (see Appendix I) based on the assumption of a constant tail pressure ($d \gg x_1$) produces an expression of predicted interaction with a number of interesting features; viz, the magnitude of the interactive force is identical for each body (in accordance with Newton's Third Law), is inversely proportional to R^2, and is directly proportional to the product of the squares of the radii of the spheres. For any case where $d<x_2$, a similar analysis of net force would require some more precise knowledge of the original number/distance distribution to be available before the quantitative magnitude of the interaction could be deduced. It can be stated, however, that in such circumstances the "expected" force would increase with decreasing R at a greater rate than that associated with a simple inverse square relationship. Of course, unless the molecular environment were of a somewhat "rarified" nature, the condition $d<x_2$ would represent extremely close proximity of the spheres in relative terms.

It is important to re-emphasise that the interactive force, F, forecast by the above model is applied to the bodies via the environmental fluid and not by the bodies themselves. Bearing in mind that constitutive facet of the general hierarchical approach relating to the presence of an "unrecognised" external quasi-fluid environment, the parametric form displayed by the expression for F based upon an assumption of constant tail pressure,

viz, $$F = k\,\frac{r_1^{\,2}\, r_2^{\,2}}{R^2}$$ (k being a system constant) 4.7

, is most significant; the immediate similarities between this and the famous Law of Gravitation formulated by Newton should be strikingly obvious. (The fact that the gravity law is typically seen in an "irreducible" light, either as a fundamental postulate or as a simple empirical statement derived from observation, need not preclude alternative developments.) Indeed, the extreme statistical aspects of the

---

* The changes envisaged here are necessarily slight. However, for the purposes of present arguments, the viability (finite "strength") of such changes - no matter how small - is all that need be recognised.

model so far described would seem to offer a plausible first-step "explanation" of the relative "weakness" of gravitational effects, a feature thereof which is often remarked upon[(302)] by those scientists who prefer to maintain a balanced global outlook on physical phenomena rather than accept a set of discrete compartmentalised local views. The full extent of potential near-equivalence, in a mathematical sense, between equation 4.7 and the gravity law can, however, only be appreciated through a logical questioning of the relevance in application of the commonly adopted "solid matter" premise* upon which much of the qualitative description offered above (and the associated analysis of Appendix I) was based.

In the context of customary measures, the terms $r_1{}^2$ and $r_2{}^2$ of equation 4.7 are, of course, proportional to the projected "solid" areas, A_1 and A_2, of the respective spheres in any particular direction. From a strictly conventional standpoint it would be quite inconceivable to treat these projected area terms as being linearly (or even near-linearly) related to the respective "quantities" of matter (i.e. the masses, m_1 and m_2) contained within the spherical volumes V_1 and V_2. Alternatively, the two r^2 terms might be associated with the respective surface areas of the spheres but, as with projected area, a linear transformation between surface area and mass is not normally considered appropriate to the case in point. Thus, the prevalent notional bias towards the continuum view of matter which finds perhaps its most popular expression in the mutually defined terminology of continuous mass density, ρ, and volume, suggests a distinctly non-linear relationship between m and A which involves the necessary presence of a locally qualifying, material-dependent (systematic) constant to maintain generality:

viz, $$m = [4\rho/3\pi^{1/2}]\, A^{3/2} \qquad (= \rho V)$$

The critical link between V and A,

i.e. $$V = [4/3\pi^{1/2}]\, A^{3/2}$$

---

* Unfortunately, in their most impressionable years, students of science are often faced with the necessity of indirectly accepting the false logic of absolutely solid matter. How else, for example, could the hypothetical molecule of an ideal gas, having no physical size, be contained by (and collide with) material "walls"?

, upon which the above relationship depends, is, however, peculiar to the continuum view. Any conceptual acceptance of "real" matter as an essentially open (spacious) structure - and a great weight of evidence, pertaining to the results of continued resolution at specific levels of discrimination, supports such an assertion - should lead to the obvious conclusion that other formulations are indeed possible. Numerous practical inadequacies of the simplistic continuum view as regards mass density are well known*. For example, in that field of concrete technology concerned with the topic of rational mix design, the volume of an aggregate particle is far from being an exact concept; the consequential emphasis placed by concrete practitioners on the different possible "meanings" of mass density, especially in relation to coarse aggregate, indicates an underlying recognition, albeit fairly limited, of the basic hierarchical structure of nominally "solid" matter.

A substantial easing of the path towards viable alternative interrelationships between m and A is afforded by a formal recognition of quantitative bulk material mass as merely a notional scaled sum of many individual, spatially distributed "units" which, allowing for the possible requirement of conversion factors to cope with unit diversity, may therefore be counted in a simple arithmetical manner. Consider, as an initial illustration, a spherical boundary volume V, of radius r, containing n identical particles each of projected solid area a: for such a system,

$$V = \frac{4}{3}\pi r^3$$

and

$$A \leqslant \pi r^2$$

The upper limit for A corresponds to a degree of projected overlap for the contributing sub-areas sufficient to produce an expression equivalent to that of the continuum view. A system completely without projected overlap is physically unimaginable, but as the degree of overlap decreases then the quantity A approaches the value na associated with this ideal condition; i.e. in a very highly "open" system,

$$A \simeq na$$

---

* Many processes of volume combination do not, of course, obey the simple additive law of arithmetic: cases in point include the combination of reacting gases and the dissolving of solids in liquids.

Since the total number of particles, n, is readily identifiable with the primitive concept of mass (the nominal quantity of matter), m_p, within the specific system described, the approximate equality deduced above naturally leads on to the relationship,

$$m_p \simeq cA$$

, where c is a systematic scale factor incorporating both the chosen "unit" of mass and the relevant arithmetic "rule" linking this and the quantity a.

Similar arguments can obviously be applied to open composite assemblies which contain within their boundary volumes various numbers (n_1, n_2, n_3 ..... etc.) of different particle types, these having correspondingly various individual projected solid areas (a_1, a_2, a_3, ... etc.). However, while such arguments require no further elaboration to immediately produce the extended approximate equality,

$$A \simeq n_1 a_1 + n_2 a_2 + \ldots\ldots \text{ etc.}$$

, the ultimate step necessary to effect an association of A and m_p can only be implemented on the condition that the respective sub-area quantities may themselves be taken as being somehow notionally representative of those "quantities of matter" contained within each individual particle type. (Had the study of matter revealed the existence of a single mass "quantum" this conditional requirement would become philosophically redundant: unfortunately, such is not the case!) In view of the relative imprecision which characterises the common "understanding" of quantitative mass, perhaps the simplest means of avoiding any potentially problematic aspects of the stated condition is to elevate this from its proviso status to that of a conceptual definition*. By doing so, a certain degree of consistency is automatically guaranteed; i.e. the treatment of projected solid area as being indicative of "contributing" mass at an elemental level within an open system yields in consequence an approximate "similarity" of the same two quantities at the level of aggregated bulk presence.

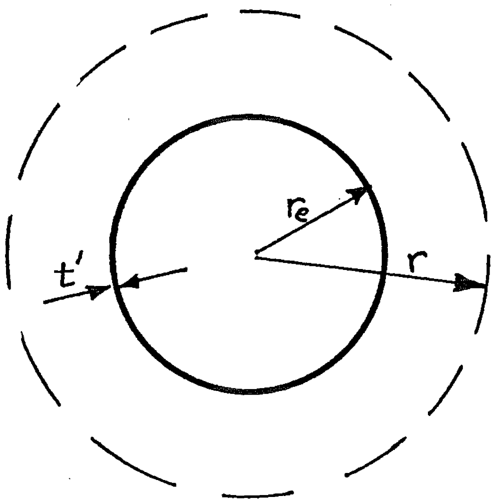
---

* The implementation of "convenient" status shifts is not uncommon within the philosophy of science. The inherent flexibility such shifts provide may be judged from, inter alia, the many interpretations and shades of meaning which Newton's Laws of Motion have sustained since their original enunciation (302).

It may have been noted (with some concern?) that the exact significance of the elemental level has not been specified; however, adopting a view of nominally solid matter which encompasses the ideas of extended hierarchical differentiation and also stipulates the existence of an essentially "open" system of "relevant" sub-areas*, the somewhat arbitrary nature of the contributing elements of mass does not, in itself, constitute a serious barrier to their notional acceptability. Of course, in hierarchical terms, the description of any projected area as "solid" is strictly relative, an absolute solid having no place within the extended schema: as an illustrative analogy consider the area of a tennis racket as "seen" by a tennis ball, as compared with that seen by a small insect, or even that "seen" by a stream of X-rays! In the model described above the quantity A is merely the net projected solid area of the sphere as "seen" by the impinging "molecules" of an external fluid-like environment. The very simple, but yet positive nature of the environmental interaction postulated only requires that the existence of a "fundamental" (primary) level of discrimination for $\underline{a}$, relative to this, be recognised. Although quite definite, such a "collision level" is only fundamental in a strictly local sense and need have no connotations of the absolute: thus, in terms of the previous analogy, while a tennis ball can not differentiate between the extent (quantity?) of a "solid" racket and that of a stringed version, a much smaller "observer" certainly can. With reference to those composite particle systems alluded to above, it should now be apparent that, for any one specific case, the degree of openness (or otherwise) will depend solely on the spatial distribution of $\underline{a}$ at the "fundamental" level throughout the pertinent system as a whole and need not therefore be governed by either the gross "size" or spatial distribution of the particles themselves unless the gross level and the "fundamental" level correspond. Thus, it should not be inferred that the gross particle level is necessarily synonomous with the elemental level already mentioned. The so-called particles may well be grouped assemblies of sub-components, the structures of which are largely open with respect to the fluid "molecules": if such is indeed the case for a mixed particle system as

---

* Since the composite atomic nucleus - as distinct from the atom itself - is not normally considered as an open structure, the primary level of discrimination pertinent to the quantitative aspects of the "actual" sub-areas involved here must clearly be taken as being below that commonly classified as "fundamental".



Mass, $m = cr_e^2$

where c is a systematic constant

FIGURE 4.12: Equivalent Shell Model

described above, each particle can be treated logically as an effective sub-system of the whole and the near-equivalence of m_1 and a_1, m_2 and a_2, etc., might be argued by simply shifting the "developed" link, $m_p \simeq cA$, to a lower level of discrimination. (Of course, such an approach does not, however, obviate the explicit requirement for either an underlying proviso or a fundamental conceptual definition at some stage in the "development" itself.) It is also pertinent to note that, in the context of a highly open assembly, the approximate equality,

$$A \simeq n_1 a_1 + n_2 a_2 + \ldots\ldots \quad \text{etc.}$$

, applies in any one direction for any bulk volume and is therefore not restricted to the spherical boundary condition originally imposed.

Despite its philosophical disadvantages the continuum view can, in effect, be modified to accommodate considerations of relative openness. Bearing in mind the degree of simplicity afforded by the continuum view in terms of analysis (e.g. converting pressure distributions to resultant forces, etc.) the immediate practical benefits to be derived from such a modification are not insubstantial. Thus, there does exist a type of spherical body for which mass and projected area would "normally" be associated - viz, the spherical shell of relatively small, constant, thickness. This leads quite naturally to the possibility of adopting an equivalent shell model based upon a resultant effective area concept, as shown in Figure 4.12. By deliberately making the equivalent shell smaller in diameter than the original sphere, recognition is given to the notion that those "relevant" areas seen by the external fluid (i.e. the projected area πr_e^2 and the overall surface area $4\pi r_e^2$) are indeed somewhat less than the "gross-body" values which would only be appropriate in the case of a truly "solid" sphere. The three-dimensional aspect of the shell serves to emphasise the distributed character of the contributing sub-areas. In turn, the finite (but relatively small) thickness shown indicates that the sub-areas themselves are derived from constitutive "elements" which are not completely devoid of physical "size"; this thickness feature also reinforces those ideas relating to the logical necessity for accepting at least a slight degree of elemental overlap. Finally, the inner volume shown in Figure 4.12 may be seen to relate (by anology) to the internal presence of the molecular environment associated with the openness of the parent system envisaged.

Of course, the equivalent shell model represents a composite abstraction (a model of a model) designed to provide immediate analytic convenience, as distinct from a general descriptive vehicle offering detailed realism. (With regard to the aspect of convenience, the use of equivalent shells in the analysis of Appendix 1 leads simply and directly to a replacement of the "r^2" terms in the resultant equation 4.7 by their respective "r_e^2" counterparts.) It is not suggested, for example, that the local geometric concentration of the would-be solid fraction of an equivalent shell be interpreted in other than a strictly "as-if" context. Similarly, the total enclosure of the inner volume should not be taken to constitute an effective barrier to continuous "communication" between the external and internal environments.

The presented arguments purporting to associate mass and aggregated sub-area, and which virtually assimilate equation 4.7 with Newton's Law of Gravitation, are effectively strengthened by those important differences commonly recognised to exist between the primitive mass concept per se and the seeming "realities" of measured mass interactions. The general principle of mass conservation has long since departed the scientific scene: numerous well known (or perhaps infamous!) experiments involving significant mass/energy conversions could be cited to confirm that measured mass does not obey the simple additive law of normal arithmetic, although in most practical cases of "matter" combination (including those which entail chemical change) the deviations from this are extremely small. Of course, in terms of purely chemical interactions, the primitive concept of mass (the notional "quantity of matter" which embraces the idea of total numbers of elementary particles) automatically guarantees the principle of mass conservation by way of definition: it therefore becomes necessary, on rational grounds, to differentiate between the primitive concept of mass, m_p, and that generally associated with measurement, namely inertial mass, m_i. Since the summed quantity A, appropriate to the "open" model described above, is largely controlled by the degree of physical overlap which occurs among the elementary sub-areas, the value of this for an effective combination of originally separate grouped systems may be greater or less than the arithmetic sum of the individual (uncombined) A values involved, depending upon the initial and final spatial distributions of contributing elements. This inherent feature of the model naturally tends to suggest the idea that A and m_i might well be considered as conceptually proportional quantities, in the conventional context of fixed interval measures; other factors further indicating the potential "suitability" of adopting (a priori) such a positive link between the two will emerge at a later stage. It may already have been noted that, for the equivalent shell of Figure 4.12, the apparent volume of "solid" matter illustrated (and hence supposedly the primitive notion of quantitative mass) is proportional to both r_e^2 and t. The lack of any absolute equivalence between the normal continuum model and the corresponding thin shell representation might therefore be attributed to the former's inherent reliance on the naive logic of the primitive mass concept - a logic which finds no ultimately consistent parallel within the realms of physical measurement. Of course, any formal recognition of a "need" to differentiate between primitive and measured mass immediately frees the equivalent shell from the requirement imposed initially that t must be considered as a relatively small quantity. However, in view of the normal lack of serious mass "discrepancies", and the prescribed openness of the simple model adopted, there is much to recommend that the notional criterion of a thin-walled equivalent shell be retained; accordingly, the initial concept will not be interfered with in that which follows.

4.2.5 An Extension of Principles

While the model developed above correctly "predicts" the unidirectional character of observed gravitational effects, it should not be overlooked that the apparent attraction forecast at the phenomenological level was seen as a direct consequence - or indeed, as a manifestation - of regional variations in the distributed magnitude of lower level "repulsive"* actions (i.e. "continuous" systematic impact forces). The prevailing variations themselves were, of course, originally associated with a statistical form of dynamic interference, the reasoning behind which was largely dictated by - or persuant to - the stochastic nature of that environmental system first contemplated. It is interesting to note in passing that the important directional aspect of unique sense displayed by mutual gravity forces is an intrinsic feature thereof frequently classified as being especially "peculiar" by field theorists[(303)]. Physical systems do exist, however, for which interactive forces of either apparent attraction or repulsion at a distance can be inferred at the phenomenological level, electrostatic systems of charged particles being

---

* Such a visualisation, being firmly based on positive ideas of interactive compression, automatically dispenses with any "need" for specific notions of absolute tension in the gravitational context: i.e. according to the fluid model described, gravitating bodies are "pushed" together, not "pulled".

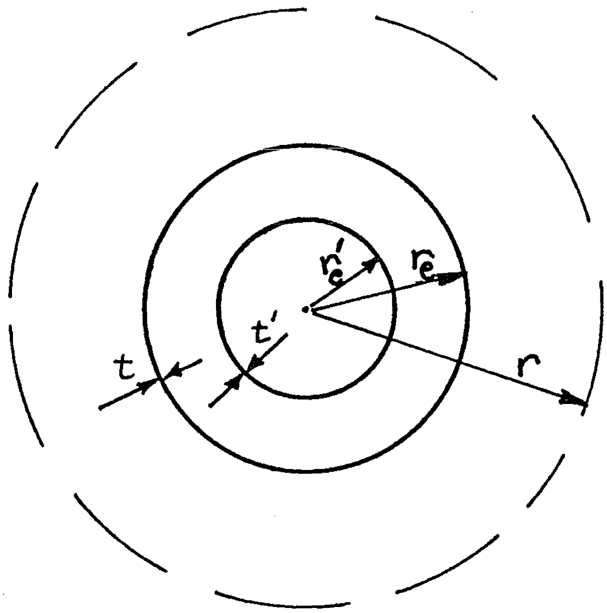
perhaps the most obvious example. If the simple fluid model is to have any descriptive generality as regards a consistent "understanding" of physical phenomena then it should also be possible to use this to "predict" the existence of such effects.

Returning therefore to the specific case of a single "material" sphere within an infinite environmental volume of the nominal "near-ideal gas", it will be recalled that the pressure of the external "fluid" upon the relatively "solid" fraction of the body was visualised, after the fashion of the kinetic theory, in terms of "continuous", but yet discrete, molecular bombardment (direct molecule/sphere interactions by way of physical contact, time-averaged momentum transfers, etc.). Although this view of the sphere's environment implicitly recognises the important aspects of both continuity and discontinuity, and the necessary degree of interplay between these, the actual concept of hierarchical differentiation, upon which such recognition ultimately depends, was only implemented at one particular level of discrimination; i.e. as a result of what constitutes strictly limited consideration, the non-molecular space was treated in essence as if it were a completely inert void, exerting no influence whatsoever on those entities within its volume. However, by adopting the extended hierarchical view which instead treats this apparent "void" as an effective lower level quasi-fluid environment (for both the molecules and the spherical body), the consequences of the initial model can now be re-applied at the foundational stage thereof, thus offering an opportunity for potential refinement to the model itself.

Such an iterative procession of ideas, whereby the initial assumptions simply comprise the nominal first estimate, does in fact lead on to some interesting areas of thought which appear ripe for further intellectual development. Thus, since the "sub-fluid", like its higher level counterparts, is supposedly amenable to differentiation, it becomes feasible to actually "forecast" those specific overall systematic effects which stem directly from the local dynamic existence of its constituent "particles" (i.e. the sub-molecules of its quasi-solid phase). For example, any first-order molecule of the near-ideal gas described will obviously be subject to "continuous" sub-molecular pressure in addition to that pressure which is derived from collisions with neighbouring first-order molecules: of course, in order to maintain consistency with the earlier model, it must be presumed that the effective "area" of the molecule as "seen" by the sub-molecules, and hence relevant to the sub-molecular pressure, is somewhat less than that "seen" by other molecules of the same order*; i.e. with respect to the sub-molecules, the structure of an individual molecule will be considered as relatively "open". The presence of a non-trivial sub-fluid also raises the possibility of interactive forces ("sub-gravity" effects) being brought to bear on the molecules within its volume. However, although the dynamic (and probabilistic) nature of both the molecular and sub-molecular fractions demands that locally unbalanced forces of this type must be considered to exist throughout the entire body of the near-ideal gas at any one "instant" in what might be termed "molecular time" (during which period an individual molecule "sees" its molecular neighbours as forming a relatively static environment), the appropriately time-averaged value of such forces on any one molecule can only be treated as a finite quantity if the equivalently valued time-averaged molecular density (number of molecules/gross unit volume) varies as some function of relative position within the system: the degree of "instanteous" existence stipulated above for the interactive sub-gravity forces essentially draws upon the same intuitive rationale as that which underlies the ergodic hypothesis of statistical mechanics, while the highlighted conditional feature pertinent to the formal recognition of resultant time-averaged effects is a direct consequence of the infinite extent bestowed upon the overall system originally envisaged. (Having referred to the dynamic nature of both the molecular and sub-molecular fractions, it is perhaps also important to stress that the locally relevant or "internal" time scales must be quite different for each; "universal" time can have little or no real associative meaning in this context of extended differentiation. Thus, for example, a so-called "instant" of molecular time is not strictly synonymous with an "instant" of sub-molecular time. In terms of analogy, the perfectly natural discrepancies which can and do occur between locally relevant time scales at different levels may be illustrated by, inter alia, the failure of the human species to appreciate the discrete aspects of fluid pressure in anything other than an intellectual sense; of course, the limited resolution of the "internal" human clock, precluding as it does any innate potential for actual physical appreciation of discrete molecular bombardment, is directly attributable to the relatively high

---

* As in the tennis racket analogy alluded to in previous arguments the character or relative "size" of an observer is not unimportant to that which is effectively "seen".



bulk volume = V = $4\pi r^3/3$

a = $4\pi r_e^2$

a' = $4\pi r_e'^2$

"sub-area density" = a'/V

FIGURE 4.13: Differentiated Shell Model

level of those particular sets of temporal circumstances upon which and from which that "clock" is based and developed.)

Now, consider the immediate consequences of extended differentiation as regards those influences likely to be experienced by the single "material" inclusion of previous arguments. First and foremost, the body will obviously be subject to an additional source of pressure - that due to the sub-molecules of the lower level dynamic environment. Since, from the sphere's position, the only significant phenomenological difference between this "extra" distributed force effect (p') and the original molecular pressure already discussed (p) concerns the particular level of relatively "solid" area upon which each is "seen" to act, it would not seem inappropriate in the interests of notional simplicity to extend the concept of the equivalent thin shell model cited earlier, as shown in Figure 4.13. Indeed, as will presently become apparent, the adoption of such a model provides a most satisfactory physical/mathematical basis from which to analyse the character, relative strength, and systematic implications of possible sub-gravity interactions between the composite body of the sphere and neighbouring molecules of its "first-order" environment.

Thus, by virtue of the plainly "relative" nature of the representative scheme devised, it can be stated initially that, corresponding to any one prevailing value of r, there must exist a particular value of r_e' (r_{eo}') for which time-averaged sub-gravity effects of the type referred to immediately above would <u>not</u> be expected; i.e. for which the distributed area of the material body as "seen" by the sub-molecules ($= 4\pi(r_{eo}')^2 = a_o'$) is identical to that of the <u>molecules</u> "contained" within an equivalent bulk volume ($\frac{4}{3}\pi r^3$) of the environment, in the absence of the material inclusion*. Of course, implicit in the rationale of such an "understanding" is the basic assumption that the differentiated environment, in its "free" state (no inclusion), may be characterised by unique single values of both molecular and sub-molecular density. These two general systematic quantities - viz, time-averaged number of molecules/gross unit volume of <u>total</u> environment, and

* The necessary condition for this "expected" lack of net sub-gravity influences, in a time-averaged sense, might equally well be expressed as a volumetric "<u>matching</u>" of resultant effective areas, at a specific level, between the composite body and its environment (a'/V must be the same for both).

time-averaged number of sub-molecules/gross unit of volume of non-molecular environment - will henceforth be designated by the symbols D_m and D_m', respectively. Since, however, in certain as yet unspecified situations, the "need" may well arise for both D_m and D_m' to be considered as variable, the particular constant values thereof peculiar to the "free" state and also to the special case where $r_e' = r_{eo}'$ (henceforth referred to as Case 1) will be seen to merit an appropriately unique form of respective notation, D_{mo} and D_{mo}'. The molecular (p) and sub-molecular (p') pressures pertinent to D_m and D_m' will be subscripted in a similar manner, where appropriate, in the subsequent text which now examines the logical consequences of the inclusion's possible non-compliance with the "neutral" criterion, $r_e' = r_{eo}'$.

If $r_e' > r_{eo}'$ (Case 2), and notional consistency with prior reasoning is to be maintained, then sub-gravity forces of apparent "attraction" must be considered to occur between the body of the sphere and neighbouring molecules of its first-order environment, their notional source being pressure/density variations within the sub-molecular system ("caused" by the same statistical shielding and interference phenomena as were alluded to in that part of the introductory presentation forming the conceptual background to the first gravity model): i.e. any neighbouring molecular "particle" will experience a finite net pressure force, via the non-molecular environment, tending to push it towards the spherical inclusion. It is worth noting that as an automatic result of system definition the overall fluid presence must remain statistically symmetrical as "seen" by the sphere. Such an inherent symmetry feature - the necessity for which is primarily dictated by the single inclusion situation at present under scrutiny - naturally legislates against the existence of any reciprocal net environmental pressure force of the reaction type being experienced by the sphere itself (i.e. $_{a'}\int p'da' = 0$*); this is not, however, like the original gravity example, a simple two-body problem and no violation of Newton's Third Law is implied. Each molecule/sphere force will be "balanced", in a statistical sense at least, by an equivalent "equal and opposite" action - also directed towards the "solid" body - on another molecule in a diametrically opposed position with respect to the centre of the sphere; the quantitative magnitude of these "balanced" sub-gravity forces will, of course, be a

* The same mathematical identity obviously applies in Case 1, where $a' = a_o'$ and $p' = p_o'$. For a single inclusion, the corresponding "molecular" identity $_a\int pda = 0$ is also quite general.

function of the particular ("instantaneous") value of radial position occupied by the molecules concerned.

Although a nominal "cause" has been invoked above, the highly interactive character of the hierarchical system described (with each of its various levels of discrimination having the inherent potential to serve as a "suitable" local reference position) strictly precludes the formal adoption of a unique cause/effect line of argument, based upon a single origin of consideration. Thus, when dealing with the immediate implications of the Case 2 situation ($r_e' > r_{eo}'$) as regards the likely nature of the prevailing molecular density in the vicinity of the material body, two alternative conceptual approaches are available. The first simply follows on from the previously established chain of reasoning which led earlier to the stipulated existence of finite sub-gravity forces acting on individual molecules,and would therefore predict regional variations in molecular density as a primary consequence thereof (i.e. as an effect "caused" by the sub-gravity forces): the second, however, is based upon the more elementary notion of sustained hierarchical compatibility and hence would treat the "induced" variations of molecular density so forecast as being <u>necessarily</u> concomitant (ab initio) to those same variations of sub-molecular density which constituted the conceptual starting point of the first causal chain employed. Each approach is naturally quite valid within the context of its own (local) rationale, the essential difference between the two being a simple matter of preferential emphasis as to the exact form of logical order deemed more appropriate (a somewhat arbitrary choice). Nevertheless, despite the possible divergence of notional interpretation - and any degree of philosophical importance which may be attached thereto - the end result of either approach is the same. In this respect at least the nominally "alternative" views actually complement one another; the ultimate reciprocity of effect highlighted in both is, of course, the predominant notional feature which, largely by design, characterises the general "understanding" of all quasi-static systems within those domains of applied mechanics purporting to describe physical phenomena. The common end-result may be expressed as the condition that systematic equilibrium (in a time-averaged sense) should prevail at <u>all</u> levels of discrimination; i.e. that the immediate potential for effective dynamic alteration of the system, concomitant to an acceptance of net sub-gravity forces on individual molecules*, must be counteracted locally through a "corresponding" variation of molecular density, the existence of which "creates" an intrinsic directional imbalance in the time-averaged number of inter-molecular collisions experienced by any one molecule.

The described local actions on individual molecules opposing sub-gravity forces may therefore, like the latter, be taken as conforming to the classification of "net pressure type", even although the specific source - and hierarchical level - considered pertinent to each is quite different (regional variations in molecular and sub-molecular density respectively). There exists a strong parallel between the ideas expounded above relating to the treatment of local "imbalance" as an effective requirement for overall equilibrium, and those of basic quantum theory where, as was mentioned in a previous section, express forms of ultimate (phenomenological) continuity are actually derived from a fundamental premise of elementary discontinuity. Bearing in mind those "discrete" aspects of the notional differentiation deliberately incorporated within each succeeding level of the hierarchical model, and also the historical background relevant to the origins of quantum theory, such obvious similarity of conceptual interplay should not, however, have been totally unexpected: the underlying physical rationale adopted by the early pioneers of the quantum approach (Plank, Einstein, et al) was, after all, founded on the implicit acceptance of a differentiated (atomic/sub-atomic) view of matter as being "suitably" representative for both descriptive and analytical purposes. Further evidence indicative of a consistent mutual correspondence of ideas could, of course, be cited. Thus, for example, by formally granting notional accord to the possibility of individual molecules experiencing finite net pressure forces, the distinct quantum overtones bestowed upon the hierarchical model during the initial stages of its development are not only maintained, but

* It may be recalled that the first-order gravity effect between two "bodies" was initially deduced from certain "properties" presumed of the hypothetical near-ideal gas in contact with both. Obviously, with regard to the molecules of a "real" near-ideal gas in the vicinity of a relatively dense body, the sub-gravity effects mentioned above (and, indeed, their immediate consequences in terms of density variation) would normally be treated as first-order gravitational phenomena, per se. However, despite appearances, no degree of inherent conflict should be inferred at this stage, since particular aspects concerning the conventional "reality" of the nominally gaseous environment of the present model have yet to be clarified.

are also essentially reinforced in a philosophical sense; i.e. had the constituent molecules of the differentiated environment not been afforded a definite physical "size" from the very outset, there would be no logical basis for the contemplation of finite net pressure forces thereon at any subsequent stage.

Having somewhat accentuated the time-averaged view(s) of systematic equilibrium appropriate to the Case 2 situation above, the essential dynamic and statistical aspects of the differentiated environment considered are perhaps worthy of re-emphasis before proceeding further. The fundamental notion of "continuous" pressure as merely a manifestation of many discrete collisions at some hierarchical boundary or other is a crucial feature of the diphase (quasi-solid/quasi-fluid) model and should be positively recognised as such. Its underlying relevance to the scheme devised is not compromised by any appeal to quasi-static principles which might be made. (The implementation of locally "averaged" quantities does not, of course, seek to effectively deny inherent statistical variation, but rather provides one means - the pun is unintentional - of allowing overall conceptual "understanding" in spite of it.) The environmental system envisaged is, by definition, self-constraining but is neither static nor strictly determinate. A necessarily high degree of uncertainty must therefore be associated with the "instantaneous" positions and temporal "paths" of individual molecules, sub-molecules, and the like. Thus, any possibility of permanent "bonding" occurring between the individual molecules of the environment and the "body" of the spherical inclusion, or indeed between individual molecules themselves by way of the sub-gravity effects described, is automatically precluded.

The systematic differences which exist between Case 1 and Case 2 are essentially "created" by the fundamental mis-match of sub-area density* which prevails within the latter; i.e. in Case 2 the effective area (a') of the spherical inclusion as "seen" by the sub-molecules is greater than that of an equivalent volume of the "normal" molecular environment. The condition $r_e' > r_{eo}'$ is, however, only a unique prerequisite for systematic change insofar as it generates a specific form of mis-match. Obviously, the converse criterion, $r_e' < r_{eo}'$ (Case 3), would also produce a mis-match situation, but of an opposite sense to that previously considered. Again, in order to comply with the logical demands imposed by an accepted need for notional consistency,

* This quantity is defined in Figure 4.13.

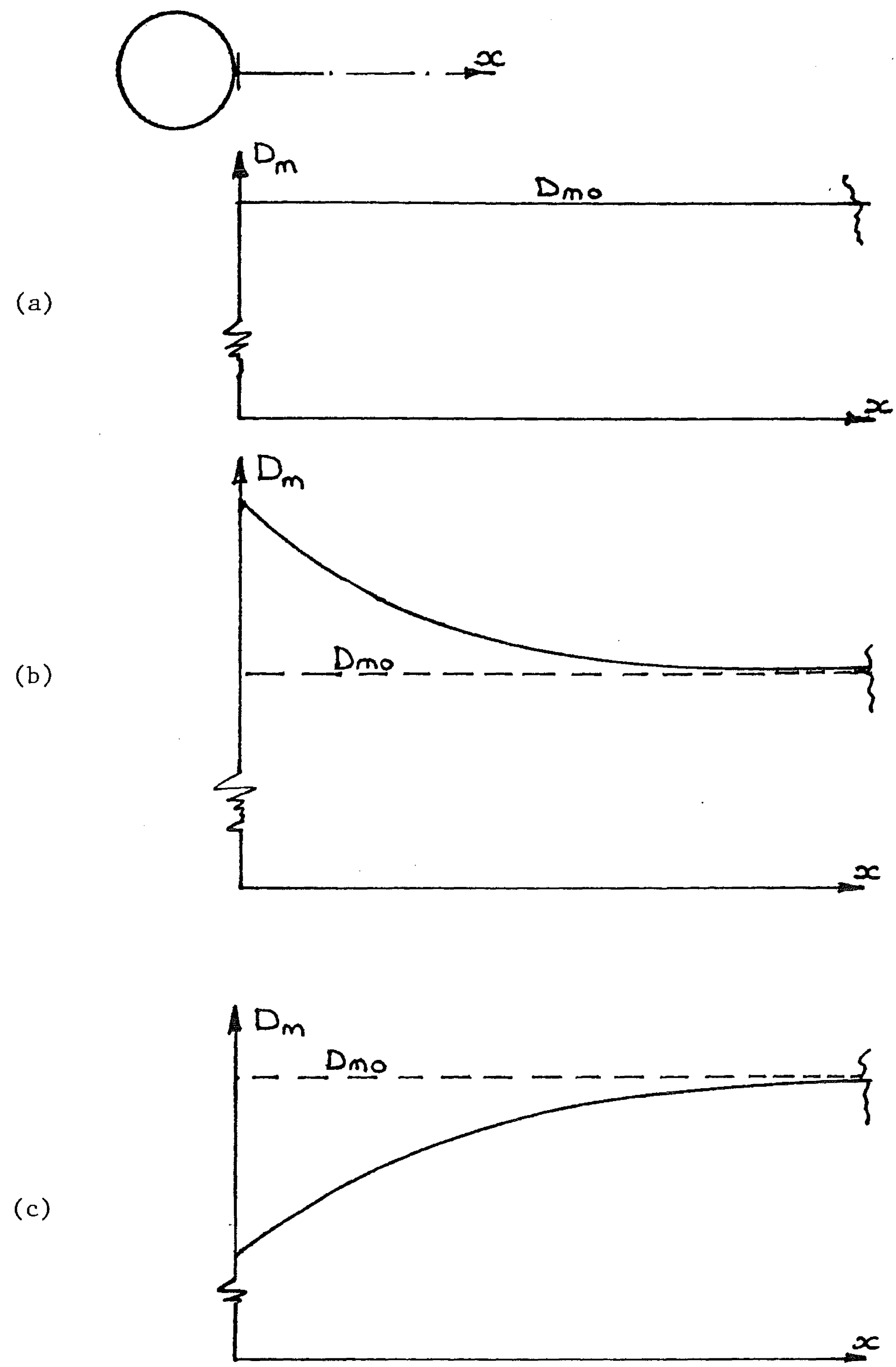


FIGURE 4.14: Variation of D_m With Distance x

(a) Case 1 body

(b) Case 2 body

(c) Case 3 body

systematic alteration of the environment from that almost "free" state originally envisaged for Case 1 must be taken to occur. In a Case 3 situation, environmental molecules in the vicinity of the material sphere will experience net sub-gravity forces, the directions of which are radially outward with respect to the centre of that symmetrical body; i.e. employing the conventional language of gravitational description, those molecules neighbouring the sphere at any one "point" in time will be "attracted" by the sphere to a lesser extent than by their more distant fellows, resulting in apparent "repulsion" from the material inclusion. As before, the conceptual requirement that overall (time-averaged) equilibrium be maintained throughout the system, at each and every level, ensures that, by whichever line of reasoning adopted as "suitable", those immediate non-equilibrium tendencies which derive from accepting the notional existence, and hence the action, of sub-gravity forces on individual molecules will be effectively balanced by appropriate (corresponding) variations in local time-averaged molecular density.

An illustrative graphical comparison, indicating the different spatial distributions of time-averaged molecular density (D_m) pertinent to each of the three cases described, is shown in Figure 4.14. Since, for two of the cases, the "instantaneous" magnitudes expected of the net sub-gravity forces on individual molecules (and, indeed, of their equilibrants) are functionally dependent on relative position with respect to the sphere, then, for those cases concerned, so too is D_m. The derivative logic of the "consequential" statement just offered relies for its overall consistency upon the application of a simple mental integration procedure. In strictly local terms it is, of course, the degree of variation in molecular density at any particular "point", rather than the absolute value thereof, which is seen as providing the counteractive influence which "balances" the appropriate net sub-gravity effect; i.e. the slope of the curves, dD_m/dx, at any particular point may therefore be taken as an implicit local "measure" of the latter. The graphical representations for all three cases naturally become asymptotic as x tends towards infinity. (This long-range feature of the specific system envisaged serves as the essential boundary condition relevant to that mental integration process mentioned above.) The quantitative magnitude of the prevailing molecular pressure, p, including the particular value thereof experienced by that relatively "solid" fraction of the material body, will necessarily depend upon exactly which of the three possible cases cited is considered to apply.

Throughout this section, from the initial foundations of the first gravity model onwards, it has been tacitly assumed that the two quantities, molecular density and molecular pressure, all but mirror one another in terms of relative magnitude. Since, however, the overall rationale of the present model leans heavily on the conceptual background of the kinetic theory of gases, it should not be overlooked that the actual degree of molecular "activity" must also have an important bearing on that local pressure likely to be experienced at any "solid" interface. For the present, and in the interests of brevity, it will simply be presumed that possible variations in the spatial distribution of molecular activity do not interfere radically with the implied general trend of "induced" changes in density and pressure both having the same sense. Thus, adopting a simple associative subscript system for the purpose of case reference, the following "relationship" emerges,

$$p_2 = p_2(x) > p_1 \ (= p_o) > p_3 = p_3(x) \qquad \ldots\ldots \ 4.8(a)$$

The conventional notation $p_2(x)$ and $p_3(x)$ signifies that both p_2 and p_3 are functionally dependent on the distance parameter, x.

Although somewhat lacking in an obvious or well documented "real-world" analogy, the apparent "anti-gravity" characteristic of the expected diminution of interactive molecular pressure associated with curve (3) is nevertheless accepted and given due recognition in certain other theoretical areas; the van der Waals correction* for the Ideal Gas Law[(304)] is perhaps the most pertinent example thereof, despite the fact that this is usually framed in terms of strictly "electrical" force interactions.

The density-related molecular pressure experienced by the material inclusion is, of course, only a partial quantity, since the body is also subject (at a different level) to the "continuous"

---

* The van der Waals correction essentially differentiates between surface pressures (i.e. those actually measured in practice) and corresponding ideal quantities. The former are reasoned to be less than those immediately forecast from an application of the Ideal Gas Law by virtue of the "additional" presence of unbalanced intermolecular forces induced systematically as a result of "missing neighbours" in the vicinity of any boundary surface.

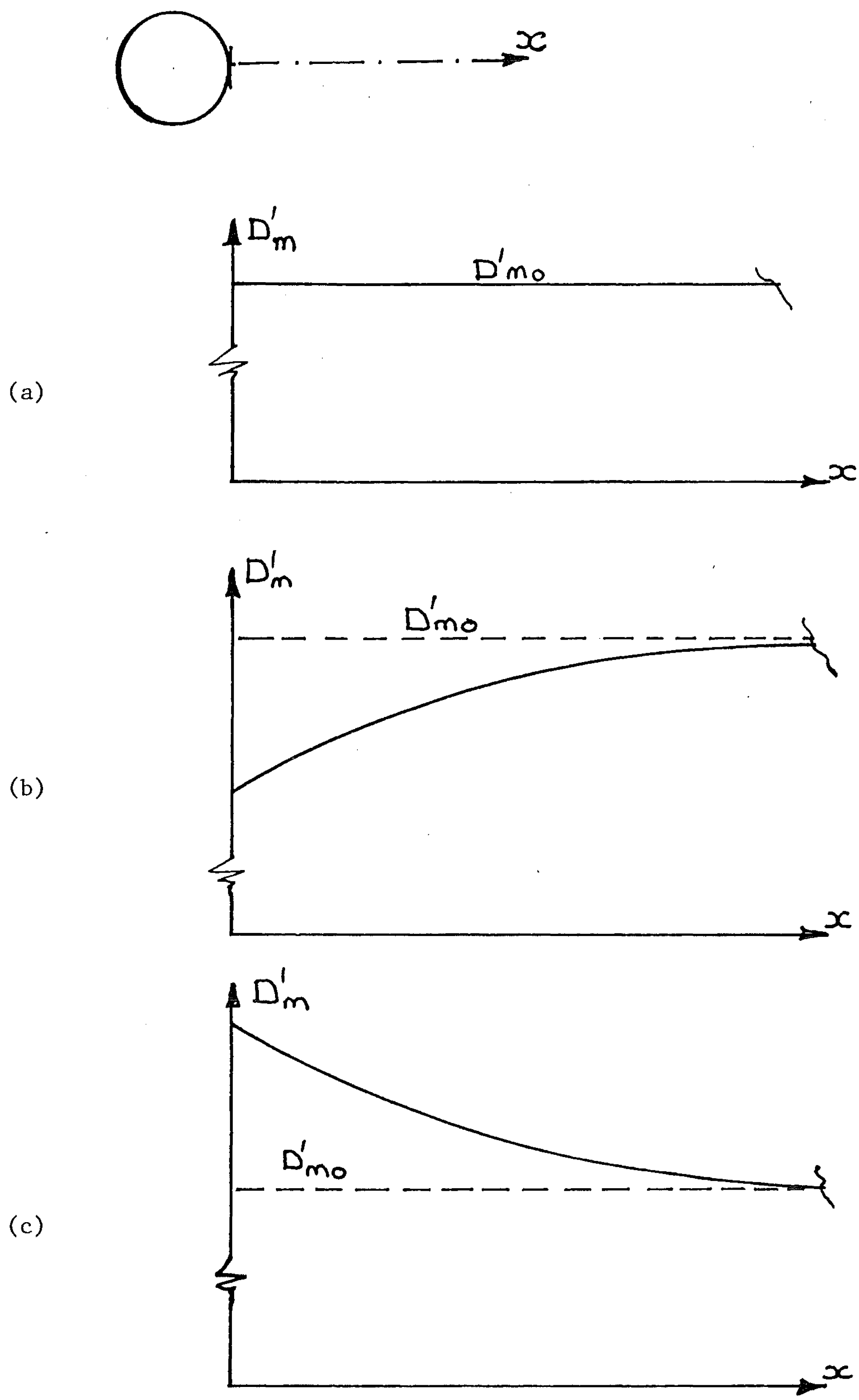


FIGURE 4.15: Variation of D'_m With Distance x

(a) Case 1 body

(b) Case 2 body

(c) Case 3 body

environmental action of impinging sub-molecules. Figure 4.15 shows the respective spatial distributions of "expected" time-averaged sub-molecular density corresponding to those mean molecular density distributions presented earlier in Figure 4.14. Similar arguments to those advanced for the "molecular" curves (1), (2), and (3), with regard to the varying degrees of relative pressure which prevail in the two mis-matched cases prescribed, can obviously be applied to the equivalent "sub-molecular" curves illustrated. As before, the resulting inferences may be expressed in the form of an appropriately subscripted inequality,

viz, $$p'_2 = p'_2(x) < p'_1(= p'_o) < p'_3 = p'_3(x) \qquad \ldots\ldots \; 4.8(b)$$

At any particular location or specific "point" within either the Case 2 or Case 3 environments, two local pressure differentials may be identified, one of which $(\frac{dp}{dx})$ relates to a non-uniform distribution of molecular density, the other $(\frac{dp'}{dx})$ to a corresponding regional variation of sub-molecular density. Since, however, both variations derive their notional existence from the same implicit requirement of overall systematic equilibrium, in a time-averaged sense, these nominally separate differential quantities can not exhibit complete mutual independence. Indeed, subject to a certain degree of constitutive definition, the precise form of this "expected" interactive association is quite amenable to relatively simple analytic techniques. Thus, having earlier stipulated that the time-averaged state of any one individual molecule neighbouring the material body may be considered as essentially conforming to the normal "understanding" of quasi-static equilibrium, the prevailing local system of "balanced" forces appropriate thereto must be governed by the fundamental principles (and axioms) of applied mechanics.

Consider the case of a differentiated spherical molecule at some distance x from a single material inclusion. (Given that the pertinent molecular "dimensions" envisaged are relatively slight compared to the reference distance, x, a working assumption as to the locally prevailing pressure gradients each being characterised by a single value may be invoked here.) From the analysis of such a case, based upon the presumed "suitability" of the concentric shell approach and linear variations of molecular/sub-molecular pressure across the body of the molecule, simple equilibrium principles (see

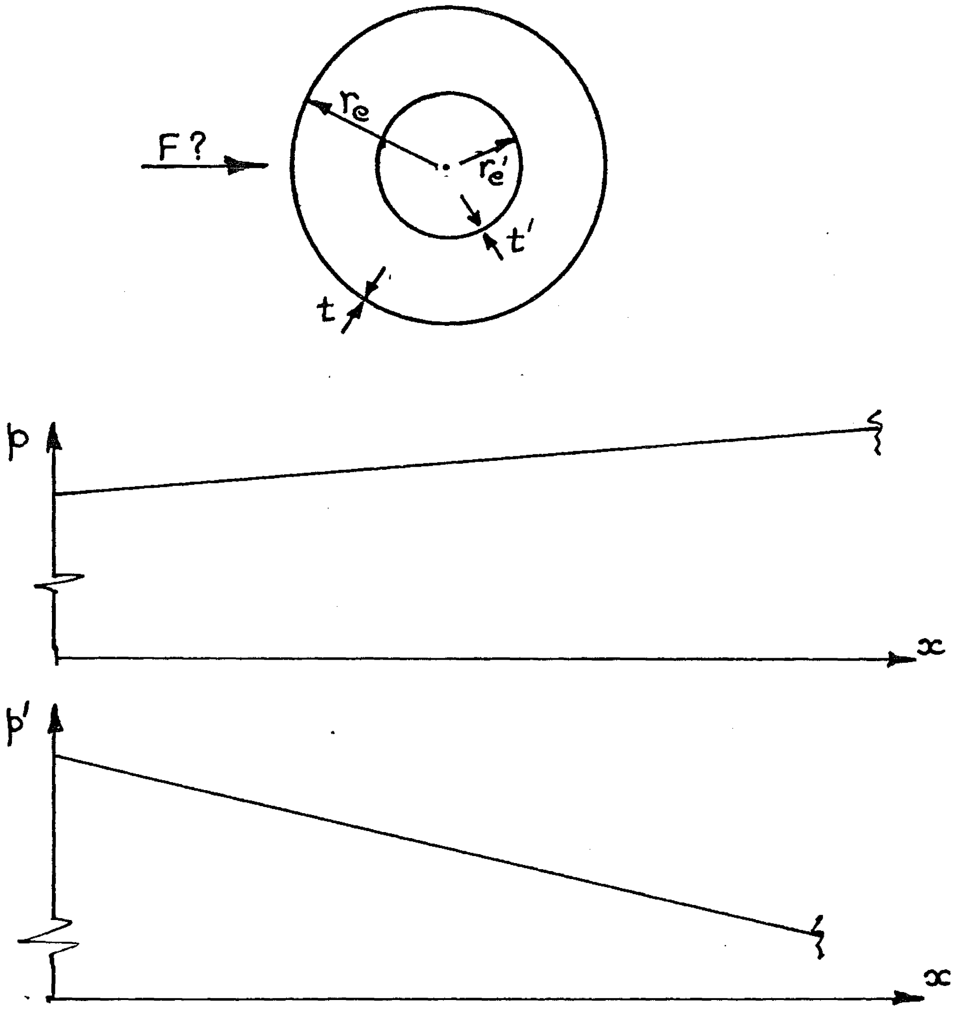


FIGURE 4.16: Equivalent Shell Model Subject to Linear Variations of p and p'

Appendix II(a)) can be shown to yield:

$$\frac{dp'}{dx} = -\Omega\frac{dp}{dx} \qquad \ldots\ldots 4.9$$

, where Ω is a systematic constant (Ω>1).

Hence,

$$p'(x) + \Omega p(x) = \text{const.}$$

But, as $x \to \infty$ then $p'(x) \to p_o'$, and $p(x) \to p_o$

Thus,

$$p'(x) + \Omega p(x) = p_o' + \Omega p_o$$

Or,

$$\Delta p'(x) = -\Omega\,\Delta p(x)$$

where, $\Delta p'(x) = p'(x) - p_o' =$ the change in p'

and, $\Delta p(x) = p(x) - p_o =$ the change in p

both due to the presence of the material inclusion

For Case 1 : $\Delta p(x) = \Delta p'(x) = 0$

Case 2 : $\Delta p(x)>0 : \Delta p'(x)<0$

Case 3 : $\Delta p(x)<0 : \Delta p'(x)>0$

Having thus established what the mere physical presence of a single inclusion can "do" to its otherwise uniform surroundings, the reaction of a spherical inclusion to an imposed quasi-static environment, devoid of that local symmetry which previously applied, may now be contemplated. Figure 4.16 shows an equivalent shell representation of a material body subject to a certain linear variation of molecular pressure and a corresponding variation of sub-molecular pressure, consistent with the quasi-static environmental condition stipulated. (The designated quantity, x, is still a relative distance measure, as before, but the origin thereof is no longer associated with the material sphere itself; i.e. the distance origin is now "external" to the sphere.) By implementing a parallel form of analysis to that used to derive equation 4.9, an expression for the resultant, F, of the two finite net pressure forces on the body, induced by the respective pressure variations, may be derived (see Appendix II(b)):

viz, $$F = -\frac{dp}{dx} \cdot 4\pi r_e^2 t - \frac{dp'}{dx} \cdot 4\pi r_e'^2 t'$$

i.e. $$F = -4\pi\frac{dp}{dx}(r_e^2 t - \Omega r_e'^2 t') \qquad \ldots\ldots 4.10$$

The immediate consequences of equation 4.10 can be envisaged in terms of three possible conditions:

Condition 1 : $r_e^2 t = \Omega\, r_e'^2 t'$

Condition 2 : $r_e^2 t > \Omega\, r_e'^2 t'$

Condition 3 : $r_e^2 t < \Omega\, r_e'^2 t'$

For an inclusion of the Condition 1 type, no resultant force would be experienced; i.e. the net molecular and sub-molecular pressure forces would "balance" one another, regardless of the degree of imposed environmental variation. If, however, Condition 2 prevails, then the direction of the finite resultant, F, coincides with that in which the molecular pressure decreases (or that in which the sub-molecular pressure increases): i.e. F acts in +x direction as shown in Figure 4.16. Conversely, the resultant force on a material body complying with Condition 3 would share the sense of the -x direction indicated. Conditions 1, 2, and 3, will henceforth be described as neutral, positive, and negative, respectively.

It may be seen that the three conditional alternatives yielded by equation 4.10 clearly mirror the three behavioural possibilities which characterise the response of a material body within an "applied" electrostatic field - hence the chosen nomenclature for the different categories of inclusion mentioned above. The conceptual basis of the former can therefore serve as a physical model for the latter, should this be deemed appropriate. Formal adoption of such a stance implies that the "molecular" pressure gradient, $\frac{dp}{dx}$, is synonomous with the electric field strength, E, and that the term $4\pi(r_e^2 t - \Omega r_e'^2 t')$ corresponds to the customary measure of quantitative charge, q. The

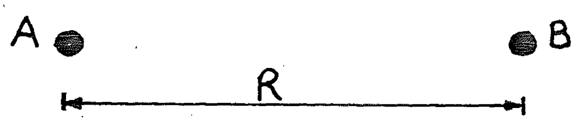


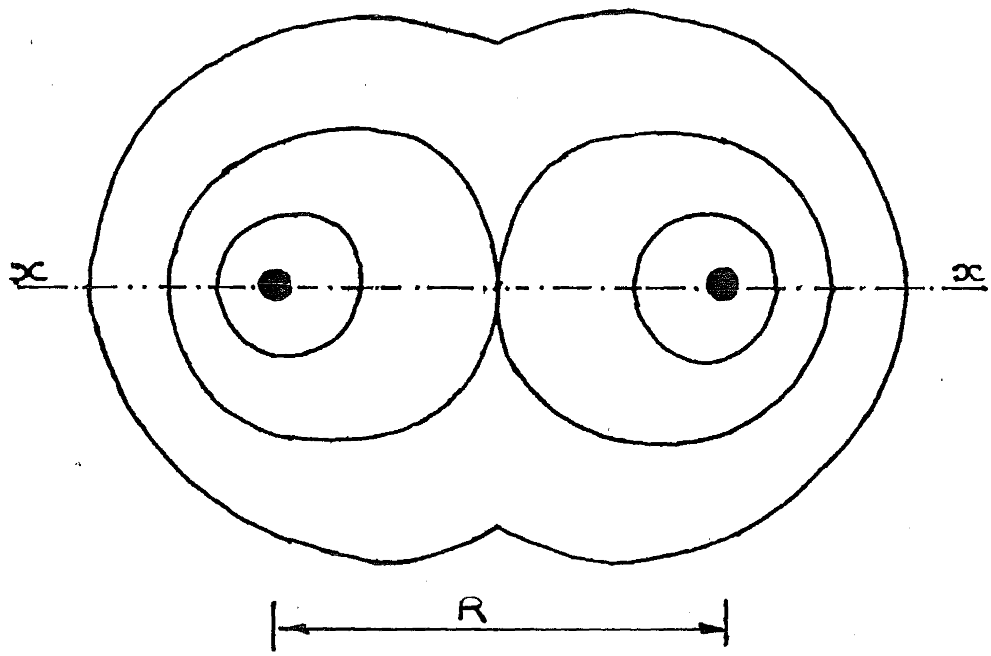
FIGURE 4.17: Two Bodies Within a Shared Environment

relatively simple physical rationale of the "new" charge parameter is worthy of some emphasis, since it embodies an underlying primary origin which bestows an element of associative meaning upon the sign of charge - cf. earlier comments regarding the vagueness of the "normal" view in this context.

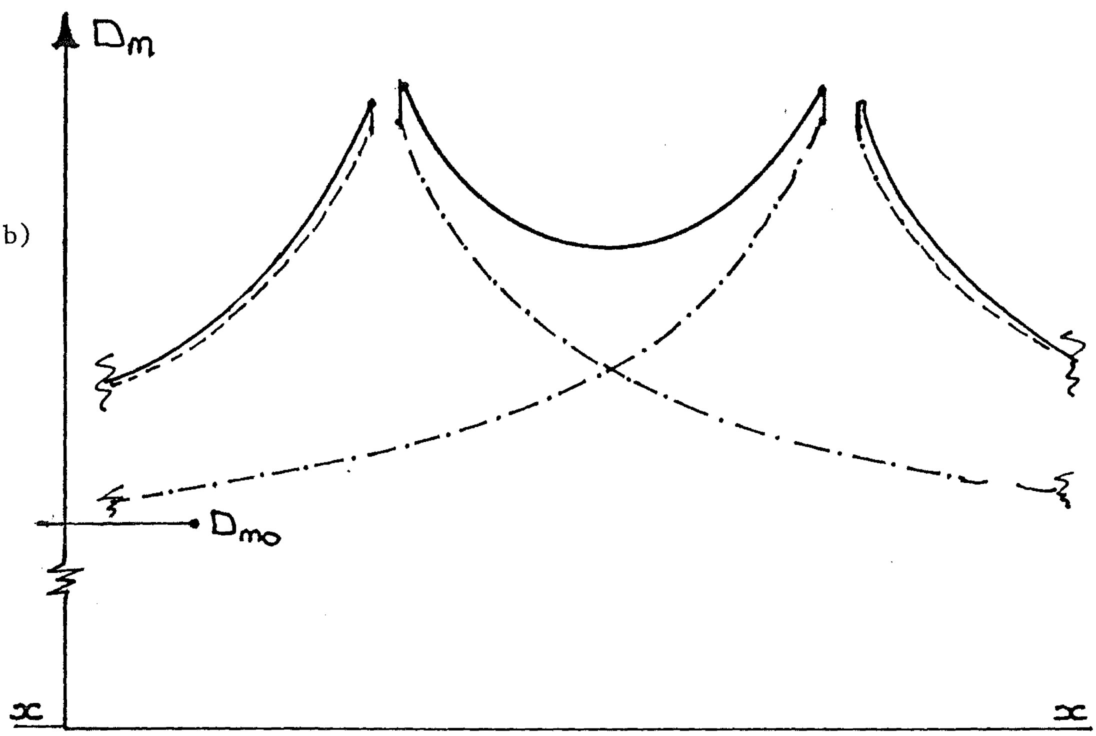
The question now arises as to the manner in which an inclusion might become subject to unsymmetrical variations of environmental pressure; it will be recalled that, while a single inclusion of either the Case 2 or Case 3 type could induce local pressure gradients, these were naturally symmetrical with respect to the material body itself. Figure 4.17 shows one obvious possibility - a situation in which two inclusions, A and B, co-exist within a "shared" environment, at some distance, R, apart. (In the arguments which follow it will be assumed that the first order gravitational interaction between the bodies is negligible.)

Consider initially that the left hand body, A, is of the Case 2 type previously described, and that the right hand body, B, is positive as defined above. Inclusion B will be subject to a pressure variation due to the environmentally "disturbing" influence concomitant to the neighbouring presence of A. The corresponding force on B (F_B) will be one of apparent repulsion from A. If the characteristic dimensions of B are relatively small compared to the separation distance, R, then the pressure gradient across B, $(\frac{dp}{dx})_B$, may be treated as a locally constant quantity, the value of which depends implicitly upon both the prevailing magnitude of R itself and the relevant "properties" of A; i.e. the quantitative magnitude of F_B is potentially amenable to solution via the use of equation 4.10. The force, F_B, can not, however, exist in isolation, since the original interactive system devised, from which this has emerged, was all but founded on an explicit "need" to recognise the concept of reciprocal effect. Thus, in order that an overall state of time-averaged quasi-static equilibrium be maintained, the body, A, must experience an "equal and opposite" force to that which acts on B ($F_A = -F_B$). Adopting the normal rule of convention that "like signs repel" implies that body A (Case 2) is effectively "positive" and that therefore body B (positive) is of the Case 2 type: if A, like B, is relatively small in physical extent compared to the separation distance, R, (i.e. if A and B are "point" charges) then equation 4.10 can be applied to both resultant forces. Similar arguments with A conforming

(a)



(b)



FIGURES 4.18: Variation of D_m Around Bodies of Equal Positive q

(a) Lines of equal D_m

(b) Variation of D_m along x

(Solid curves indicate net effects: dashed curves show variations due to either body in absence of the other.)

to either the Case 1 or Case 3 designation would yield a corresponding mutual association between Case 1 and Condition 1 (neutral), and Case 2 and Condition 2 (negative), respectively.

For two "point" charges, A and B, at distance R, equation 4.10 gives:

$$\left.\begin{aligned} F_A &= -\left(\frac{dp}{dx}\right)_A q_A \\ F_B &= -\left(\frac{dp}{dx}\right)_B q_B \end{aligned}\right\} \quad \text{where } q = 4\pi\,(r_e^{\,2}t - r_e'^{\,2}t') \qquad \ldots\ldots 4.11$$

But, since $F_A = -F_B$

$$\left.\begin{aligned} \text{, then } \left(\frac{dp}{dx}\right)_A &= Cq_B \\ \text{and } \left(\frac{dp}{dx}\right)_B &= -Cq_A \end{aligned}\right\} \quad \begin{array}{l}\text{where C is a local constant} \\ \text{("fixed" R)}\end{array} \qquad \ldots\ldots 4.11(a)$$

The nature of the differentiated external environment shared by two separate charged bodies will necessarily be influenced by the physical character and relative "properties" of <u>both</u> inclusions. There will thus be an important element of underlying compromise with regard to the quantitative and directional aspects of local net sub-gravity effects and the like. The final "balance" may, however, be inferred through a simple mental combination of those <u>individual</u> influences previously contemplated (see Figures 4.14 and 4.15), this logical "summation" process being effected in such a manner as to guarantee the inherent pre-requisite for overall consistency.

Consider, for example, the likely distribution of local molecular density, D_m, around two <u>small</u> charged bodies of equal positive q. Figure 4.18(a) shows this in a two-dimensional representation as "lines of equal density" within the vertical plane. (Mathematical "surfaces" of equal time-averaged density could obviously be produced by simply rotating the given series of curves about the straight-line axis xx which passes through the centre of both spherical bodies.) An infinite set of essentially one-dimensional (linear) representations, each displaying the important feature of <u>statistical</u> "continuity", is equally viable; Figure 4.18(b) takes the particular form of one such "continuous" distribution - that along the central axis xx; as an aid to comparative reasoning, the respective molecular density distributions which would be associated with the surrounding environment of either sphere in the absence of the other are also included, these being designated by the appropriate (locally symmetrical) dashed curves. Two important qualitative aspects of the diagrams which follow on directly from the established basis of prior arguments are deserving of some note. Firstly, the immediate region between the spheres is an environmental "transition" zone of strictly limited extent and is hence characterised by relatively "high" molecular density. Secondly, the changing slope of the molecular density/position curve of Figure 4.18(b) is consistent with expected differences in the "instantaneous" magnitude of the net sub-gravity effect* on individual molecules; a brief visual comparison of the full and dashed curves should serve to confirm this latter aspect, the central point of zero slope on the full curve between the two bodies being especially significant. Corresponding "sub-molecular" curves could also be deduced along similar lines. Bearing in mind the prescribed link between pressure and density - a constitutive facet of the model crucial to any extension of its fundamental rationale - the general form of the "resulting" environment forecast by Figures 4.18 is, of course, in complete accord with that physical concept of an electric field adopted above.

In the absence of either charged body, there would be no pressure gradient across the other (i.e. a locally symmetrical field would prevail). This feature, which might logically be termed a "null-effect" of sorts, was in fact alluded to during the earlier discussion of solitary inclusions of either the Case 2 (+ve) or Case 3 (-ve) types: of perhaps most significance in the present context is the important characteristic that, where appropriate, this conditional null-effect must pertain regardless of the relevant charge value, q, involved. Thus, although the total quantitative magnitudes of the partial pressure distributions to which A and B are notionally subject are necessarily controlled by the intrinsic "balanced-influence" principle, those net pressure gradients from which the forces F_A and F_B actually derive their

---

* As was mentioned earlier, the slope quantity, dD_m/dx, may be taken as an implicit local "measure" of the prevailing time-averaged net sub-gravity effect.

"being" are not. For any particular value of q_B, the quantity $(\frac{dp}{dx})_A$ is therefore quite independent of q_A itself: similarly, for any one value of q_A, $(\frac{dp}{dx})_B$ is independent of q_B. Or, more specifically,

$$(\frac{dp}{dx})_{A\ q/q} = (\frac{dp}{dx})_{A\ o/q}$$

and

$$(\frac{dp}{dx})_{B\ q/q} = (\frac{dp}{dx})_{B\ o/q}$$

where $(\frac{dp}{dx})_{A\ q/q} = (\frac{dp}{dx})_A$ for $q_A = q$, $q_B = q$, etc.

and $(\frac{dp}{dx})_{A\ o/q} = (\frac{dp}{dx})_A$ for a <u>single</u> charge, $q_B = q$, etc.

The right-hand sides of the two systematic equalities presented above have been chosen deliberately of course, since these relate to the somewhat "special" cases of respective isolation already examined. As was previously highlighted, the regional variations of molecular pressure, $\frac{dp}{dx}$, appropriate thereto, <u>and</u> the corresponding sub-molecular pressure gradients, $\frac{dp'}{dx}$, may each be attributed to the immediate consequences of a certain "mis-matched" condition, whereby the physical nature of the single inclusion concerned is fundamentally incompatible with that of the normal (free) environment; i.e. the "strength" and direction of the local sub-gravity effects (and of their equilibrants) so created may be simply envisaged as the results of an "induced" systematic response governed by both the degree and sense of the prevailing mis-match. Thus, having formerly associated in this manner the transformed environmental quantity, $\frac{dp}{dx}$, for the case of a single inclusion, with a category of physical interaction which involves an appeal to a nominal derivative of the <u>gravity</u> concept, the explicit requirement that the local constant, C, in equation 4.11(a) must be inversely proportional to R^2 in order to secure an overall equivalence between equation 4.11 and the empirical electrostatic law of Coulomb, can hardly be classified as "unexpected".

In the light of earlier comments on the continuing trend towards the abstract which epitomises 20th century science, it is perhaps appropriate to note that the obvious similarity of form shared by the gravity and electrostatic "laws" was once considered an extremely important factor, especially by those such as Priestley and Faraday whose powers of physical reasoning first shaped the growth of rational scientific "understanding" as regards the latter. The proponents of more "modern" science, who now typically adhere to the abstract field approach, have seen fit, however, to sever any notional link between gravitational and electrostatic forces which might possibly be construed as indicating an element of positive association in a physical sense*. This conventional relegation of form similarity to the level of almost fortuitous coincidence is not, of course, dictated by any demands born of strict logical necessity, but rather by the adoption as "suitable" of a quite arbitrary preference; given that the underlying rationale of those well-known indirect experiments which have established the "accuracy" of the electrostatic inverse square relationship to within exceptionally fine limits was originally based upon a gravitational analogy, the ultimate wisdom of such a choice is not beyond question. (The indirect experimental apparatus of Plimpton and Lawton which, in 1936, yielded the electrostatic power exponent as lying within the range $2 \pm 10^{-7}\%$ is merely a somewhat refined version of the "silver pint cann" first used by Benjamin Franklin almost two centuries before: it was, of course, from Franklin's results that Priestley actually deduced the electrostatic "law" in 1767, thereby pre-dating by nearly 20 years Coulomb's memoir to the French Academy of Sciences involving its direct verification.)

It may be seen that Figures 4.18 exemplify the important principle of limited interference, so crucial to the formal implementation of practical electrostatics. Thus, while the intruding presence of a small charged body necessarily alters the overall character of any pre-existing field, the interactive force to which the former is subject may be used as a consistent measure of the latter at the "point" occupied by the intruder.

Partial analogies of the electrostatic model can be developed by examining the "hydrostatic" response of an infinite volume of a real near-ideal gas to the gravitational influences of an inclusion having any particular mass density characteristics. The writer has been able

---

* The search for consistently general mathematical links goes on, but, despite many interesting developments, the formulation of a sensible unified field theory still appears remote. In many ways the seeming elusiveness of the latter might well be likened to that of the 19th century aethers!

to show that the "equivalent" versions of equation 4.11(a), with C being inversely proportional to R^2, give rise to pressure distributions for such cases which are compatible with the mutual satisfaction of elastic strain energy and thermodynamic considerations (see Appendix III).

4.2.6 Molecular Status

Having now presented both the gravity and electrostatic models, the potentially tortuous question of comparative molecular "reality" can be broached (with minimum complication) by effecting a slight refinement of the somewhat loose descriptive language so far employed. Within the context of the extended hierarchical scheme, the so-called "molecules" of the background environment postulated are no more or less real than the conventional (recognisable?) molecules of say, carbon dioxide. In these terms, earlier references to conventional molecules as "real" and to background molecules as "hypothetical" carry little significance. However, the associated levels of relative existence seen to be occupied by each are quite dissimilar; i.e. a rational basis for effective distinction between molecular types does prevail. Thus, although envisaged as being inherently substantive (non-trivial), the "new" molecules can not be designated as "matter" in the customary sense, since, by definition, they form part of an external environment which constitutes the hierarchical complement to matter itself. As the notional source (rather than the subject) of first-order interactions between material bodies (including conventional molecules), they necessarily lack the qualitative attribute of primary gravitational mass, that most characteristic property of all matter*; were this not so, the initial logic of implementing the matter/non-matter differentiation could not be sustained in a consistent manner. Of course, from the more normal view, based upon a typically unquestioning acceptance of "empty" space dogma, the idea of "real" non-matter represents

* Uncertainties with regard to the gravitational mass of a neutrino leave the status of this enigmatic "particle" very much in limbo.

complete antithesis and is therefore tantamount to a form of "scientific" heresy. However, given the purely arbitrary nature of the "absolute void" premise underlying the latter, such inevitable conflict between fundamental ideas (a simple clash of opposing methaphysics) should not present a significant intellectual barrier to the contemplation of viable alternative developments. (Emotional barriers, born of familiarity and resistance to change, may unfortunately prove to be more formidable.) It is pertinent to note that the implicit substantive aspect first bestowed upon the "new" molecules during the earlier stages of conceptual development is essentially reinforced by extending the interactive principles of hierarchical differentiation to include lower levels of the "real" non-matter complement: i.e. the "new" molecules are also seen as having a non-trivial external environment, and hence possess what might reasonably be termed <u>secondary</u> gravitational mass.

4.2.7 <u>And Beyond?</u>

Perhaps the most embarrassing feature of the hierarchical scheme proposed is the considerable potential offered by its obvious generality. There would, for example, seem little to suggest that its immediate suitability for the purposes of consistent physical "explanation" is necessarily limited to the confines of the simple gravity and electrostatic models described in prior sections. While it is outwith the primary aims of the present work to examine the broad field of further possibilities in any great detail, a number of relevant points and comments are worthy of special mention at this juncture.

(i) The "basic" (ideal) electrostatic model deliberately ignores any finite contribution from the environmental first-order gravity effect between "point" material bodies. Its ultimate forecasts of <u>net</u> resultant forces are therefore somewhat restricted in application; i.e. additional allowance must be made for appreciable gravity effects, should these also be expected to prevail. One particularly interesting case arising from this "extra" consideration is that which concerns two discrete bodies within <u>very</u> close proximity of one another.

In the specific set of circumstances to which the given electrostatic representation purported to refer, the constitutive assumption of a negligible first-order gravity effect was, of course, quite reasonable. Coupled with those extreme statistical aspects afforded to the initial gravity model itself, a "designed" combination of small primary masses, significant charges, and relatively long-range separation distances ($R>>r$), guarantees adequate local justification. Indeed, the ensuing rationale, whereby the mere presence of a "mis-matched" (charged) material body was associated with extensive environmental disturbance, might well be seen as serving to emphasise the relative weakness of the first-order interactive "mechanism" originally postulated. At first sight it would therefore appear that an "addition" of the appropriate first-order gravity component to the restricted forecasts of the ideal electrostatic model should entail only minor quantitative modifications to the predicted overall effects. However, despite its inherent long-range viability, the supposition of small change (and the degree of understanding it accommodates) is completely untenable in the context of a very close proximity situation. The extra gravitational component is not universally "weak" by prescribed definition; to presume it so would be erroneous. Mention was in fact made during the preliminary discussion of the gravity model that there must exist a systematic lower limit for R below which the "extreme" condition of constant "tail pressure" - the important proviso governing the descriptive suitability of the inverse square law, equation 4.7 - is no longer valid. Thus, in any close proximity situation for which $d<x_2$ (see Figures 4.10 and 4.11), a relatively "strong" gravity component, capable of radical interference with strong electrostatic/weak gravity predictions, becomes a distinct notional possibility. The implications of this radical interference as regards establishing a sensibly consistent "explanation" of nuclear bond forces should be abundantly clear. Another domain in which the idea of a short-range strong gravity component has obvious connotations is that of physical contact bonding (van der Waals forces).

(ii) During the development of the differentiated electrostatic scheme, two quite separate conditional representations were examined in turn. Although a notional combination thereof was eventually effected, no unified physical model for a charged material body - incorporating the joint attributes of the two would-be alternatives - was in fact offered. The overall compatability requirements imposed by such a convenient assimilation of the various "Case" and "Type" bodies undoubtedly provides scope for further study. (A more definitive single "picture" could well have many conceptual advantages.) It may be recalled that, unlike their "Type" counterparts, the "Case" bodies were specified in terms of a relative condition which did not invoke any immediate associations with primary mass*. As an inherent partial feature of a combined model, this particular aspect of local charge/ mass independence embodies significant potential for consistent "explanation" at the so-called "elementary" level of conventional physics; it is, of course, a well known characteristic of sub-atomic particles that their charge is quantitised to a much finer degree than is their mass.

(iii) Despite having placed frequent emphasis on the extended nature of the generalised hierarchical system envisaged, neither of the interactive models presented take the fundamental principle of progressive differentiation very far. However, it should not be thereby inferred that the models are necessarily incompatible with an extended hierarchical scheme. Thus, for example, the limited molecular considerations of the two-body gravity model are certainly within the bounds of justifiable reason, bearing in mind the prescribed statistical basis which underlies its long-range "mutual shielding" rationale; i.e. lower level environments, below that of the first-order molecules, would "see" the composite material bodies as merely constituting two elements of an infinite system of discrete "inclusions" and, in this context, effective long-range submolecular shielding is virtually inconceivable. With appropriate submolecular "design", similar lines of argument, by which the potential

* In all three "Cases" ($r'_e/r'_{eo} = 1$, $r'_e/r'_{eo} > 1$, and $r'_e/r'_{eo} < 1$) the quantitative primary mass, m, was seen as proportional to $r_e^{\,2}$, where $r'_e < r_e < r$. Apart from a systematic lower limit of unity, no restriction was laid on the ratio r_e/r'_e.

contribution of the "lower levels" is essentially deprived of significant local relevance, may also be applied to the electrostatic representation*. It is not suggested that environmental pressure gradients, other than those described as being pertinent to the electrostatic model, do not (or can not) "exist", but rather that, as a possible source of action on a composite material body, any hierarchical variation within the differentiated non-matter complement is subject to defined (chosen) systematic constraints. While more complex interactive models could undoubtedly be created through exercising alternative choice, the deliberate sacrifice of notional simplicity without due cause has little to recomment it: cf. earlier comments on dualism.

(iv) The preliminary single material body of the original gravity model, like its later electrostatic equivalent, was considered to occupy a static position with respect to the differentiated external environment - hence the natural all-round symmetry of active pressure distributions and the like. This aspect almost begs the question as to whether the logical consequences of possible non-compliance with such an ideal condition do not necessarily compromise the claimed general "suitability" of the overall representation. Given a statistical "picture" of the environment, the concept of relative (body/environment) motion poses no great difficulty in itself; the associated loss to the body of its earlier symmetrical "surroundings" is, however, distinctly more problematic. (Historically, the absence of observable "drag" effects in the specific context of planetary motion has always constituted a major stumbling block to the realistic acceptance of an external medium**.) Had the discrete components of the "unrecognised" environment been formally designated as matter, a serious clash of theory and related experience would be inevitable; i.e. in the conventional language of elementary mechanics, a state of uniform velocity would require the application of an "impressed" force to counteract the drag resistance induced on the body concerned by the varying relative motion of its dynamic surroundings. Observation does not substantiate such a requirement which, if valid, would essentially guarantee implicit recognition of

* All theories rely on a measure of convenient "tailoring" at some stage of their development. Such is the hallmark of science!

** Of course, by converse reasoning, more modern ideas alluding to the propagation of waves through "nothing" must be treated with equal suspicion, unless only a one-sided doctrinaire form of scepticism is regarded as healthy.

the external environment! (It is pertinent to note in passing that the term "drag" has unfortunate tensile connotations which are _not_ borne out by the normal understanding of the physical effect it serves to describe.)

While the positive stipulation of a differentiated (multi-level) _non-matter_ complement precludes the automatic rejection of an external environment on the grounds of kinetic incompatibility, this is hardly sufficient to justify it as immediately suitable beyond the static context. (It does, however, tend to cast a somewhat different light on the would-be "obvious" character of the notional stumbling block referred to above.) Further development of the material body/environment scheme is undoubtedly required before any real degree of confidence as regards "understanding" the consequences of relative motion could be established. Nevertheless, with a view to ultimately "forecasting" the absence of drag phenomena, a number of relevant points emerge from previous arguments. Thus, for example, although prescribed relative motion necessarily removes the erstwhile viability of considering a single material body as being subject to the influences of an isotropic environment, there would seem no compelling reason to presume that this _interactive_ change must be seen to manifest itself in the form of an unbalanced net pressure force; cf. the null reaction of a neutral body to the local imposition of an electric field. Furthermore, the static model also made reference to effective (probabilistic) areas, at different levels of internal structure, within a material body. Any extension of the model to accommodate ideas of relative motion would demand a critical reappraisal of such area concepts. Probabilistic areas with an inherent dynamic quality*, are not insensitive to the kinematic variations of interactive "observers". (As an illustration of this principle, compare the effective area of a rotating spoked wheel as "seen" by an impinging high-speed bullet with that "seen" by a similarly sized but relatively low speed "observer".)

If the schematic alteration concomitant to relative motion does not manifest itself as drag, then alternative forms of "reaction" must be considered to occur: the model would be quite trivial otherwise. Such _expected_ systematic change finds no place in classical mechanics but is, of course, very much part of modern physics (mass, length, time transformations, etc.).

---

* Earlier descriptions placed no restriction on the nature of the internal effective areas. Taking into account that which is already known about constitutive matter, an assertion of locally dynamic properties is far from contentious.

(v) The extensive environmental disturbance associated with the static "presence" of a single charged material body is especially significant to any contemplation of further possible effects due to relative motion; i.e. a "moving" non-uniform disturbance necessarily gives rise to the concept of environmental flow (flux?). Thus, it might well be stated that the hierarchical model essentially "predicts" (albeit in a very crude manner) the physical phenomenon of magnetic field induction. (With regard to the magnetic properties of matter, the apparent absence of monopoles serves to emphasise the potential suitability of the flow "picture".) Since the external environment is itself differentiated, it can sustain the simple notion of complementary internal flow, commensurate with the maintenance of local hierarchical compatibility.

4.2.8 A Brief Reflection

Many of the arguments so far advanced in favour of the "new" model have appealed to philosophical considerations. Some retrospective comment on the need for such recourse to philosophy would therefore seem apt.

The somewhat stifling classical links between science and philosophy which once existed - originally forged within the Ancient Greek Schools - have long since been severed, and rightly so. While not wishing in any way to see these re-established (to the probable detriment of both) the writer would maintain that the resulting "freedom" from the arbitrary constraints of speculative philosophy should not - as often seems the case - be regarded as an open license for the justification of ambivalence and/or logical inconsistency. Science claims to deal with models of reality (the pygmalion syndrome notwithstanding) and thereby creates its own local philosophy either directly or indirectly. The writer does not seek to contest the apparent suitability of the mathematics proferred by the modern scientific community, but rather the implied philosophy of the new Establishment with regard to "fields" devoid of substantive associations.

As was highlighted earlier, the concept of a substantive external medium is no more or less metaphysical than the assertion of an absolute void. It does, however, have many features which commend it to the physical thinker. Consider, for example, the phenomenon of "radiation pressure", first observed by Fresnel[305] some 150 years ago. Given the notional premise of a "real" non-matter environment, a sensible rationalisation of Fresnel's largely unheralded* discovery that small "hot" bodies repel one another, even in a nominal vacuum[306], presents no problems of consistent interpretation: i.e. if a material body interacts with its non-matter environment the effects of its presence may be transmitted to other bodies which share these external surroundings (cf., the electrostatic model - governed by a different form of interaction, but essentially the same principle). On the other hand, the modern "explanation" of this action at a distance requires a convenient (but hardly plausible) shift from the waves-in-nothing "picture" to that of "packets" of nothing! The heuristic photon, as a prescribed energy packet, defies substantive classification, despite its frequent inclusion in the lists of elementary "particles" provided by introductory physics texts. The concept of energy as a substance per se (as distinct from a systematic property) died its first death with Rumford's experimental refutal of the caloric theory; many more have since followed[287, 307].

Although loose descriptions which invoke the "flow" of heat tend to perpetuate the caloric myth, they are, of course, relatively harmless providing a more rational description ultimately prevails. According to the continuous "nothing" philosophy of modern electromagnetic field theory the phenomenon of radiated heat must be treated separately from the phenomena of conducted and convected heat, since the latter ostensibly pertain to physical forms of systematic energy transference. The stipulation of a differentiated external medium, other than mathematics, removes this arbitrary distinction and imparts a background system to which

---

* Apart from eliminating certain possibilities, Fresnel was not prepared to speculate further on the would-be source of these "calorific repulsions". Taking into account the historical timing of his finding, his guarded reticence was, however, quite understandable. As Cardwell(294) so astutely remarks:

> *Twenty or thirty years earlier these observations would have been hailed as positive and final confirmation of the truth of the orthodox caloric theory; in the climate of opinion of the 1820's they were of little more than technical interest.*

radiated heat can sensibly refer. A substantive field environment also gives associative "meaning" to the concept of energy density, the Poynting vector, etc.

The physical implications of the alternative hierarchical scheme may therefore be seen as having the potential to de-mystify the mathematical "wonderland" of relativity. Even a simple physical "understanding" of the creation (anti-particles) and annihilation of matter is not beyond conjecture. These aspects are, however, beyond the scope of current arguments but, in the light of previous comments, one immediate consequence of Einstein's famous $E = mc^2$ deserves a brief mention. The equation of energy/mass equivalence* asserts that the mass of a material body is increased through heating. While the traditional view of mass as an absolute "quantity of matter" can obviously not sustain such an interpretation, the primary mass/effective area rationale of the hierarchical model is thereby unimpaired (if not strengthened); i.e. an increase in the dynamic component of the primary effective area is an <u>expected</u> (defined?) manifestation of the heating process.

The "new" model undoubtedly resurrects some old ideas, as may be judged by a quote from Whitaker[280] on Newton's largely <u>undeveloped</u> aether notions:

> *This aether pervades the pores of all material bodies and is the cause of their cohesion; its density varies from one body to another, being greatest in the free interplanetary spaces. It is not necessarily a single uniform substance but just as air contains aqueous vapour so the aether may contain "aetherial spirits" adapted to produce the phenomena of electricity, magnetism and gravitation.*

Newton did not of course consider electromagnetic radiation (or, more specifically, light) within an aether context: he took light as a quite separate phenomenon by which corpuscles ("old" photons?) were propagated from "lucid" bodies. The distinct historical precedents are perhaps unfortunate in that scientific references to the past generally allude to theories which have outlived their usefulness and hence been discarded (e.g. phlogiston, caloric, etc.). In a scientific society dedicated and accustomed to "natural" advancement, conceptual revivals are not notable for their frequency. However, as even Truesdell[308] - ever the mathematician - has stated:

> *We do not have to equate "progress" with every* $\delta f(t)$ *if* $\delta t > o$, t *being the time.*

---

* $E = mc^2$ is most definitely an equivalence, <u>not</u> an identity.

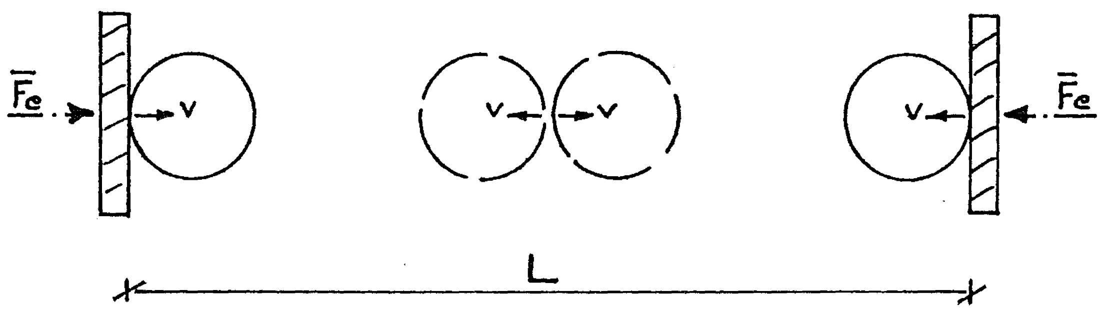


FIGURE 4.19: A Simple Dynamic Arrangement

The adoption of a physical rationale (thinking in objects) is itself somewhat old-fashioned. The abstract axiomatic approach so favoured by Truesdell has infiltrated most former bastions of physical thought, including Engineering Science. Nevertheless, the author would assert that, as a useful model of reality, a mechanics devoid of physical reasoning is all the poorer for it; in this, he shares the view of Aristotle that mathematics alone is not enough. Or, to quote Lande[291]:

> *Skepticism about thinking objects is an old story. Every ostrich and every babe is a Berkeleyan idealist to whom things exist only when seen, and discontinue to exist when covering his eyes.*

4.3 THE MODEL IN APPLICATION

4.3.1 A Simple Two-Body System

To demonstrate some of the principles which have been raised above, the interactive properties of an extremely simple composite system will now be examined briefly. Figure 4.19 serves to define the actual system involved. The inherently dynamic arrangement illustrated comprises two "hard" spherical bodies of equal diameter, d, equal mass, and "instantaneous" equal-and-opposite velocity, v: the extended motions of the bodies are restricted internally by mutual elastic collision and externally by symmetrical "hard" barriers. The mean (time-averaged) value of the external impact forces provided by the barriers is symbolised as $\overline{F}_e$.

If the absolute value of v is considered a constant quantity <u>within</u> the system, the "linear" model is strongly analogous to those typical representations offered by elementary kinetic theory from which the derivation of the Ideal Gas Law (with or without the Clausius correction) is ultimately effected. In such a case, it can therefore be shown with relative ease that the constraining barrier forces, $\overline{F}_e$, are directly proportional to the combined travel distance, L-2d. Also, as the only external forces prescribed in this instance, $\overline{F}_e$ may obviously be equated with the mean (time-averaged) internal impact force, $\overline{F}_i$.

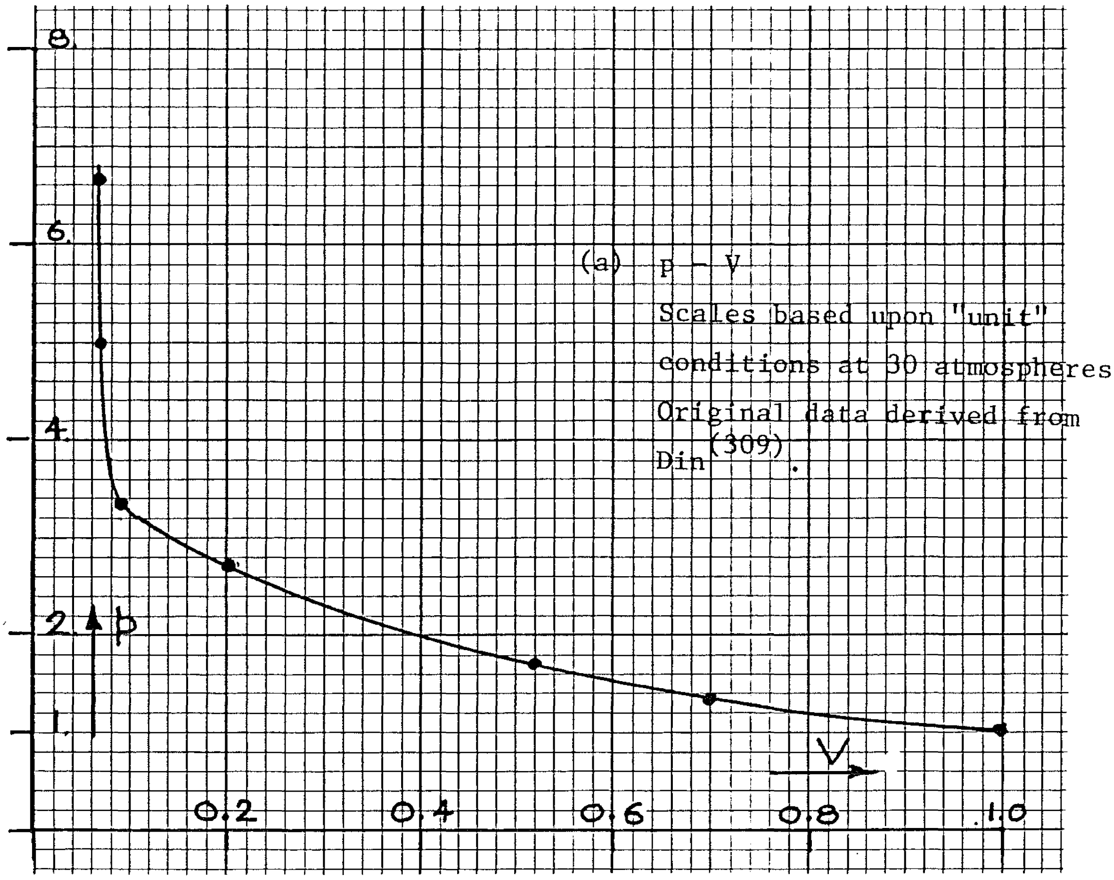
If, however, the two discrete bodies are seen to interact to a significant degree, via their mutual background environment, and are thereby subject to net pressure forces, the internal constancy of v becomes an untenable premise. Consider, for example, that the bodies experience "attractive" forces which vary according to some inverse function of separation distance. This definitive stipulation immediately requires that the internal impact velocity, v_i, must exceed the impact velocity, v_e, at the external barriers; hence $v_e<v<v_i$, and $\overline{F}_e<\overline{F}_i$. Indeed, with an appropriate selection of quantitative system parameters the occurrence of barrier impacts may be even rendered a locally redundant feature ($v_e = 0$), in which case $\overline{F}_e = 0$. Such a situation of apparent "self-constraint" - should it prevail - does not, of course, warrant the misleading interpretation that the two-body system is thereby "unloaded" or entirely free from external influence; the prescribed source of the "continuous" mutual interaction which sustains this degree of overall systematic integrity - viz, the shared environment - is external to the bodies themselves and thus transcends the nominal internal/external boundaries alluded to earlier. Consistent internal/external descriptions of effective environmental constraint will be developed in due course.

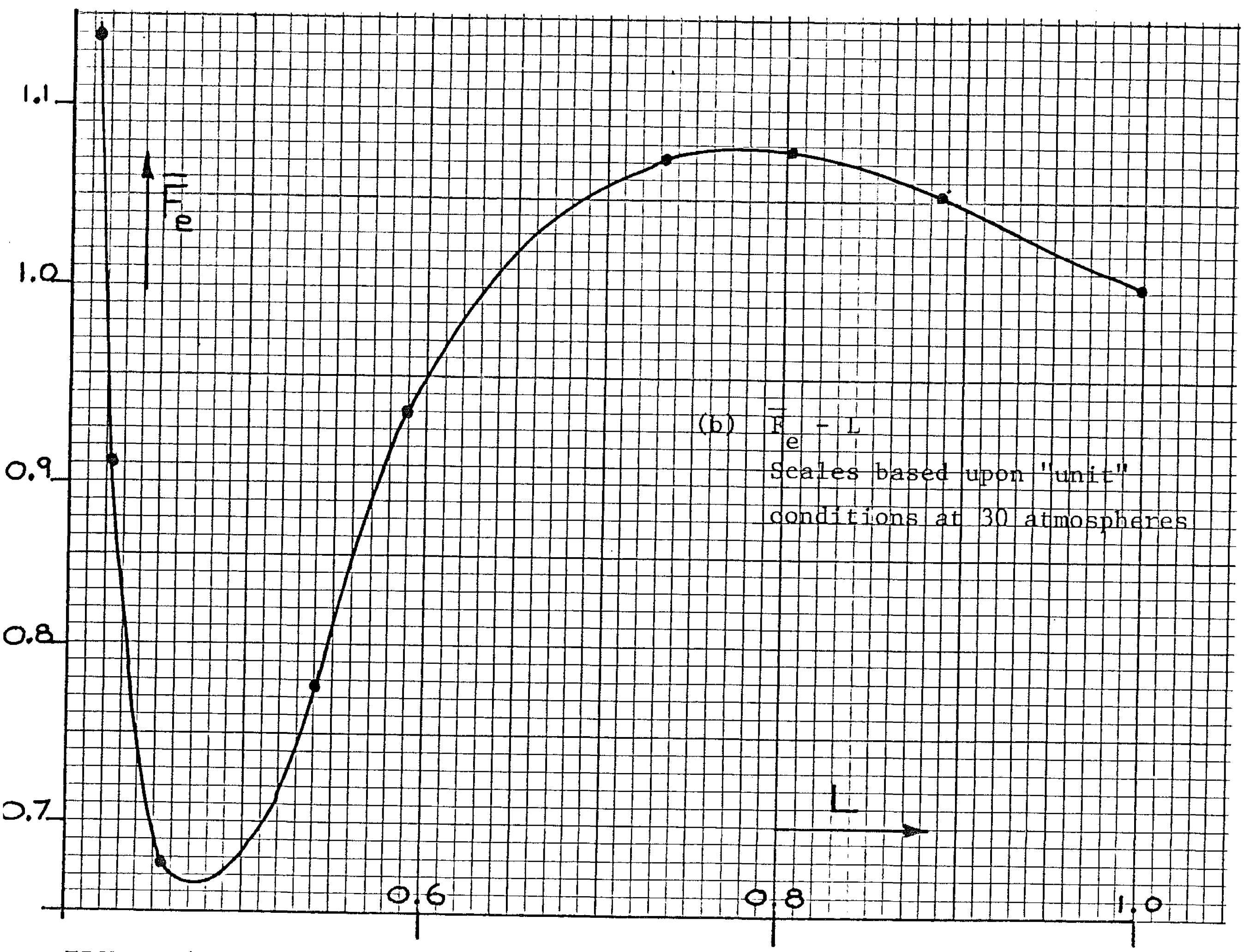
Despite its merely purporting to illustrate the fundamental concept of composite systems being "held together" by compression from without rather than by tension from within - <u>and</u> its obvious crudeness* - the one-dimensional vibratory model undoubtedly embodies some potential usefulness as regards "explaining" the typical thermodynamic behaviour of bulk matter in simple qualitative terms. There is, for example, no reason other than established tradition which dictates that customary pressure and volume are <u>uniquely</u> "suitable" measures of the relative existence manifested by the composite "body" of a confined fluid. Although the writer is not aware of this ever having previously been given serious consideration, conventional pressure-volume (p-V) data is in fact quite amenable to an elementary transformation which yields associative force-length ($\overline{F}_e$ - L) values;

i.e. $$\overline{F}_e = pV^{\frac{2}{3}} \quad \text{and} \quad L = (V)^{\frac{1}{3}}$$

Recorded experimental thermodynamic data for fluids so transformed exhibits many interesting traits, the very existence of which is

* The notional hardness ("instant" recoil) granted to the discrete bodies greatly simplifies quantitative analysis but necessarily detracts from their inherent "realism". A more sophisticated model could, if desired, be designed to account for this discrepancy.





FIGURES 4.20: Plots of Isothermal (40°C) Data for Carbon Dioxide

essentially hidden by the more standard manner of presentation. Figure 4.29(a) shows part of the p-V isotherm for carbon dioxide at the super-critical temperature of 40°C, indicating a seemingly steady trend; the equivalent section of the $\overline{F}_e$-L isotherm shown in Figure 4.20(b) dramatically reveals somewhat contrasting "hidden detail". It would appear from the examination of many other constant temperature p-V curves that such transformed "behaviour" is certainly not peculiar to the specific case cited, and that a stable upper turning point is a probable characteristic of all gaseous $\overline{F}_e$-L isotherms, both above and below the critical temperature: for sub-critical isotherms, the upper turning point occurs before the onset of the unstable vapour stage which marks the transition from gas to liquid. Further possible comment on this topic is inappropriate to the study in hand.

4.3.2 The Elements of Composite Structure

By definition, structure is the most characteristic - and perhaps the most important - property of any hierarchical system; in its absence (cf. the alternative philosophy of the undifferentiated continuum approach) the non-trivial identification of discrete interactive systems within systems would be rendered a somewhat meaningless exercise. There is, of course, much related evidence in support of the claims to suitability of both selective and progressive "internal" differentiation: an assertion of ordered (systematic) structural form is far from an arbitrary precept in the materials context. (Conversely, such accumulated evidence also tends to highlight the inherently limited scope of the idealistic continuum approach* as an appropriate basis for the rational description of "real" materials and/or the behavioural traits these exhibit.)

The twin concepts of structure and composite integrity are closely allied. One of the major benefits which derives from adopting an extended hierarchical view of materials is the deliberate (a priori) emphasis this places on the systematic aspect of composite integrity at different levels. Thus, although material systems can (and do) disintegrate when subjected to certain critical loading regimes, the associated loss of their initial composite structure is rarely extensive in a hierarchical sense: atomic nuclei, for instance, are not at risk during

* Regardless of any local justification for the use of continuum mechanics in specific sets of circumstances, its undoubted mathematical convenience does not legislate on behalf of its global suitability.

routine mechanical testing! The "failure" (or hierarchical degeneration) of a system of components need not always be seen to entail a breakdown of the components themselves. Interpretations of breakdown are naturally quite sensitive to the exact manner of system/component definition. Since such definition is largely governed by notional preference, the descriptive flexibility of the general hierarchical approach offers a substantial measure of possible choice. One particular option - to be developed more fully in that which follows - is simply to interpret "failure" per se as a definitive process of structural alteration which serves to identify the locally "relevant" components of the original system. In this way, material breakdown may therefore be envisaged to involve an effective shift in local hierarchical status, by which particular constituent sub-systems of the original "whole" undergo significant (recognisable) structural dissociation and hence cease to interact in a composite fashion. As an analogy, the typical collapse mechanism of an unbonded voussoir arch - whereby some critical load combination acts to deprive the constituent blocks of that stable configuration which normally sustains their mutual (as distinct from individual) integrity - illustrates the underlying principle of limited degeneration extremely well.

Having stipulated (chosen) an essentially local failure process (as an ultimate response to as yet unspecified loading regimes), there is no strict need to consider the structural integrity of each and every hierarchy of a composite material system on an individual basis; i.e. stable hierarchies may be conveniently grouped in such a manner that a selective single level of internal differentiation, based upon highlighting those "relevant" components which are first susceptible to structural dissociation, becomes quite adequate for most purposes. According to the simple diphase model which results therefrom, any critical region of a composite material system can be represented by two appropriate fractions - one consisting of discrete interactive quasi-solids (which may or may not be permenantly contiguous), the other comprising their complementary quasi-fluid at the corresponding level of internal discrimination. The relative physical extent of the so-called critical region (or regions) will be an intrinsic property of the particular system and loading regime involved, and in some instances may actually embrace the entire material "body". Adoption of the diphase model for the purposes of local description does not compromise the implied generality of the extended hierarchical view of materials in any way, since the former is but a logical derivative of the latter. Thus, for example, while the primary degeneration of an original "whole" into a partial assembly of largely non-composite elements is certainly indicative of overall systematic breakdown, the onset of such structural transition need <u>not</u> be accompanied by immediate and/or total instability (gross failure at the macroscopic level): a manifestation of unstable bulk behaviour may well require the subsequent (progressive) degeneration of those dissociated elemental systems first created by "earlier" local failure mechanisms. As will shortly be demonstrated, sequential alteration processes of this type can be envisaged without much difficulty in the context of a stepped (ordered) diphase model.

The formal introduction of a specific failure concept prior to any detailed consideration of less critical behaviour marks a radical departure from most conventional approaches to the mechanical properties of materials. There are, however, several areas in which the prime requirements for systematic integrity are given (at least by implication) a positive measure of background recognition. Among the more obvious of these is the commonly inferred "understanding" of materials in the gaseous state.

A finite "body" of gaseous matter relies for its separate physical existence on the restricted mobility of its molecular constituents: without some degree of effective confinement, gross composite stability (in a sustainable sense) is quite impossible. The "failure" of a gaseous body through a loss of confinement involves a simple breakdown of restrained dynamic structure; here, the quasi-solid "components" which cease to interact are, of course, the discrete molecules. Such failure is manifestly limited in extent and purely systematic by nature. The molecules themselves do not cease to exist; i.e. the physical integrity of the erstwhile components is maintained after failure despite the obvious shift of hierarchical status which thereby occurs. (The mere opening of a pressure vessel which previously contained a gas does not, under normal circumstances, precipitate a molecular degeneration to the plasma state!) In essence, this positive identification of the locally relevant quasi-solid components and the pertinent restraint requirement establishes the notional limits of overall system applicability and provides an important conceptual origin from which the behavioural characteristics of the system itself (i.e. the gaseous body) become amenable to rational description, both qualitative and quantitative.

The behaviour of gases constitutes a convenient starting point. Thus, with a view to formulating the basis of an expression linking distributed composite restraint, r, and bulk density, ρ, for a body of gaseous matter, the following has complete generality and is unrestricted by those underlying conceptual inadequacies which severely limit the viable implementation of the Ideal Gas Law:

viz,
$$\frac{dr}{r} = n \frac{d\rho}{\rho} \qquad \ldots\ldots\ 4.14$$

, where n may best be designated as an "appropriate" scale factor* relating to two systematic measures of incremental change. Despite possibly misleading first appearances, this equation is <u>not</u> being offered unconditionally as a special case of the descriptive equation 4.5 presented earlier. No suggestion is being made here - nor should be inferred - that the associating scale factor, n, is <u>necessarily</u> a constant for any particular system and/or change regime.

The deliberate adoption of a separate symbol, r, to signify quantitative restraint, rather than p as before, is prompted by (or indeed, reflects) the need to recognise molecular interactions of other than the elementary direct collision type; i.e. in conventional terminology, the molecules of a gas may exert mutual forces of either effective attraction or repulsion "at a distance" upon their neighbours. Those molecules which actually impinge on the container therefore experience "continuous" actions tending to either complement or oppose the external barrier restraint (discrete impact forces) provided by the container walls. Of course, from an extended hierarchical viewpoint this "additional" feature of contributary "internal" effects is merely a manifestation of local interactions between the quasi-solid molecular constituents and their quasi-fluid non-matter environment (<u>not</u> strictly between the molecules themselves).

The bulk restraint term, r, of equation 4.14 may conveniently be written as an <u>equivalent</u> "continuous" total of two partial time-averaged actions, both distributed quantities being expressed in

* The scale factor, n, should not be confused with the molar number, n, of the Ideal Gas Law. Similarly the chosen restraint parameter, r, and the various radius terms, r, of previous arguments must be clearly distinguished between.

relation to some suitable common reference area. A "unit" of the container boundary area would seem a most appropriate choice for this latter purpose since the barrier restraint, p, is normally so referenced.

i.e. $$r = r_E + p \qquad \text{..... 4.14(a)}$$

, where r_E is the restraint contribution which derives from the "unrecognised" background environment. Having implicitly adopted restraint as a positive concept (no net restraint - no system), the potentially disruptive influence concomitant to effective "inter-molecular" repulsion at a distance, where applicable, will find quantitative description within equation 4.14(a) as a negative r_E value.

The bases of would-be equations of state for gases vary somewhat in theoretical content[310]. Some adopt ostensibly "rational" modifications* to the Ideal Gas Law, the quantitative magnitude of the corrective parameters being determined from known experimental results; others, totally lacking in any semblance of a priori justification, simply employ rather arbitrary curve fitting techniques. All ultimately rely to some extent on the empirical approach. In this respect at least the present treatment of gaseous "properties" is no exception, although, for reasons now to be discussed, the empirical approach will not be formally implemented as such.

Despite their appealing simplicity of form, equations 4.14 and 4.14(a) contain two unknown system parameters, r_E and n, which are not amenable to obvious solution from the routine manipulation of available p-V data for any particular gas. The main practical difficulty arises because neither can be attributed the status of an unqualified constant: both may well be functions of bulk density and/or temperature. For an isothermal process the only definite characteristics which can immediately be inferred are that,

$$r_E \to 0 \quad \text{and} \quad n \to 1 \quad \text{as} \quad \rho \to 0$$

---

* Although fundamentally different in concept from the well-known Clausius/van der Waals equation, equations 4.14 and 4.14(a) together embody the essential spirit of the former.

Of course, unless the environmental restraint contribution, r_E, and the scale factor, n, are extremely sensitive to relatively small changes in the bulk density, ρ, then it should be possible to obtain local estimates thereof, providing that there exists sufficient p-ρ data for the specific gas, temperature, and small density range under scrutiny; i.e. if r_E and n are assumed virtually constant over a very limited range of ρ, the integrated version of equation 4.14 is of the power-law type, $r = r_E + p = B.\rho^n$, to which statistical "best-fit" techniques can be applied for the "solution" of all relevant unknowns, including the local constant B. Unfortunately, the significant degree of isothermal compressibility displayed by most gases has led to a situation of some scarcity with regard to small density change p-ρ data. The relative ease with which gaseous matter can be compressed or expanded also means that small density changes are induced by correspondingly small changes in p; under such circumstances it is difficult to clearly establish the suitability of one form of correlation over that of some other quite different form. Experience has shown that a three-constant power expression possesses considerable flexibility for empirical correlation purposes. There would therefore seem little to be gained from demonstrating this here with isolated small samples of experimental gaseous p-ρ data. What then of corresponding data for liquids?

Consider, for example, the controlled compression of a liquid body under prescribed thermodynamic conditions. (For obvious practical reasons, the response to isothermal regimes makes an excellent special case.) As before, the "relevant" quasi-solid components of the bulk system are its discrete molecules. While the "internal" mobility of the molecular constituents is somewhat more restricted in a short-term sense than that which pertains to the gaseous state, the general fluid characteristic of effective long-term positional freedom within the boundary volume occupied by the body is retained. However, unlike gases, liquids typically exhibit a relatively low degree of isothermal compressibility. Only at temperatures approaching the critical value do regions of what might be classified as "medium" compressibility be readily identified. An appropriately selective examination of available p-ρ data for liquids therefore offers a welcome opportunity to assess the validity of a "local" power law expression (the integral form of equation 4.14 with r_E and n constant) as a suitable indicator of associated interactive change within particular isothermal systems. Grimer and Hewitt[110] have in fact already demonstrated that just such a form* can be fitted most successfully to known pressure-volume data for liquid water. They found that the value of r_E, although sensitive to the actual temperature under consideration was typically of the order of thousands of atmospheres (hundreds of MPa): e.g. at 60°C a best-fit value of 3750 atmospheres (380 MPa) was determined. The data employed - that generated experimentally by Bridgman[311] - involved applied pressures of up to 1200 atmospheres (1216 MPa). In qualitative terms, the relatively large magnitude of the "additional" environmental restraint may be seen as explaining the apparent near-incompressibility of water under more "normal" circumstances of comparatively small applied pressure variations. Grimer and Hewitt also reported that, unlike r_E, the quantitative magnitude of n - the isothermal scale factor of associated incremental change - did not display a notable variation with local temperature, and hence suggested a unique value thereof for liquid water, $n = 6$, based upon their correlation calculations.

Further investigation by the writer has revealed that isothermal p-ρ data for other liquids is equally amenable to "high agreement" numerical description via r-ρ expressions of the power law type. Examples of findings are given below:

	Pressure range (atmos.)	Temp. (°K)	r_E (atmos.)	n
Ammonia	30-1100	320	490	8
	40-1100	340	315	8
		370	85	8
Carbon dioxide	50-1000	283	24	9
	60-1000	293	-27.5	9

---

* The terminology and symbols of the present treatment differ slightly from those originally adopted by Grimer and Hewitt in this context, but the effective end-result is identical. A number of factors prompted these minor alterations. For example, the "internal stress" parameter, i, is rather loosely defined in their introductory paper on the new hierarchical approach to material behaviour; it is therefore unfortunately liable to some degree of misinterpretation. One of the principal aims of this work is, of course, to remove any such element of potential ambiguity which could ultimately prove detrimental to the hierarchical "cause".

The temperature-dependence of r_E is clearly shown here. Of course, a diminution of the environmental restraint contribution with increasing temperature is not beyond the realms of logical expectation. In terms of the hierarchical model, a positive value of r_E merely signifies that the "continuous" environmental forces exerted upon boundary molecules are "attractive" in a net time-averaged sense with respect to their fellows deeper within the fluid body. Because the system is inherently dynamic, the forces which prevail at any one "instant" may be either attractive or repulsive. As the level of molecular activity increases with temperature, the period of the mean free path naturally decreases, and a relative gain in the overall influence of close proximity "repulsive" effects - manifesting itself as a decrease in r_E - is a reasonable consequence thereof*.

Although certain qualifying statements have already been made, it is perhaps important at this point (in the light of the claimed "success" described above) to emphasise most strongly the inherent practical limitations of the r-ρ power expressions which can unfortunately render these somewhat inappropriate for extrapolation purposes. Consider, for example, the appropriate regression equation for liquid carbon dioxide at 20°C, with p in atmospheres and ρ in kg/l;

i.e. $$(p - 27.5) = 291.4\rho^9$$

At first sight the equation would appear to "predict" the failure of any finite liquid body of carbon dioxide at 20°C, if the applied pressure, p, is reduced to 27.5 atmospheres. Such a prediction is, however, invalid on two quite separate counts. Firstly, it ignores the possibility of a liquid/gaseous phase change taking place if p falls below the relevant vapour pressure; in fact, at 20°C with p = 50 atmospheres, carbon dioxide exists as a gas! An isothermal liquid/gaseous phase change is not, of course, a "bulk" phenomenon: neither can it be classified as instantaneous, since its controlled occurrence

---

*The previously mentioned alteration in terminology from that originally adopted by Grimer and Hewitt is especially beneficial here. While a negative environmental restraint contribution - signifying partially disruptive "background" tendancies - presents no serious conceptual difficulties, a negative "internal pressure" is a distinctly more problematic notion.

requires the elapse of a finite time interval*. The process of isothermal transition between different states (systems) happens in a progressive manner as the original body of liquid gradually loses its physical identity (and quantity) and is replaced by a "new" body of gas. During the potentially unstable in-between period the separate phases co-exist in varying proportions (an individual molecule may "instantaneously" belong to either) within a changing overall boundary volume. Under these conditions the bulk density parameter, ρ, is deprived of its erstwhile significance. Only upon the ultimate emergence of the new "single" system (a confined body of gas), with its correspondingly "new" interactive characteristics, is the phenomenological relevance of ρ effectively reinstated. Secondly, and perhaps more importantly from a general cautionary viewpoint, as p decreases from 60 to 27.5 atmospheres the "predicted" bulk density, ρ, changes from 0.784 kg/l to zero - a significant alteration for an applied pressure drop of only 32.5 atmospheres (cf., the experimentally recorded density change of 0.365 kg/l over an applied pressure range of 940 atmospheres between p = 60 and p = 1000 atmospheres). A rate of change of this order is totally outwith the physical scope of the rationale which first suggested the local suitability of a constant r_E power expression as an integral form of equation 4.14. (Indeed, it is even very probable that the relative change in density pertinent to the experimental data employed to determine best-fit estimates of the parameters r_E and n of the regression equation cited - the largest relative density variation of any examined with a view to establishing such r-ρ correlation - is itself a little too great to fully justify the underlying premise involved in the trial adoption of the latter. Significantly perhaps, the overall degree of fit obtained with that particular equation, while not unsatisfactory, was the least impressive of all those subjected to scrutiny in this context.)

The findings given above (like those of Grimer and Hewitt) are characterised by single-valued integers for each liquid. In the

---

* While the classical theories of thermodynamics are notably devoid of any allusions to temporal matters - a rather curious fact which has not escaped previous comment (307, 308) from philosophers of science - the importance of time has long been recognised in the more practical aspects of the subject.

interests of fair reporting it must be remarked, however, that this "result" is influenced to some extent by a measure of interpretive license assumed on the part of the writer. Thus, the best-fit regression procedure, which actually investigated for the most suitable estimates of r_E and $\frac{1}{n}$ compatible with the experimental data to hand, yielded in each case a value of the latter for which the inverse was extremely close to an integer. Since the maximum and root mean square deviations were found to be relatively insensitive to small variations in $\frac{1}{n}$, the value of the integer was adopted, principally on the grounds of simplicity. (Bearing in mind the immediate nature of the inquiry, there appeared little point in the formal establishment of statistical confidence limits.) As regards the question of "constancy", all that can strictly be stated is that, for the particular sets of data examined, the local values of n derived therefrom displayed no degree of appreciable dependence on temperature. Of course, having originally designated n as a systematic scale factor, there would seem no essential a priori requirement to demand that this need be an integer* (see earlier comments on scale factors) or necessarily exhibit temperature independence.

From a phenominological viewpoint there is little - apart from degree - to differentiate the isothermal p-ρ (or p-V) response of an isotropic solid from that of a fluid body. (With very few exceptions the solid state represents the most dense and least compressible form of material existence.) Thus, if the ideas of scaled incremental change and "additional" environmental restraint expressed earlier are to have generality, these should be especially applicable to quantifying the isothermal behaviour of isotropic solids under applied hydrostatic-type stress or pressure regimes. For obvious practical reasons, ρ (or V) is too insensitive a parameter to cope with any quantitative description of mechanical response pertinent to non-isotropic bodies. In terms of those aspects of the hierarchical model so far emphasised, the severely restricted mobility of molecular or atomic constituents which epitomises the common "understanding" of material bodies in the solid state may be interpreted as effectively limiting the bestowed status of "boundary molecule" to only a small fraction of the "relevant" quasi-solid

---

* The writer does not suscribe to the suggestion of integer significance proferred by Grimer and Hewitt.

components; i.e. the boundary status is not shared among the individual molecules or atoms on a time-averaged basis, as it was previously considered to be with those of contained fluid bodies. This somewhat static feature of solid bodies does not detract, of course, from the important dynamic aspects of their components at the local level, associated with thermal activity. Unfortunately, unlike the bountiful situation which obtains with fluids, "raw" p-V data for solids over sizeable ranges of p is not generally available in any great quantity. Virtually all experimental p-V information gathered from the testing of solids has been processed in one form or another before its published presentation in the guise of empirical relationships. The variation of the elastic-theory parameter "bulk modulus" with applied pressure has been a particularly popular topic for empirical study, and accounts for much of the data-processing which has occurred. (In the elementary theory of linearly elastic solids, the bulk modulus, k, finds formal definition via the expression,

$$p = -\frac{k}{V_0}\,\Delta V \qquad \text{or} \qquad dp = -k\,\frac{dV}{V_0}$$

, where V_0 is the volume corresponding to $p = 0$, and $\Delta V = V-V_0$. Experimental measurement has shown however that k is not a constant* for real materials, although the extent of variation within "normal" ranges of p is so slight that an assumption of constancy is quite "safe" for most practical applications.)

During its early stages of development, the topic of high pressure physics was largely dominated by the pioneering efforts and experimental work of one man - P.W. Bridgman. His overall contribution to the field, which ultimately earned him the Nobel Prize in physics (1946), was immense, especially in the realms of experimental technique. It was Bridgman who first suggested the following relationship for isotropic solids;

---

* The two definitive expressions listed are only equivalent if a constant k prevails. In the absence of apparent linearity the latter differential form is generally preferred.

viz, $$\frac{1}{k} = \text{"compressibility"} = \frac{1}{V_0}\cdot\frac{dV}{dp} = a - bp$$

, where a and b are material constants, each implicitly dependent on temperature. Thus, at the integrated level,

$$\frac{\Delta V}{V_0} = (a - \frac{b}{2}\cdot p)p$$

Bridgman found that this expression, in which $\frac{\Delta V}{V_0} \ll 1$ and $b \ll a$, could "in most cases" be fitted to the p-V data he accumulated to within the estimated limits of experimental error.

Now, the compressibility equation may be written as,

$$\frac{dV}{V_o} = \frac{-dp}{(a - bp)^{-1}} = \frac{-dp}{\frac{1}{a}(1 - \frac{b}{a}\,p)^{-1}}$$

$$\rightarrow \qquad \frac{dV}{V} \simeq \frac{-dp}{\frac{1}{a}(1 + \frac{b}{a}\cdot p)}$$

since $$V_0 \simeq V \text{ and } \frac{b}{a}\cdot p \ll 1$$

i.e. $$\frac{dV}{V} \simeq \frac{-dp}{\frac{b}{a^2}\,(p + \frac{a}{b})}$$

Equation 4.14 with constant r_E and n may also be rearranged to produce a similar end-result;

i.e. $$\frac{dr}{r} = n\,\frac{d\rho}{\rho}$$

$$\rightarrow \qquad \frac{dp}{p + r_E} = -n\,\frac{dV}{V}$$

$$\rightarrow \qquad \frac{dV}{V} = \frac{-dp}{n(p + r_E)}$$

Of course, the empirical success of Bridgman's formula* coupled with the near-equivalence of basic form demonstrated above does not, in itself, constitute adequate "proof" of suitability with regard to a constant r_E power expression linking r and ρ. Plausible arguments of notional preference are rarely convincing enough in such circumstances to generate the necessary impetus for any need for change to be so recognised. Fortunately, however, Bridgman's very honest qualifying statement *"in most cases"* offers an opportunity to gauge the benefit of an alternative formulation because included in his paper[(312)] on the compressibility of metals are two sets of "raw" p-ΔV data which do not conform well to the suggested equation of quantitative interaction.

The writer has determined that the would-be anomolous data sets in question (pertaining to sodium and potassium) are amenable to excellent correlation via equations 14 and 14(a). For sodium at 30°C a "best-fit" combination of R_E = 32200 atmospheres and n = 2 was obtained. For potassium at 45°C the corresponding figures were R_E = 14400 atmospheres and n = 2.2. In the light of previous arguments, the lack of a near-integer index for the latter case was not seen as particularly strange. Conversely, the relatively high R_E values (as compared to the previous findings for liquids) offered "confirmation" of the restrained state seen to epitomise the nominal solid condition.

---

* The approximation of a shortened expansion of the binomial theorem has often been employed to invert Bridgman's compressibility equation. Indeed, the inverted form,

$$k = V_0 \frac{dp}{dV} = a' + b'p$$

, where $a' = \frac{1}{a}$ and $b' = \frac{b}{a^2}$, is the more commonly quoted in texts.

4.3.3 The Development of a Diphase Solid

An attempt to model/explain/forecast the general load-induced behaviour and breakdown of nominally "brittle" solids will now occupy the greater part of that which remains in this chapter.

Fortunately perhaps, by comparison with certain of the earlier arguments involved in establishing the necessary conceptual groundwork, the logical steps which lead towards the application of hierarchical diphase principles to describing the behavioural traits of particular material systems (including the concrete family) are relatively direct. While some recourse to previous deliberations will prove necessary, this will not be such as to hinder the progress of development. The first point to be built upon here concerns the importance of appropriate systematic characterisation.

Consider a cubical specimen of an isotropic solid material for which a pertinent r-ρ response relationship of the form $r = p + r_E = B.\rho^n$ is known to hold with a highly positive r_E. If eight "identical" versions of this "elemental block" are arrayed three-dimensionally in the contiguous geometric form of a composite cube (2 x 2 x 2) and the latter configuration is subjected to an externally-applied isotropic compressive stress regime, $\sigma(\equiv p)$, then the <u>same</u> response relationship as that of the individual components might reasonably be expected. If instead, however, a "tensile" ($\sigma < 0$*) regime were to be imposed on the grouped assembly, the blocks could not remain together (unless the influence of gravitational, electrostatic, and/or mating-surface interactions assumed non-trivial proportions); i.e. the composite structure would fail!

While the dangers of long-range extrapolation have already been illustrated, and although only the action of compressive ($\sigma > 0$) external regimes has been formally discussed up to this point, it does not demand

---

* The exact manner in which this effectively "negative" restraint can be applied in practice will become the subject of closer scrutiny at a subsequent stage. The recent symbol change from p to σ has in fact been implemented to avoid any suggestion of negative pressure - a quite meaningless concept from a physical viewpoint.

too great a measure of conjecture to suppose that the known r-ρ response relationship for the individual material bodies implies at least some isotropic "tensile strength" capacity on their part. This being so, it becomes evident that the behavioural characteristics of the component blocks are <u>not</u> transferred to the composite assembly for the case of applied "tension". The shared response of components and composite in compression and the divergent response in "tension" may be rationalised in simple hierarchical terms as follows. Under applied isotropic compression the internal block boundaries have no immediate phenomenonological significance as regards the overall behaviour likely to be exhibited by the composite; i.e. the quasi-solid components "relevant" to the r-ρ interaction are not the individual blocks per se but rather the molecular or atomic constituents thereof. The external boundary restraint, $r(= \sigma + r_E)$, was in fact originally prescribed as referring specifically to the latter. Conversely, under applied "tension" the most relevant physical quantum of component existence is quite definitely a single block; here, the presence of internal block boundaries determines the actual mode of composite structural breakdown. The essential difference as described might well be likened to that which obtains between the contrasting arithmetic notions of lowest and highest common factors.

A similar line of argument (although, of necessity, less specific) can now be applied to the "tensile" performance of a single block. For obvious reasons, the pertinent question as to exactly what might be taken as the "relevant" quasi-solid components of a solid material specimen in such circumstances is unfortunately outwith a strictly definitive answer. However, through logical recourse to that store of recorded "experience" which derives from both direct and indirect physical observation, it is possible to identify certain components which do <u>not</u> qualify as possessing an immediately relevant status. Thus, with the notable exception of sublimation, the breakdown of a solid body rarely involves significant disintegration at the molecular/atomic level; the "pieces" which remain after the failure of a solid specimen are typically several (if not many) orders of magnitude greater in size than average molecular/atomic dimensions. While the process of fragmentation (or composite degeneration) requires that certain molecules or atoms, which were once adjacent, must separate, this does not entail their "escape" from the system on an individual basis. As a consequence, failure of the total composite via the mutual dissociation of lower order composite groups of molecules or atoms is well beyond the predictive capacity of the bulk "molecular" expression, $p + r_E = B.\rho^n$, even allowing for the latter's aforementioned limitations with regard to the restricted range of viable extrapolation. This lack of "tensile" predictive power is of course neither embarrassing nor hardly surprising. (It will be recalled that the rationale of the r-ρ interaction was originally introduced to describe a system for which "internal" failure as just portrayed was quite inconceivable: the subsequent limiting of any considerations of systematic restraint to that experienced by external boundary molecules* was therefore <u>not</u> inappropriate in the earlier context.) Indeed, the <u>local</u> "credibility" of the suggested r-ρ relationship is essentially preserved by highlighting <u>a priori</u> the logic of such inherent deficiencies. Had r_E been designated as a <u>total</u> restraint parameter, the r-ρ expression might have been interpreted as predicting a reserve strength capacity under the action of isotropic "tensile" stress regimes which produce only relatively small density changes. Bearing in mind the very slight alterations of density typically displayed by nominally brittle materials before "tensile" failure, and remembering the order of magnitude of the two particular r_E values previously determined, this "prediction" - were it not able to be discredited - could most definitely have proved embarrassing! A distinct similarity exists here with the unrealistic strength magnitudes forecast by the conventional molecular theories of "intrinsic cohesion". However, unlike the latter, the present treatment has no need to introduce an additional (<u>a posteriori</u>) concept of "flaws" to explain the obvious discrepancies between experimentally recorded and would-be "real" tensile strengths; an ordered succession of internal boundaries is a natural (ab initio) feature of <u>any</u> hierarchical system. It is interesting to note, as a supplement to certain remarks made earlier on the somewhat suspect philosophy underlying the intrinsic strength approach, that while the conventional explanation of those quantitative strength differences between theory and practice adopts the descriptive "picture" of a material as a <u>continuum</u> with critical flaws, the original forecast it purports to sustain is actually founded upon definitive notions of <u>discrete</u> molecular existence.

---

* Having explicitly mentioned the phenomenon of sublimation in passing above, it is probably apposite to remark also that the "bulk" expression $p + r_E = B.\rho^n$ does not embrace the possibility of a solid/gaseous (or a solid/liquid) state transition - see previous comments relating to liquid/gaseous phase changes.

The only conclusion to be formally reached so far is that, in hierarchical diphase terms, the most pertinent quasi-solid components of a composite solid material as regards its ultimate performance when subject to an applied "tensile" loading regime are of a higher order than molecules or atoms; i.e. the "relevant" components are themselves solid composites of the latter. Nevertheless, despite its rather vague appearance, this super-molecular designation - a statement of perspective - is quite sufficient to now allow significant progress to be made. Surprisingly perhaps, in view of the "inherent complexity" which tends to be ascribed to the critical behaviour of material systems under general loading conditions, the seeming need to bestow more specific identity on the "relevant" components seldom arises and is thus largely redundant to the predictive applications of the various governing principles developed.

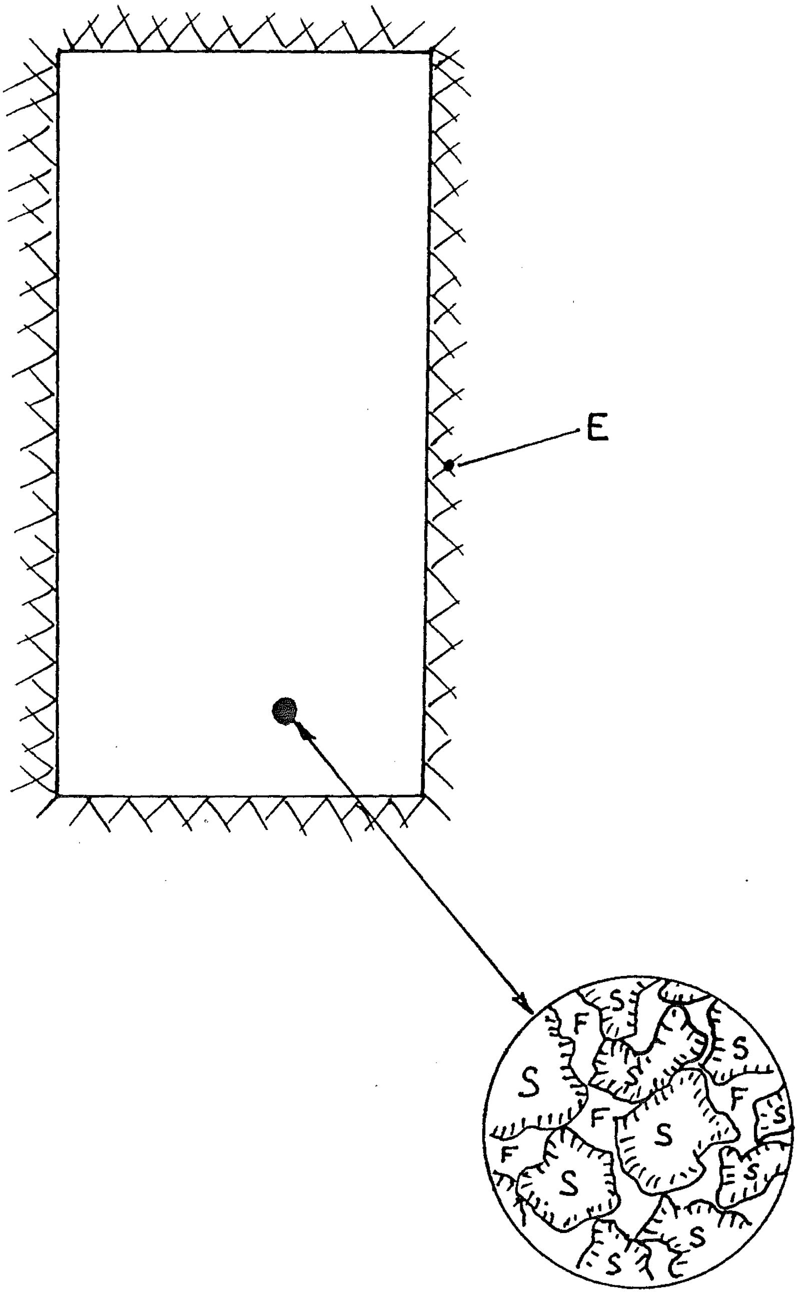
Because its roots are firmly entrenched in the realms of physical "understanding", the hierarchical approach rejects outright the concept of action at a distance, be it in any guise. There is therefore no place within the diphase model for the conventional notions of "bond", whether these be presented as foundational axioms or as part of an all-too-frequent exercise in linguistic tautology*.

Composite existence demands that the components be subject to a certain degree of restraint: in colloquial terms, the parts of the whole must be "held together" in some manner. Their component status would be meaningless otherwise. For a body of matter in the solid state, the intrinsic rigidity thereof implies that the pertinent systematic restraint must be such as to maintain the components in a relatively fixed configuration with respect to one another and also with respect to the "bulk" of the body itself. It is, of course, this property of rigidity which enables a primary differentiation between solid and fluid matter to be effected.

---

* Elementary textbooks in material science and engineering must shoulder much of the blame for the unhelpful perpetuation of nonsensical circular arguments. To attribute, on one hand, the phenomenon of solid failure to the intensity of applied loading exceeding the material's "strength", while defining strength, on the other, as the applied loading intensity which causes failure is hardly a sound basis for rational development. Were the unfortunate misunderstandings such absurd arguments invariably engender not so widespread, the intellectual self-deceipt which they represent might have been a subject for mild amusement!

The built-up cube alluded to previously was then rather loosely described as a "composite". Its bulk integrity was, however, only strictly assured through the external application of an isotropic compressive stress, σ, the action of such a regime being transferred between the individual blocks by way of mutual physical contact. In the absence of applied external constraint, the eight subsidiary elements of the would-be whole could not interact in this direct and positive manner and hence would merely constitute a fortuitously grouped assembly, rather than an integral system as such; i.e. the condition $\sigma = 0$, at which point no "inter-block" pressure prevails, marks the effective lower limit of composite systematic potential with regard to the particular built-up structural form and class of loading considered. It is precisely this notion of "forced" bonding from without which epitomises the strength rationale of the hierarchical approach. According to the latter, the conventional designation of an "unloaded" material body (or of a "free" state) is a basic contradiction in terms ; i.e. if the components of the body were indeed unloaded, neither the body nor the components could exist as stable physical entities. Of course, having stipulated the presence of an "unrecognised" background environment, an additional source of external restraint, with which to supplement conventional conceptions (and measures) of applied load, poses no logical problems. In order to avoid any risk of ambiguity the term regime-free will henceforth be used to denote the status of a material system before its "loading" in the normal sense. (To be fair, it must be remarked here that not all branches of conventional "understanding" with regard to material systems subscribe explicitly to the notion of an unloaded free state. Thus, in those areas of science which operate below the level of bulk material existence, and focus upon the "binding" and structure of constitutive matter, concepts of equilibrium involving the balance of various <u>finite</u> force effects may be seen to prevail. Unfortunately, the interdisciplinary nature of material science is not such as to promote a consistent interpretation in this context. The would-be forum of material science is highly compartmentalised, with the result that the "message" deriving from one area or topic need not be taken up in another. Whether through deliberate choice, ignorance, or custom of habit the spurious connotations of the "unloaded" premise continue to escape notice.)



E = first-order quasi-fluid complement

F = second-order (internal) quasi-fluid phase

S = second-order quasi-solid components

FIGURE 4.21: A Diphase Solid

Figure 4.20 shows a possible representation of the hierarchical diphase model as it will be seen to apply to the exterior and interior conditions of solid material bodies in the regime-free state. No specific internal geometry is assumed, only that the discrete quasi-solid components (S) take up a mutually contiguous structural form. (Inferences with regard to a relative scale of sizes should not be drawn from the figure: the proportions of the diagram were adopted primarily for the purposes of adequate illustration.) Each individual S component is subject to two environmental influences - viz, that which derives from its complementary second-order quasi-fluid (F), and that which results from "continuous" physical contact with its immediate neighbours. Both are of crucial importance to the model's viability. In view of the recent attention given to the composite performance of the built-up cube, the significance of the latter influence is probably the more obvious; i.e. the forced mutual contact between the S components is a necessary pre-requisite for the skeletal arrangement thereof to possess inherent structural stability - a feature which imparts its associated rigidity to the system as a whole. However, were it not for the deliberate recognition afforded to the influence of the "interstitial" F component, the manner in which the forced contact is "understood" to be generated (and thus how it might be altered) would remain somewhat obscure, and hence the potential for any further development of the model would effectively vanish. Some amplification of this point is perhaps required before moves to realise that potential are instigated.

During earlier discussions of the built-up cube, no mention was made of "normal" atmospheric (air) pressure. This omission does not seriously jeopardise the logic of the arguments then advanced because atmospheric pressure rarely plays a significant role in the loading or behaviour of solid materials; nevertheless, those slight differences which arise from its inclusion do merit some examination - at least in principle. Consider, therefore, the consequences of placing two of the elemental blocks (material cubes) together to form a prismatic "bar". If the mating surfaces, each of area A, were "perfectly" plane it is conceivable that such an operation could be achieved to the complete exclusion of the local atmospheric environment previously existing between them, in which case a truly composite structure with inherent "tensile" stability would result; i.e. an "inter-block" pressure or forced contact (F/A) would be maintained by the action of the surrounding air on the "end" external surfaces. (In the process of creating the new composite system, one sixth of the original external area would be lost to the internal interaction.) Denoting the atmospheric pressure as p_e and the solid contact pressure as p_s, the internal/external static equilibrium condition for the forced composite bar described may be written as,

$$p_s \; (= F/A) = p_e$$

It is pertinent to note that this expression represents a quantitative equality, not an identity. Unlike p_s, p_e is a fluid pressure. While continuum mechanics makes no such distinction between "types" of pressure, the difference is important in a hierarchical context.

Of course, smooth perfect planes belong only to the ideal world of geometric abstraction. The degree to which real surfaces may be rendered plane is eventually limited by the discrete (and fundamentally dynamic) nature of constitutive matter. Apparently smooth planes at particular levels of discrimination can never survive the close scrutiny of higher resolution. However, although this encroachment of reality necessarily interferes with certain of the previous assertions regarding the composite bar held together by external air pressure, it does not eliminate the latter per se as a conceivable entity. Thus, providing that during the initial creation process a finite contact area, A_c, is established between the mating surfaces at some level which excludes the action of local air pressure, a forced bonding effect, F, may thereby be attained despite the "natural roughness". A finite A_c is a necessary condition for stable bonding but, as will emerge very shortly, not a sufficient one. If $A_c<A$ air could well be present within the interfacial region, between the "points" of contact: to cover this possibility an interfacial fluid pressure, p_i, acting over an area A_i, will be supposed. Specific details of the elemental blocks themselves (including their surface characteristics) and of the manner of their "placing together" have been deliberately kept to a minimum in the interests of generality. Therefore, although it is perhaps tempting to presume that $p_i = p_e$ and/or that

$A_c + A_i = A$ there is no sound basis upon which to uniquely justify either. An elementary longitudinal force balance yields the combined equilibrium/ stable bonding condition for the general case;

i.e. $$F = p_s A_c = p_e A - p_i A_i > 0$$

Notwithstanding their deceptively simple appearance, both internal area parameters, A_i and A_c, suffer from an unavoidable aspect of vagueness. It should be remembered that air is an intimate mixture of gases and, since the local geometry of the interfacial region is quite arbitrary, there is no unconditional guarantee that each molecular fraction even "sees" the same area! Fortunately, such vagueness and any associated problems of interpretation it might raise can be easily circumscribed by invoking the concept of partial pressures; viz, where uncertainty exists as to the "actual" (net) area over which a system of distributed forces is understood to apply, the statically equivalent distributed effect spread over a known but larger (gross) reference area serves as a convenient substitute for "absolute" pressure. (In fact, because real matter is not continuous, this element of uncertainty always prevails, though is seldom formally recognised unless rationalisation depends upon it; e.g. to "explain" osmotic phenomena. All practical measures of stress and pressure are therefore strictly partial quantities*.) Adoption of the partial pressure concept allows the useful conventions of continuum mechanics to be implemented without having to subscribe to the universal suitability of the implied philosophy which generally underlies them. Expressed in terms of partial rather than "absolute" pressures (symbolically, p' rather than p), the combined equilibrium/stable bonding condition for the composite bar structure,

viz, $$\frac{F}{A} = p_s' = p_e' - p_i' > 0$$

, is much more manageable as a descriptive tool than is the previous version given above. Thus, for example, while a "knowledge" of the condition $p_i' > p_e'$ automatically precludes the occurrence of stable structural association between the would-be component blocks, the condition $p_i > p_e$ offers insufficient information to enable general inferences with regard to composite potential - or the lack of it - to be drawn.

---

* Nowhere is this more apparent than in the field of soil mechanics.

It may be seen that a notional expansion of the parameters p_e' and p_i' to include the effects of an additional quasi-fluid background environment is perfectly feasible. (The stipulation of a "normal" atmosphere of other than just air need not, in principle, alter the important systematic difference term, $p_e' - p_i'$, to any significant degree.) By retaining the use of partial pressures there is no requirement for immediate concern regarding the "actual" areas upon which this "extra" source of both internal and external pressure might prevail. Although the gravity and electrostatic models presented earlier focussed particular attention on "relevant" or effective areas "seen" at different levels of environmental discrimination, it is pertinent to remark here that each is equally amenable to the manipulative convenience of the total environment/ partial pressure approach*.

The thin-walled box analogy (Section 4.2.1.2) and the composite bar discussed above obviously have much in common. (The forced integrity of the former may also be described via partial pressures, p'. Indeed, had a thick-walled structure been originally specified, the general descriptive insufficiency of the $p_e > p_i$ criterion would have emerged much sooner.) Only one apparently small feature differentiates them. Whereas the composite box arrangement was nominally closed to any "communication" between the external and internal fluids, the interfacial region between the mating surfaces of "rough" blocks was <u>not</u> so prescribed. Of course, a total lack of communication is quite foreign to the philosophy of the general hierarchical approach; that "perfect" closure is not an essential pre-requisite for forced bonding from without is therefore a significant point to have demonstrated in the structural analogy context. The same principle may also be inferred from the gravity and electrostatic models. (Historically, physical notions of an "aether" have been dominated** by the "all-pervading" form.)

An appropriate return can now be made to the diphase model of a solid material. (By virtue of the partial pressure concept formally introduced during the interim period since its initial presentation, the

* The partial pressure approach was not implemented during the initial stages of concept development because, at that point, its use of the conventions (but not the fundamental rationale) of continuum mechanics could have fostered unintentional ambiguities.

** One of the few notable exceptions to this trend is the "captive" aether theory (313) formulated by John Bernoulli - and later adopted by Euler - to account for the elastic properties of material bodies.

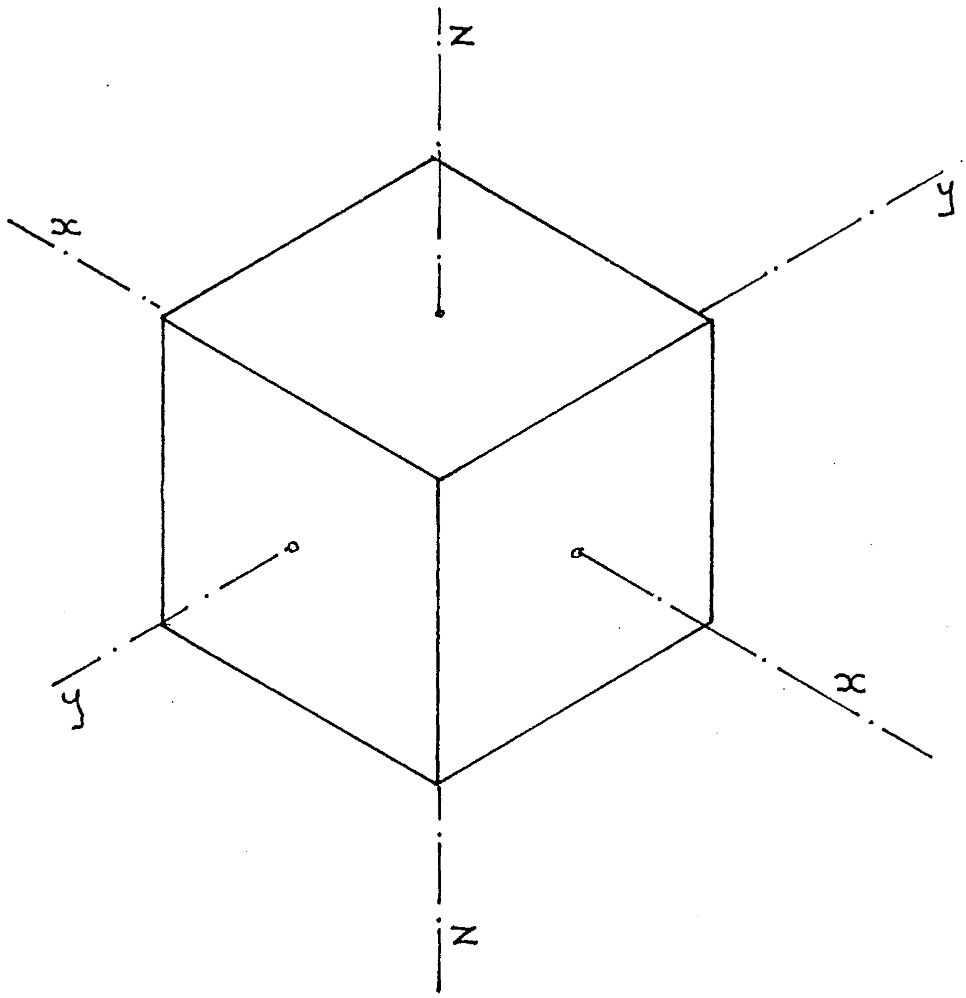


FIGURE 4.22: Coordinate Reference System

basic task of its systematic description has been somewhat facilitated.) Figure 4.22 shows a cubical specimen of the bulk solid in the regime-free state, so orientated that its external surfaces are normal to the orthogonal reference directions, x, y, and z. The fundamentals of the diphase model are not, of course, tied specifically to any particular specimen shape: the present choice of a cube is governed primarily by the immediate geometric convenience which this offers. Adopting the primed (p') notation for partial pressures, the external/internal equilibribium conditions pertinent to the three principal directions, x, y, and z, may be written as,

$$\left.\begin{aligned} p'_{Ex} &= p'_{Sx} + p'_{Fx} \\ p'_{Ey} &= p'_{Sy} + p'_{Fy} \\ p'_{Ez} &= p'_{Sz} + p'_{Fz} \end{aligned}\right\} \quad \ldots\ldots 4.15$$

, where p'_E = the partial pressure of the external quasi-fluid environment (E),

p'_S = the partial pressure of the internal contact between quasi-solid components (S),

and p'_F = the partial pressure of the internal quasi-fluid environment (F)

, each being subscripted according to the actual direction considered. The corresponding conditions for the composite stability of the quasi-solid (S) substructure are,

$$p'_{Sx} > 0 \;,\; p'_{Sy} > 0 \;, \text{ and } \; p'_{Sz} > 0$$

The internal partial pressure terms p'_S and p'_F may, or may not, exhibit quantitative directional sensitivity. For the case of a material specimen which is isotropic, then,

$$p'_{Sx} = p'_{Sy} = p'_{Sz}$$

and

$$p'_{Fx} = p'_{Fy} = p'_{Fz}$$

Comments with regard to the external partial pressure terms p'_E, and the question of possible directional variation will be witheld for the meantime.

According to the diphase model, material systems in their "natural" state are effectively pre-loaded. Thus, although the term "additional" has frequently been used above when referring to the unrecognised background environment, if viewed in a proper perspective it is perhaps more fitting that the effects of applied loading regimes be qualified in this manner. The external boundary restraint approach adopted earlier went as far as considering the volumetric compressibility of isotropic solids. This would therefore seem an excellent starting point for an investigation of those systematic changes concomitant to the additional effects of applied loading regimes in the context of the current diphase model.

The general symbol R' will henceforth be employed (and subscripted where necessary) to denote any external partial pressure which derives from an applied loading regime. Corresponding changes in either p'_S or p'_F with respect to their values in the original regime-free state will be signified by a conventional "$p'_0 \pm \Delta$" notation. For the case of an isotropic diphase material subject to uniformly forced volumetric compression ($R'_x = R'_y = R'_z = R'$), a single equation of external/internal equilibrium is quite sufficient since it obtains regardless of reference direction;

$$\text{i.e.} \qquad R' + p'_E = p'_S + p'_F = [p'_S]_0 + \Delta p'_S + [p'_F]_0 + \Delta p'_F$$

It may be seen that the effect of the change in external conditions, like that of the original external environment itself, is partitioned between the internal parameters p'_S and p'_F. As the former increases, the stability of the skeletal substructure of S components is thereby enhanced. Providing the individual S components retain their integral status, the possibility of load-induced systematic failure is non-existent. (A brief examination of Figure 4.21 should reveal that each S component could well be treated as a "body" under load in its own right, the composite stability of which could then be described via a lower-order diphase representation, etc., etc. However, for the moment at least, the break-down of S components - as distinct from their structural dissociation from one another - will not be contemplated.)

Having previously alluded to the case of applied isotropic compression when dealing with the p-ρ response of solid material bodies, it is important to emphasise that the distributed time-averaged environmental restraint parameter, r_E, then adopted is not equivalent to the initial systematic restraint, $[p'_S]_0$ ($=[p'_E]_0 - [p'_F]_0$) understood to prevail above - this despite the common "interactive" basis of the rationale which links them. The external boundary approach, which might aptly be described as a "fluid" treatment of solid behaviour, was concerned solely with the containment of a dynamic molecular or atomic system within a finite bulk volume; internal characteristics beyond the immediate boundary region were of no special significance. The quasi-solid components "relevant" to that treatment belong to a somewhat lower order of discrimination within the general hierarchical scheme envisaged for materials than do the S components of the present "solid" diphase model. It may, however, be safely inferred from the fundamental "nesting" feature endemic to the extended hierarchical view that $[p'_S]_0 < r_E$.

Before leaving the isotropic compression case and proceeding to examine the action of various applied regimes for which the S components do have some "relevance" as regards systematic failure, the pertinent question of possible changes in the local value of p'_E due to R' must be broached. Expressed as a partial pressure, the external distributed load contribution which derives from the presence of the background quasi-fluid can not be presumed totally insensitive to any dimensional changes experienced by the bulk material specimen; i.e. if the fraction of the gross external surfaces "seen" by the "unrecognised" environment increases, then so too does the equivalent partial pressure. Thus, the sum of the external partial pressures (the left-hand-side of the previous equilibrium condition) could strictly be written as,

$$R' + p'_E = R' + [p'_E]_0 + \Delta p'_E$$

Of course, if the dimensional changes induced by R' are relatively small, a "rigid body approximation" of $\Delta p'_E \simeq 0$ is not unreasonable (cf. the assumed local "constancy" or r_E employed during earlier developments).

Consider now the likely performance of the diphase model specimen when subjected to a uniaxial compressive regime, R'_z. Largely for reasons of convenience, the suitability of a rigid body approximation will be assumed; any secondary interactions between R'_z and p'_{Ez}, p'_{Ey} and/or p'_{Ex} are thereby rendered insignificant, and will thus be treated as negligible effects. For many materials (including concrete) the loss of "realism" entailed in such an assumption is quite minimal. Furthermore, although the exclusion of secondary effects necessarily limits the validity of any quantitative statements which might be elicited, it does not interfere with the basic principles involved in their derivation.

Equilibrium conditions:

$$\underline{z} \qquad R'_z + p'_{Ez} = p'_{Sz} + p'_{Fz} = [p'_{Sz}]_0 + \Delta p'_{Sz} + [p'_{Fz}]_0 + \Delta p'_{Fz}$$

$$\underline{x} \qquad p'_{Ex} = p'_{Sx} + p'_{Fx} = [p'_{Sx}]_0 - \Delta p'_{Sx} + [p'_{Fx}]_0 + \Delta p'_{Fx}$$

$$\underline{y} \qquad p'_{Ey} = p'_{Sy} + p'_{Fy} = [p'_{Sy}]_0 - \Delta p'_{Sy} + [p'_{Fy}]_0 + \Delta p'_{Fy}$$

or, by implementing the "original" conditions contained in equations 4.15,

$\underline{z}$ $$R'_z = \Delta p'_{Sz} + \Delta p'_{Fz}$$

$\underline{x}$ $$0 = -\Delta p'_{Sx} + \Delta p'_{Fx}$$

$\underline{y}$ $$0 = -\Delta p'_{Sy} + \Delta p'_{Fy}$$

Stability conditions:

in any direction, $p'_S > 0$ (general)

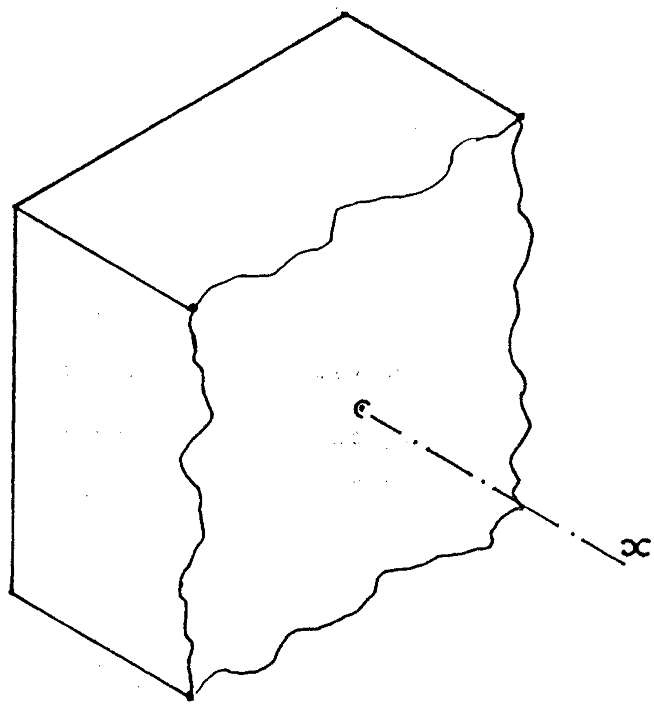
$\rightarrow$ in any direction perpendicular to z, $\Delta p'_S < [p'_S]_0$

$$(\text{e.g. } \Delta p'_{Sx} < [p'_{Sx}]_0 \;,\; \Delta p'_{Sy} < [p'_{Sy}]_0\ )$$

With the possible exception of the decreases in p'_{Sx} and p'_{Sy} forecast as being induced by the additional regime, R'_z, the various equations given above are self-explanatory. Only in the z direction does the solid specimen experience a change of external partial pressure, and this increase, R'_z, is shared between p'_{Sz} and p'_{Fz}. While the adoption of partial pressures for descriptive purposes precludes the general identity of $\Delta p'_{Fz}$, $\Delta p'_{Fx}$, and $\Delta p'_{Fy}$ (as it also precludes the general identity of p'_{Fz}, p'_{Fx}, and p'_{Fy}), the important fluid characteristic bestowed upon the internal quasi-solid complement, F, ensures that the changes must all share the same sense. (The partial pressure approach merely modifies the quantitative aspects of that fundamental hydrostatic principle relating to an omni-directional transmission of effect; the essence of the principle itself remains unaltered.) For static external/internal equilibrium to prevail in the x and y directions, the associated increases in p'_{Fx} and p'_{Fy} must be "balanced" by corresponding decreases in p'_{Sx} and p'_{Sy}, respectively - hence the pattern of signs employed.

At this juncture, it becomes necessary to expand a little on the exact significance of internal partial pressure terms such as p'_{Fx} and p'_{Sx}. Bearing in mind the spatial distribution and physical character of the systematic diphase "picture" envisaged (Figure 4.21), it would be somewhat inconsistent to suggest that these parameters offer a unique description of internal actions. Consider, for example, the first of the regime-free equations, 4.15,

viz, $$p'_{Ex} = p'_{Sx} + p'_{Fx}$$



EXTENSION OF FIGURE 4.22: Example of a General x Surface

In principle at least, a practically infinite number of internal x surfaces, each exposing a certain fraction of the S components, could be identified. (To qualify as an x surface, only the "corners" thereof need lie in an x plane - see Figure left.) For any one of these surfaces both p'_{Sx} and p'_{Fx} are gross averaged quantities by virtue of their definition. Each represents a notional mean of supposedly very many distributed local effects. To presume the latter (and hence the respective means for different surfaces) free from inherent statistical variation contravenes the spirit of generality deliberately embodied within the diphase model. The expression cited should not therefore be seen as merely constituting a single external/internal static balance relationship but rather as alluding to an almost infinite set of such equations, in which case the terms p'_{Sx} and p'_{Fx} can best be interpreted from a systematic viewpoint* as referring to ranged stochastic quantities. Of course, the arguments in favour of this width of interpretation with regard to internal partial pressure parameters may be equally well applied to all the other equations presented above in association with the diphase model solid.

Unlike the previous case of isotropic compression where the additional loading enhanced the inherent stability of the skeletal substructure, the action of a uniaxial compressive regime, R'_z, effectively threatens sustained composite integrity. In all directions orthogonal to z (of which x and y are particular examples) the degree of internal forced contact between S components is reduced. (As was highlighted during initial discussions of the box analogy, this reduction of interactive restraint has obvious connotations in relation to "predicting" a Poisson's ratio effect.) There is therefore a limiting value of R'_z beyond which the system as a whole can not maintain its bulk identity as a single body; i.e. as R'_z increases, a point will be reached where, in some "critical" direction, c, orthogonal to z, the element of forced internal contact is eventually eliminated on a particular c surface, thus contravening the general stability criterion for total composite integrity. At that point, and on that surface,

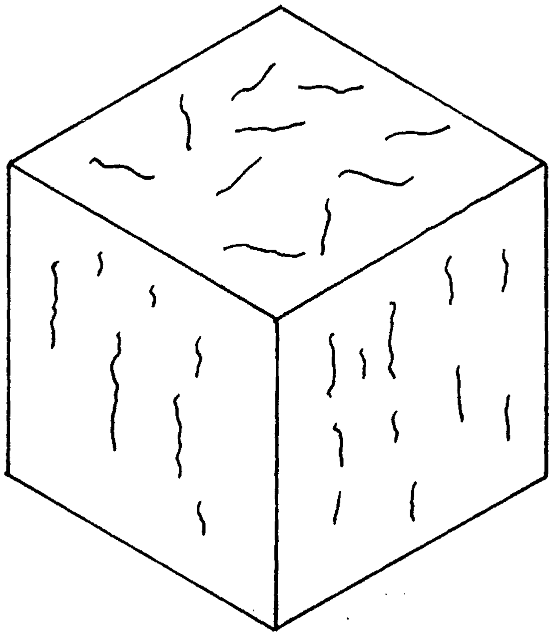
$$p'_{Sc} = 0 \quad (\Delta p'_{Sc} = [p'_{Sc}]_0)$$

and $$p'_{Fc} = p'_{Ec}$$

---

* Were the elementary mechanics of equilibrium the sole consideration, it would obviously be feasible to re-define p'_{Sx} and p'_{Fx} as overall means (i.e. as means of means) and thereby establish a single equation, but, since potential instability at the local level is also of concern here, the would-be convenience of such multiple averaging has nothing to recommend it.

(a)



(b)

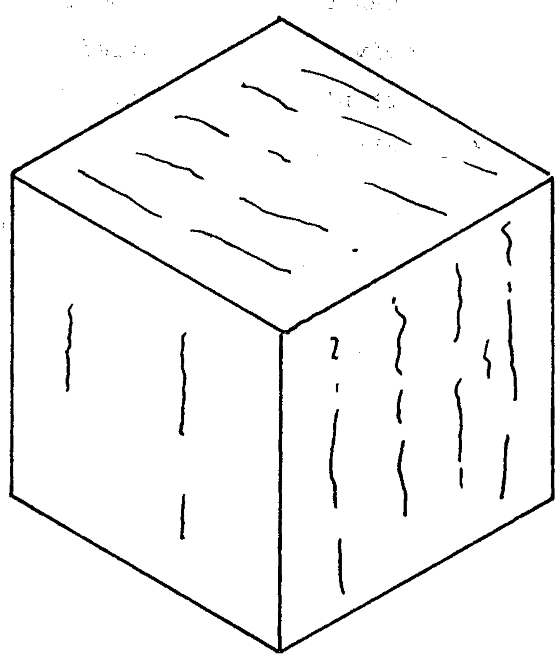


FIGURE 4.23: Early Crack Options Under Uniaxial Compression, R'_z

(a) No directional preference

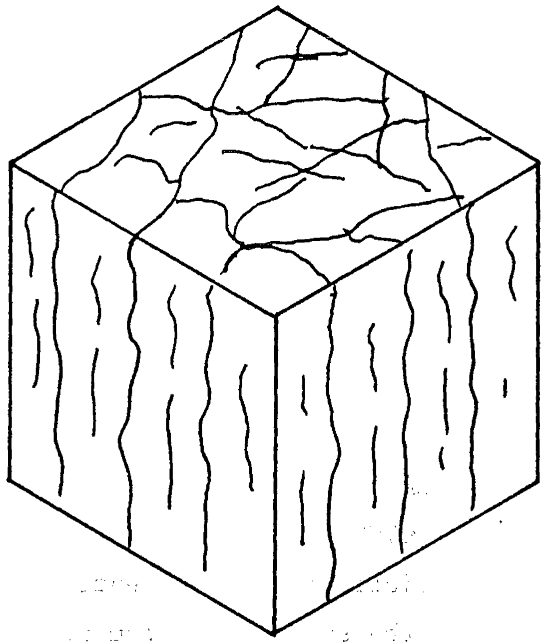
(b) Distinct directional preference

In the light of certain earlier comments, the two non-composite bodies thereby created could, if desired, be labelled as the "relevant components" of the original specimen as regards its performance under uniaxial compression, and the occurrence of their dissociation treated as a "failure" of sorts. This approach will not, however, be adopted here because the "new" (degenerate) two-part specimen system is ostensibly still capable of withstanding the action of the applied regime in a stable manner.

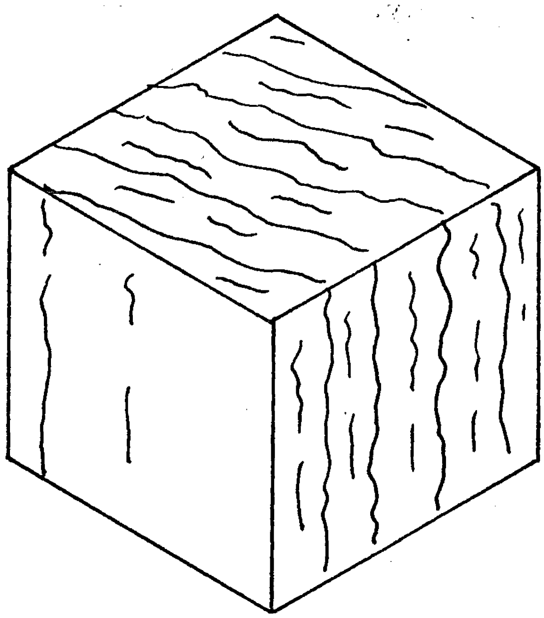
The formal recognition previously afforded to the innate statistical aspects of the diphase model implies that, at the point of transition from a single to a two-part specimen system, there exist other particular c surfaces for which $p'_{Sc} \neq 0$ ($\Delta p'_{Sc} < [p'_{Sc}]_0$). However, while this condition signifies a lack of total dissociation on these surfaces, it does not signify a total lack of dissociation there. As a result of local variations in the overall distribution of effects, some loss of forced contact between individual S components is to be expected. (Even a change to forced separation is not inconceivable on a limited regional basis.) This line of argument has two important corollaries. The first is that the localised dissociation which culminates in the degenerative transition from a single to a two-part specimen system is not sudden, but rather proceeds gradually as the pertinent p'_S value decreases with increasing R'_z. Secondly, the feature of partial dissociation, which obtains throughout the specimen, need not be restricted to the c direction; some loss of forced contact between individual S components may well occur in any direction orthogonal to z if local conditions are so favourable. Thus, to summarise, no special significance can be attached to the occurrence of the so-called transition in the context of forecasting the likely ultimate performance of the specimen system, unless by way of that transition, some further form of unstable mechanism were to emerge.

It may be inferred from the "gradual" feature of partial dissociation that "visible" cracks will begin to appear on the external surfaces of the specimen before the onset of systematic degeneration, growing in both extent and number as R'_z increases. The form of the crack pattern displayed will obviously depend upon the degree to which the appropriate ranged parameters, p'_S, (orthogonal to z) vary according to direction. The lower p'_S becomes, the greater is the probability of losing part of the "original" forced contact at the local level; however, if p'_S is significantly less in one particular direction than in any other, this will establish a directional preference for dissociative tendancies. In

(a)



(b)



FIGURES 4.24: Degenerate States Under Uniaxial Compression R'_z

(a) No directional preference

(b) Distinct directional preference

the absence of inherent directional preferences, as would be the case for a statistically isotropic specimen*, a crack pattern such as that shown in Figure 4.23(a) might reasonably be expected. The contrasting crack pattern of Figure 4.23(b) illustrates the likely consequences to a specimen of limited regional dissociation maintaining a particular directional preference.

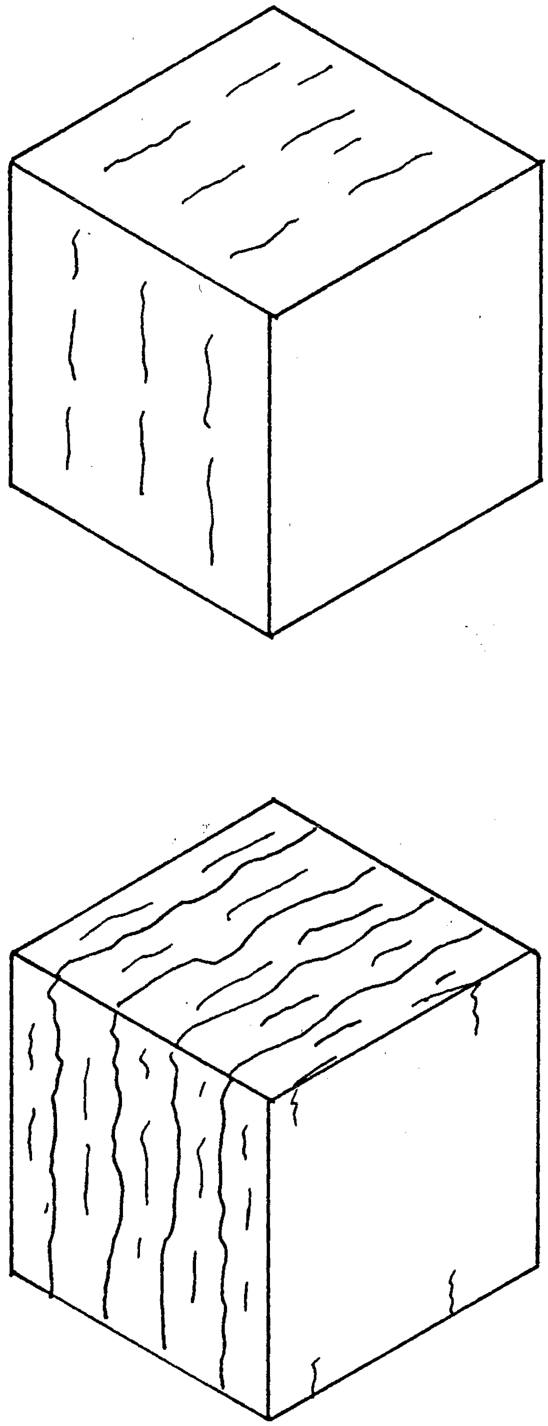
Figures 4.24(a) and (b) show the "predicted" appearance of the same two specimens after considerable systematic degeneration has taken place. It should be noted from the multi-part character of both that the diagrams refer to a stage of loading beyond that at which the first loss of "total" integrity is sustained. The predictions themselves are, of course, based upon the implicit acceptance (as suitable) of a "stepped" diphase model - an approach which simply involves successive reapplications of the transition rationale employed earlier. There is, however, one important factor limiting the ultimate suitability of such an approach. Since the S components were prescribed as discrete entities of a finite "size", the seeming trend whereby the specimen system merely undergoes progressive subdivision into smaller and smaller grouped elements can not be expected to continue indefinitely. Thus, for example, the "columnar" elements of Figure 4.24(a) and the "plate" elements of Figure 4.24(b) must remain stable if they are to resist the additional action of the applied loading regime; but, as their minimum dimensional "size" decreases towards an order of magnitude approaching that of the S components, the likelihood of their skeletal substructures retaining a stable form of internal arrangement decreases also. A stage will therefore be reached where the degenerate elements (which together now loosely comprise the specimen system per se) begin to "fall apart". Although the immediate consequences of such local instability will not be insensitive to the physical nature of the applied loading regime** it is possible to make certain qualified "predictions". For example, under uniform plane-strain conditions, individual elements on the verge of collapse may be "shielded" temporarily through the adjacent parallel presence of more stable elements but, as degeneration proceeds, bulk instability of the structural system will eventually manifest itself by way of an ultimate (maximum) load carrying capacity.

---

* For the purposes of general description, the qualification of a diphase specimen as "statistically isotropic" alludes only to the directional insensitivity of its ranged mean internal conditions (p'_S, p'_F) in the regime-free state.

** Further comment on this topic will be offered shortly.

(a)



(b)

FIGURES 4.25: Systematic Responses To Equal Biaxial Compression $R'_y = R'_z$

(a) Early cracking

(b) Degenerate state

From the descriptions of the progressive breakdown and failure process given above, the significant potential of the statistically isotropic diphase solid as regards its forming the basis of a behavioural model for that class of nominally brittle bulk materials devoid of particular directional characteristics (e.g. uniformly compacted concrete) should already be abundantly clear. Comparisons with reality will, however, be withheld for the moment until the likely performance of a statistically isotropic diphase specimen subject to biaxial compressive loading regimes is investigated. Having previously dealt with the case of uniaxial compression in some detail, this next inquiry can now be approached with much less need for step-by-step elaboration. A cube will again be chosen as the specimen shape and, as before, a rigid body approximation will be deemed operable. To begin, consider the case of equal biaxial compression, $R'_z = R'_y$.

Equilibrium conditions:

$$\left.\begin{array}{l}\underline{z}\\ \\ \underline{y}\end{array}\right\} \quad R' + p'_E = p'_S + p'_F = [p'_S]_0 + \Delta p'_S + [p'_F]_0 + \Delta p'_F$$

$$\underline{x} \qquad p'_{Ex} = p'_{Sx} + p'_{Fx} = [p'_{Sx}]_0 - \Delta p'_{Sx} + [p'_{Fx}]_0 + \Delta p'_{Fx}$$

or,

$$\left.\begin{array}{l}\underline{z}\\ \\ \underline{y}\end{array}\right\} \quad R' = \Delta p'_S + \Delta p'_F$$

$$\underline{x} \qquad 0 = - \Delta p'_{Sx} + \Delta p'_{Fx}$$

Stability conditions:

$$p'_{Sz} = p'_{Sy} > 0 \ , \ p'_{Sx} > 0$$

$$\rightarrow \qquad \Delta p'_{Sx} < [p'_{Sx}]_0$$

In this instance, and despite the "original" statistically isotropic feature, the action of the applied loading regime establishes a distinct and sustained directional preference (x) for dissociative tendancies. The pattern of "early" cracking may therefore be easily inferred - see Figure 4.25(a). If a line of argument similar to that adopted for the uniaxial regime is then followed, the ultimately unstable degenerate state shown in Figure 4.25(b) is predicted. At first sight it would appear that the diphase model must forecast a biaxial compressive strength, $[R'_z]_{max.} = [R'_y]_{max.}$, which is less in quantitative magnitude than is the strength value, $[R'_z]_{max.}$, likely to be derived from the same specimen through the application of a uniaxial compressive regime, R'_z. (Although the notions of linear superposition have no place here, some form of superposition principle governing the "addition" of effects must prevail, in which case the systematic inequality,

$$\Delta p'_{Sx} \ (=\Delta p'_{Fx}) \text{ due to } R'_z = R'_y \ > \ \Delta p'_{Sx} \text{ due to } R'_z \text{ alone}$$

, is eminently reasonable.) However, while the diphase model undoubtedly suggests a greater tendancy for dissociation in the x direction under a biaxial regime than obtains under an equivalent (equal R'_z) uniaxial regime, this does not in itself justify a similar inference in relation to ultimate strengths. The stability implications of the diphase model are closely allied to its essential statistical character, and therefore this too merits corresponding scrutiny. Thus, for example, it may be seen that the systematic preference for primary dissociation in the x direction under biaxial compression is offset by a reduced potential for extensive breakdown in any other direction orthogonal to z - hence the degenerage state illustrated in Figure 4.25(b) consists of "plate" rather than "columnar" elements. The emergence of plate elements in the present isotropic specimen/biaxial loading context is not of course symptomatic of an inherent directional "weakness" as it was in the case cited earlier during discussions pertaining to uniaxial compression. Indeed, since local "weaknesses" within a statistically isotropic specimen must be distributed in some near-random fashion throughout its bulk, the action of a biaxial compressive regime might well be interpreted as effectively suppressing the greater majority of these. The would-be prediction of a uniaxial compressive load capacity in excess of the biaxial compressive strength fails to take such factors into account - factors having an important bearing upon the stability of the degenerate state and hence upon its ultimate resistance to collapse. Appropriate modifications of the "prediction" to allow for these statistical aspects could therefore lead to its reversal!

It has been implicitly assumed above that the first transition from a single to a two-part specimen system has no special significance as regards overall stability. If, however, the biaxial loading regime takes the physical form of a contained fluid under pressure, this assumption is quite unwarranted; i.e. the presence of the pressurised fluid on a completely penetrated internal x surface automatically precludes external/internal equilibrium in the x direction, and the dissociated elements

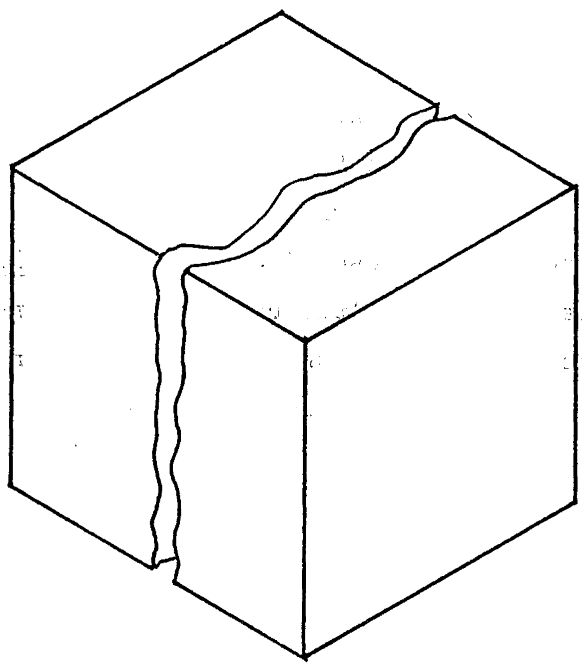


FIGURE 4.25(c): Ultimate Response To Equal Biaxial Compression Via Fluid Loading

are thereby forced apart - see Figure 4.25(c). Of course, were the fluid prevented from entering the specimen (by means of an impermeable sheath or such like) this mechanism could not occur, and the previous arguments of progressive degeneration and ultimate collapse would still apply in principle. Nevertheless, it is pertinent to remark here that the "constant stress" action of a biaxial fluid pressure regime, transmitted to the specimen system via a flexible membrane, poses a far greater threat to the structural stability of individual dissociated elements than does that of an "equivalent" uniform plane-strain regime which tends to shield the weaker of these from immediate collapse. In the context of unprotected specimens, it is possible to envisage a physical distinction between those which exclude a fluid loading medium in their uncracked state and those which the fluid naturally penetrates to some extent regardless of cracking: an alternative local distinction could be expressed in terms of a single specimen and fluids of varying penetrative capabilities. However, from a hierarchical diphase viewpoint which already recognises the effects of an internal quasi-fluid environment, F, neither form of distinction is of fundamental importance, both being concerned only with a question of degree. It is a basic concept of the diphase model that the effects of any external changes are communicated to the internal S and F fractions. The exact manner of the communication is not crucial. Therefore, whether or not the $\Delta p'_F$ terms include a "pore pressure" contribution from the loading medium, the "understanding" which the model provides is essentially unaltered. (Obviously, since pore pressure influences may have considerable quantitative significance in certain instances, the same will not be generally true of relative strength predictions.)

Attention will now be focussed briefly on the case of unequal biaxial compression, $R'_z > R'_y$. This will enable relevant comment on recorded experimental information (biaxial strength envelopes, failure modes, etc.) to be offered at a later point.

Equilibrium conditions;

$$\underline{z} \qquad R'_z + p'_{Ez} = p'_{Sz} + p'_{Fz} = [p'_{Sz}]_0 + \Delta p'_{Sz} + [p'_{Fz}]_0 + \Delta p'_{Fz}$$

$$\underline{y} \qquad R'_y + p'_{Ey} = p'_{Sy} + p'_{Fy} = [p'_{Sy}]_0 \pm \Delta p'_{Sy} + [p'_{Fy}]_0 + \Delta p'_{Fy}$$

$$\underline{x} \qquad p'_{Ex} = p'_{Sx} + p'_{Fx} = [p'_{Sx}]_0 - \Delta p'_{Sx} + [p'_{Fx}]_0 + \Delta p'_{Fx}$$

Stability condition(s):

$$\Delta p'_{Sx} < [p'_{Sx}]_0 \ , \ (\Delta p'_{Sy} < [p'_{Sy}]_0)^*$$

The early crack pattern displayed by a statistically isotropic specimen subject to unequal biaxial compression will be greatly influenced by the ratio R'_y/R'_z ($0 < R'_y/R'_z < 1$). Thus, although as in the previous case ($R'_y/R'_z = 1$) the x direction again emerges as the most likely for dissociative tendancies, the lower is the ratio R'_y/R'_z then the less likely is a distinct directional preference to be established. With $R'_y/R'_z \simeq 0$ the situation will be little different from that which obtains for the case of uniaxial compression.

4.3.4 Comparisons with Concrete Reality

Until the various micro-crack studies of the late 1950's/early 1960's (see Chapter 2) some confusion prevailed among concrete technologists as to the manner of concrete failure under "simple" compression. Up till then, a sizeable (but steadily diminishing) body of opinion had attributed the multiple splitting phenomenon so characteristic of standard compression specimen breakdown to the "secondary" wedging action of "shear cones" (cylinders) or "shear pyramids" (cubes/prisms), the formation of the latter being wrongly assumed as a primary effect. (The general absence of "shear pyramids" in the failed compressive zones of over-reinforced concrete beams appears to have been conveniently overlooked; it is also interesting to note in passing that even this "old" understanding, which is still unfortunately perpetuated by some modern elementary texts, has no logical connotations of "crushing" whatsoever!) The discrediting of the bulk wedging view - and the consequent reversal of interpretational emphasis as regards primary and secondary effects - is now generally accepted by concrete technologists but not, it would seem, by some of their colleagues in other related disciplines concerned with brittle material performance. Thus, for example, the following rather incongruous statement derives from a state-of-the art type material science text[314] published in 1970:

> *In compression tests at atmospheric pressure, rocks often show a characteristic longitudinal splitting,*

---

* The stability criterion for the y direction is not unique because the general sense of $\Delta p'_{Sy}$ is indeterminate (±). That shown above in parenthesis relates to a situation in which $p'_{Sy} < [p_{Sy}]_0$.

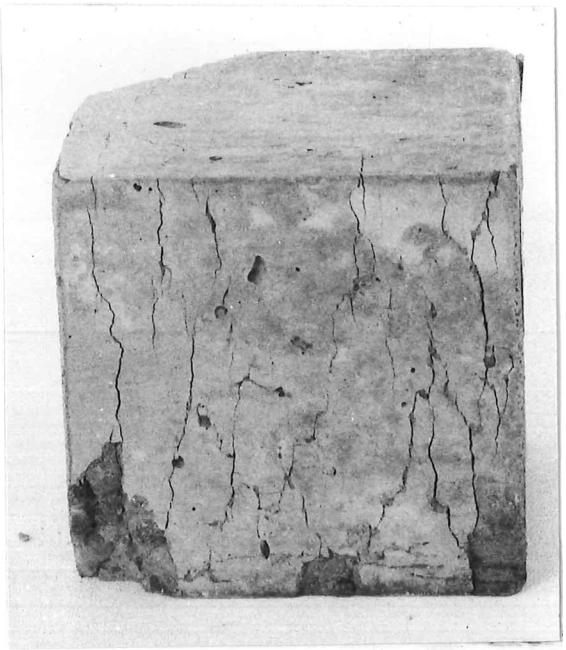
> *i.e., extension fracturing parallel to the axis of compression. This seems to be a secondary effect, associated with inhomogeneity of stress due to friction at the ends or with a "splitting action" of earlier-formed, wedge-shaped pieces resulting from initial shear fractures. Confining pressures of even a few bars can suppress the effect in jacketed specimens and lead to clearly-defined shear fractures at nearly the same stresses as those for the longitudinal splitting failures at atmospheric pressure. The longitudinal splitting can also be avoided by taking special precautions at the ends of the specimen to ensure uniform stress.*

As may easily be demonstrated in practice, the emergence of shear pyramids and shear cones during the routine compression testing of concrete specimens is a simple manifestation of the frictional boundary effects which almost inevitably arise as a somewhat indeterminate by-product of those various forms of locally-standardised test procedure (cf. the barrelling of cylindrical ductile metal specimens). While the quantitative magnitude of such "normal" secondary end-effects is hard to gauge (thus belying the would-be simplicity of most standardised compression tests), the consequences to the specimen of deliberate attempts to secure their elimination, or at least to minimise them, can be quite dramatic. Figures 4.26(a)-(c) show the typical pyramid-free results of testing concrete cubes to failure under applied uniaxial (vertical) compression when positive measures of this type are implemented.* Figure 4.26(d) pertains to the specimen (c), from which a proportion of the loose exterior material has been removed in order to expose the physical appearance of its disrupted interior. The immediate similarities with the previous Figure 4.24(a) and the associated descriptions thereof should need very little emphasis. Nevertheless, to reinforce the obvious parallels even further, Figure 4.27 shows a selection of free-standing (but severely cracked) columnar elements extracted from the remains of concrete cubes after "no-friction" uniaxial compressive failure; the full height of the "columns" corresponds to that of the laterally fragmented specimens from whence they came.

Providing a bulk concrete specimen is uniformly compacted and evenly cured, its description as statistically isotropic in the regime-free state would seem most reasonable: any internal features of locally random anisotropy are not, of course, thereby excluded. (While "original"

---

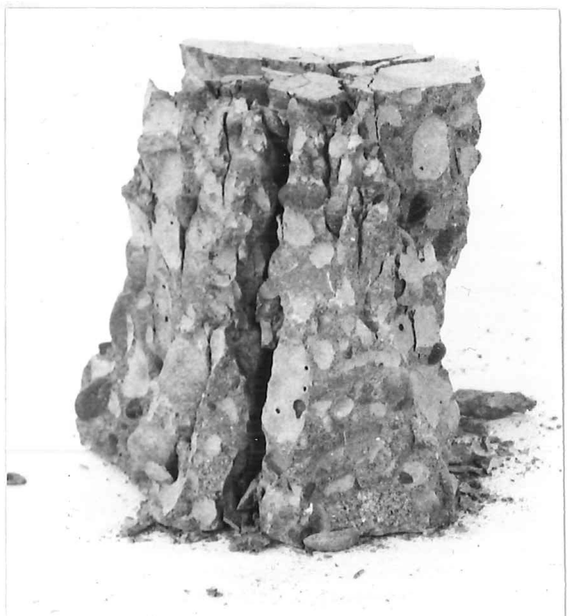
* The tests to which these photographs relate were undertaken using brush platens.



(a)

(b)

(c)



(d)

FIGURES 4.26: Concrete Cubes Tested to Failure in Uniaxial Compression

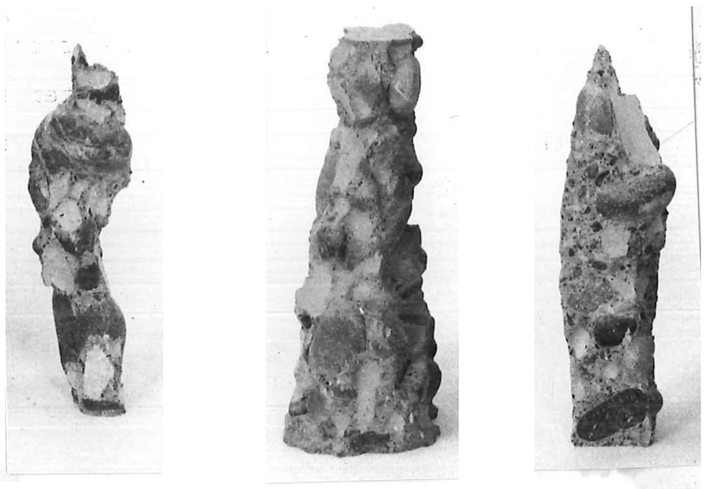


FIGURE 4.27: Degenerate Columnar Elements

bulk anisotropy in no way interferes with the general principles of the diphase model, it can render their implementation as regards the "prediction" of ultimate specimen performance a slightly more problematic exercise, involving "either-or" type arguments.) The manifestations of early cracking forecast by the diphase model were initially described as being "visible" on the external surfaces of the loaded specimen. In applying the model to the load-induced behavioural traits displayed by real concrete specimens (and drawing upon the vast store of recorded observation available in this context), it becomes possible to effect a qualification of such would-be visibility. Thus, although the first signs of regional dissociation within real concrete specimens are not, in fact, discernable to the naked eye, these can be - and have been - detected ("seen") by other means[13-16, 97, 98]; i.e. the "early" crack patterns of the diphase model may be considered representative of microcracking phenomena. Only when the specimen system becomes structurally unstable (in either a local or total sense), and alters accordingly, will obvious visual evidence of prior dissociations clearly emerge as surface macrocracking. Whilst on the topic of "reality", another aspect of the previous description offered for the behaviour of the diphase model - viz, that relating to the "gradual" spread of regional dissociations with the increasing intensity of the applied loading regime - should perhaps also be qualified a little before proceeding further. The appeal to statistical concepts which led to the first use of the term "gradual" in connection with <u>local</u> dissociative tendancies does <u>not</u> require that the progressive loss of internal forced contact be necessarily envisaged to occur in a quasi-continuous fashion, by way of infinitesimally small steps; i.e. the "gradual" designation accorded to the systematic changes might equally well encompass finite (spontaneous) steps at the local level. Experiment has revealed that the actual movements of microcracks in real concrete systems subject to increasing applied load generally defy a unique descriptive classification; <u>various</u> forms of movement are typically exhibited, ranging in character from slow steady (quasi-continuous) growth to periodic fast propagation via a series of distinct quantum jumps.

In common with those mechanical analogies of concrete behaviour reviewed in Chapter 3 (Section 3.5), the diphase treatment of ultimate specimen performance under applied uniaxial compression places considerable emphasis on the criterion of sustained <u>structural</u> stability as an essential prerequisite for composite strength. (Of course, since the majority of such structural analogies also tend to embody subsidiary strength concepts of the defined axiomatic type*, any further elements of rationale they might share with the diphase model are far from extensive.) Many of the unfortunate misunderstandings fostered by the traditional (but still very prevalent) "intuitive" view of strength undoubtedly stem from a general failure to fully appreciate the phenomenological significance of structural form. While the fact that structure is as relevant to a material specimen as it is to an articulated space-frame or to a multistorey building might seem blatantly obvious in the light of pertinent information made available (in ever greater quantities) by the progressive development of material science, such is the heritage of the strength topic that a number of rather nonsensical "blind spots" continue to resist rational illumination. The most glaring (and certainly the most persistent) of these historical enigmas is the somewhat naive interpretation of material strength as a form-insensitive property, devoid of all structural associations. (Despite its complete lack of factual corroboration, this almost groundless supposition still commands a remarkably wide degree of implicit acceptance within various sections of the "scientific" community.) The "consequent" distinction often tendered between structural failure and material failure is, however, of a quite arbitrary nature. Both failure "types" are invariably manifestations of the same physical phenomenon - viz, structural instability - albeit at different levels. It is not being suggested here that descriptively distinguishing features of scale and form are unimportant as regards a categorisation of the various possible classes of systematic instability, only that the presumption of some fundamental difference between macrostructure and microstructure is demonstrably erroneous. The basic non-equilibrium principles of structural instability are few in number but, because the potential for system diversity is so great, their range of practical application is correspondingly wide.

In the engineering field, the original motivation behind the mechanical testing of materials was generated not so much by any specific interest in the "actual" mechanisms of material breakdown but rather by a need for simple quantitative strength measures which could then be utilised in conjunction with working-load type analyses to ensure "safe"

---

* Perhaps the only mechanical anology to effectively predict the onset of overall (specimen) instability, without presuming some subsidiary "strength" to have been exceeded during prior loading, is that due to Uppal and Kemp (238), which involves the elastic buckling of a two-dimensional (arrayed) assemblage of structural components.

designs. The primary concern was not therefore with failure itself (or its understanding) but with the implicit numerical requirements for the latter's complete avoidance. Although, with the passage of time, this design-orientated (factor of safety) approach to material "properties"* has undergone significant modification, the qualitative aspects of specimen performance under applied loading regimes still tend to be treated as being of passing secondary importance compared to "useful" quantitative strength data.

The characteristic failure state of the specimens shown in Figure 4.26 lends itself most readily to a structural interpretation; the visual "evidence" could hardly be more plain! Such obvious manifestations of systematic instability under applied uniaxial compression are by no means restricted to the ultimate performance of cubical specimens, but it would often appear that observers of material behaviour see (and report) only that which they wish or expect to see. Thus, for example, the once common belief that concrete specimens subject to uniaxial compression "should" fail by shear was sustained for many years by the "confirmatory" observation of shear cones and shear pyramids, while the parallel observation that the onset of multiple cleavage cracking actually <u>preceeded</u> the emergence of the latter was conveniently ignored**; the observation that the "uniaxial" regimes producing such "shear failures" invariably included an indeterminate (but finite) contribution from an <u>applied</u> shear component also escaped widespread attention. Similarly, although many strength studies have highlighted the potential <u>quantitative</u> significance of varying specimen aspect ratios, few have offered much pertinent commont on the qualitative influence of different specimen geometries as regards exhibited failure traits. This restricted vision/limited reporting feature which represents, in effect, an unconcious suppression of information, has created a regretable situation where, even today, the structural connotations of concrete specimen failure are <u>still</u> not generally appreciated. However, on some occasions, the visual

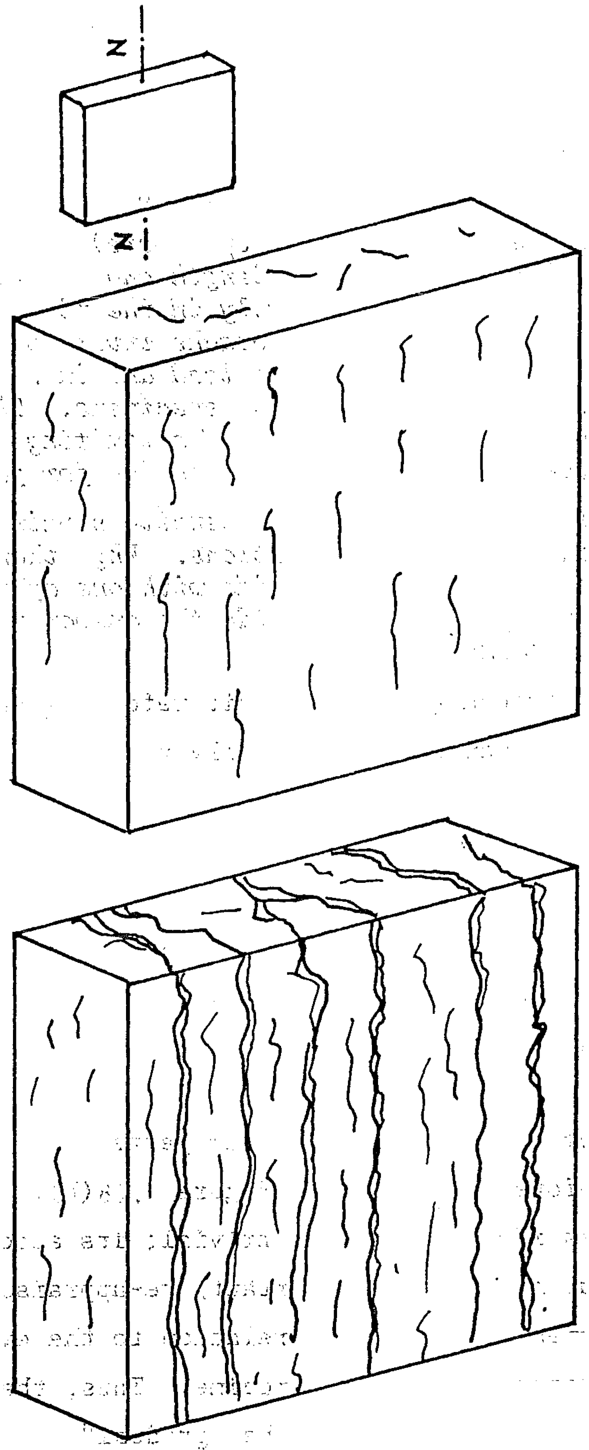
* The term crushing strength undoubtedly belongs to the "intuitive" (preconceived) vocabulary of the design-orientated approach. The choice of nomenclature is so utterly inappropriate that it must be supposed that those who first coined it had never witnessed the consequences of a uniaxial compression test in any great detail. Those who continue to offer the term as a rational description are presumably afflicted by a similar blindness.

** That the temporal sequence of the two breakdown phenomena was unnoticed is beyond all credibility.

(a)

(b)

(c)



FIGURES 4.28: Plate-Like Specimen Subject to Uniaxial Compression, R'_z

(a) Orientation

(b) Early cracking

(c) Degenerate state

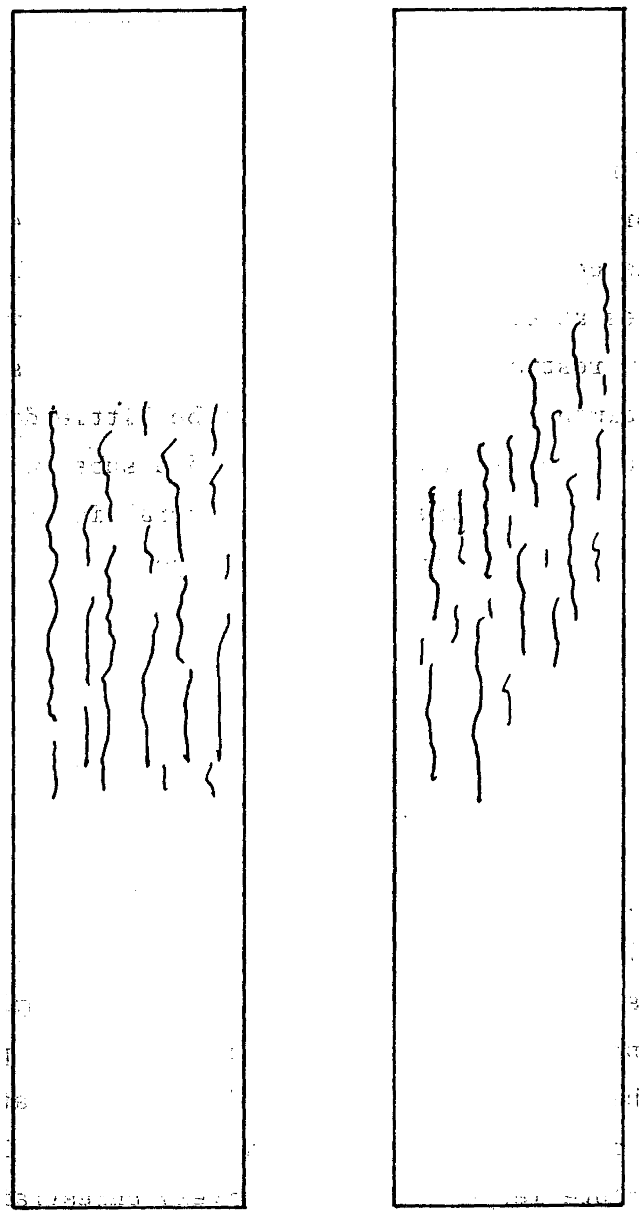
evidence has been heeded, despite its obvious non-conformity with certain other preconceived theoretical notions. For instance, the following observation and related comment are taken from G.S. Pandit[315]:

> *Tests on concrete tiles (plate specimens) whose thickness is small compared to the length and width, have shown that when loaded uniaxially in the plane parallel to the faces of the tile, the cracks invariably occured in the direction of the applied load and in planes perpendicular to the face of the specimens. This observation cannot be explained if a limiting tensile strain is taken to be the sole criterion for failure.*
>
> *Under uniaxial compression, the tensile strain is the same in both the lateral directions. Why, then, should the failure plane always coincide with one of them? Can the shape of the tile provide the answer to this observed phenomenon?*

Since the phenomenon to which Pandit refers is not commonly observed with other specimen types, the reply to his second query must naturally be in the affirmative. As to his first enquiry regarding a preferential direction of visible structural dissociation, the statistically isotropic diphase model furnishes an extremely simple rationalisation of the sustained coincidence which he cites. The basic equilibrium and stability criteria for a plate-like sample of the diphase solid subject to an applied uniaxial compressive regime, R'_z (see Figure 4.28(a)), are of course identical to those listed previously for a cubic specimen under the same loading condition. A similar pattern of early microcracking* would therefore be predicted (Figure 4.28(b)). The difference in specimen geometries is not, however, trivial; its accommodation requires a slight (but nonetheless important) re-appraisal of those previous (cubic) arguments advanced in relation to the eventual degenerate state induced by a progressive loading regime. Thus, the manner of formation of columnar elements through the "gradual" spread of regional dissociations is necessarily influenced by the relative thickness of the specimen; although local losses of forced contact between S components are equally probable in all directions orthogonal to z, the ultimate degree of spread required to produce a complete dissociation of internal surfaces may vary markedly according to exactly which direction is involved. For a "thin" plate there is a far greater likelihood of complete dissociations first occurring on particular internal y surfaces than on any internal x surface. Hence, as the specimen system becomes unstable and the columnar elements begin to "emerge" (fall apart), a

---

* The "cracks" alluded to by Pandit are of the immediately visible variety - i.e. macrocracks rather than microcracks.



(a) (b)

FIGURES 4.29: Failure of "Long" Specimens Under Uniaxial Compression

(a) Non-inclined

(b) Inclined

macrocrack pattern such as that shown in Figure 4.28(c) is to be expected.

Pandit's observations were derived from a study in which direct steps had been taken to minimise frictional transfer between specimen and loading regime. The more conventional approach to countering the end-effect "problem" associated with the "normal" application of uniaxial compression is indirect; i.e. by design, the test specimen (generally of either square or circular cross-section) is given an elongated geometric form so that the central portion thereof is far enough removed from the loaded boundaries at each end to be considered relatively free from superfluous secondary restraint. In the context of "understanding" the process of material/specimen breakdown, there can be little doubt that the qualitative results of such tests also lend a substantial measure of credence to the claimed aptness of a structural interpretation which places deliberate emphasis on particular aspects of composite stability. The supportive evidence is both clear and plentiful; the only apparent shortage is in the rather limited number of those who have been prepared to grant it formal recognition as regards theoretical developments. Perhaps the most significant piece of evidence relates to the ultimate behaviour displayed by "long" concrete cylinders and prisms which have an aspect ratio of sufficient magnitude to preclude the visible formation of cones/pyramids*. For such specimens the emergence (and subsequent collapse) of columnar elements in the central portion, as illustrated in Figures 4.29, is typical[120, 316-318]. If those simplistic arguments which treat compressive strength as a purely material property were indeed valid, then alterations to a specimen aspect ratio would cause the physical extent of the failure zone to vary; i.e. the latter would reflect the absolute length of the relatively unrestrained central portion. However, this would-be expectation of corresponding change is unrealised in practice: the visible signs of failure shown by "long" concrete specimens having a fixed constant cross-sectional area but of different lengths are remarkably consistent, the failure "volume" exhibiting little or no dependence upon aspect ratio. That behaviour of this type bears the hallmark of systematic structural instability is

---

* Under normal testing conditions, this effective suppression of secondary phenomena generally requires an aspect ratio somewhat greater in magnitude than that associated with most "standard" concrete specimens. However, although the minimum aspect ratio necessary to achieve an observed failure state for which shear cone/shear pyramid manifestations are only notable through their absence is not insensitive to material, testing, and specimen details, it would seem that the lower boundary value rarely exceeds 3.5.

beyond question. By way of comparison, it is interesting to note that the "primary shear" interpretation commonly applied to (or inferred from) inclined crack patterns such as that shown in Figure 4.29(b) is dubious in the extreme; thus, while the overall pattern is certainly inclined, the cracks themselves are not!

Despite much misdirected (and largely wasted) effort, the performance of concrete specimens subject to applied loading regimes which provide biaxial compression has received a considerable amount of sound experimental study. (Unfortunately, because of an inadvertant, but not infrequent, failure to make due allowance for the influence of induced secondary restraint, the results of many early investigations - and, indeed, of a few more recent studies - claiming to deal with the behavioural response to "pure" biaxial compression actually bear very little relation to that topic; at best, such results and the "conclusions" derived therefrom are highly suspect - at worst, quite misleading.) However, as with the case of uniaxial compression just discussed, an appreciation of the structural aspects of material/specimen breakdown under applied biaxial compression is far from general. Again, those forms of restricted vision arising from an inflexible adherence to certain "fundamental" preconceptions often appear to have clouded the powers of both direct physical observation and consistent logic. Thus, for example, although it only requires a brief objective comparison of recorded experimental findings pertinent to biaxial compression to reveal most forcibly that unqualified references to <u>the</u> biaxial strength of concrete have no sustained practical basis upon which their justification might be argued, allusions to this mythical "material property" still persist in current literature. What is, perhaps, an even greater cause for concern (and despair!) is that these allusions somehow continue to escape question and hence, by default, be deemed rationally acceptable by the majority of concrete technologists*; while such unwitting assent is rarely explicit, the misunderstandings it fosters (and perpetuates) are nonetheless real.

Microcrack studies undertaken in conjunction with experimental programmes to examine the behaviour of concrete specimens subject to increasing regimes of <u>equal</u> biaxial compression have shown that the

---

* It is not, of course, being suggested that the impairment of observational facilities wrought by an over-reliance on preconception is necessarily unique to those who work in the concrete field. The blinkered approach to strength is very widespread in material and engineering science.

"early" phenomenon of progressive internal dissociation, which occurs throughout the body of material under test, is essentially as would be predicted by the statistically isotropic diphase model (Figure 4.25(a)). Also in accordance with the latter's forecasts, the ultimate form of systematic failure displayed by concrete specimens under equal biaxial compression is demonstrably influenced by the precise manner of load application. This feature naturally implies therefore that an external loading regime described solely in terms of an average stress is inadequately specified. If it were only the qualitative traits of specimen failure which were sensitive to the actual details of the loading technique adopted, the need for a more discerning specification would not be especially crucial in a practical sense; however, it is a well-established experimental <u>fact</u> that quantitative biaxial strength measures exhibit a similar sensitivity to the exact method of applied load generation employed and, furthermore, that the relative magnitude of the differences involved can be quite significant.

In the context of direct load application, it is generally possible to distinguish - at least in principle - between various classes of regime. Thus, for example, distinctions may be effected on the basis of the pertinent boundary conditions which different forms of regime impose upon the specimen under test. Some regimes can be reasonably classified as "hard"; these subject the loaded surfaces of the specimen to approximately plane-strain conditions. Others distribute the applied load in a more uniform fashion, almost independently of local surface deformations, and are commonly designated as "soft". For the purposes of practical description, the terms "soft" and "hard" are typically far from absolute. The spectrum of "real" loading regimes is wide. Nevertheless, most tend positively toward either one of the ideal extremes and hence the nominal hard/soft classification serves a useful descriptive function, even if, in some instances, it only provides a means whereby different regimes may be ranked according to the relative orders of their hardness/softness.

The physical nature of the medium which produces the surface loading on a solid specimen under test may also be used as a categorising feature. Pressurised fluids impart characteristically soft loading to the external surface(s) of a solid specimen; solid-to-solid regimes, on the other hand, may be either soft or hard. Although it is not uncommon for test systems to incorporate subsidiary load transferance via a fluid medium, the "applied load" to which the material specimen is ultimately subject is more often than not of the solid-to-solid variety. In this regard, it is interesting to note that the fluid-activated platen of a conventional concrete testing machine performs the same primary role of load-type conversion as do the impermeable membranes generally employed in routine soil and rock testing when "jacketed" specimens, surrounded by a contained fluid, are subject to "confining" biaxial cell pressures. However, despite the fundamental equivalence in the manner of such conversion, the final outcome - as it affects the tested specimen - is not completely identical for each case. Thus, while the stiffness of a typical testing machine platen is not infinite, its relative rigidity compared to that of a membrane (and, indeed, to that of most material specimens) is sufficiently great to adequately justify a hard/soft differentiation; of course, if the machine platens are capable of rotation during specimen testing, and/or if the mating surfaces of the platens and the specimen are geometrically incompatible, the "actual" loading transmitted to the specimen by the machine will be inherently softer than that which might be expected in the absence of such features.

The chief motivation behind the use of sheaths and membranes in the experimental realms of soil and rock mechanics is generally one of test specimen "protection", rather than of deliberate load-type conversion; i.e. the main aim is to nulify any would-be penetrative capacity (and hence the associated potential for internal disruption) on the part of the pressurised cell fluid by effectively isolating those specimens which are permeable to the latter in their natural state. However, the flexible membranes which are employed in practice to achieve this aim may do much more than simply eliminate natural surface porosity. Thus, for example, if cracks open up within the specimen, extending to its external surfaces, a typical membrane prevents (or, at least, severely inhibits) the cell pressure from being brought to bear on the new "areas" thereby created*. In addition, "real" membranes provide the "protected" specimen with a possible source of external boundary (surface) restraint. While, by design, this somewhat indeterminate secondary restraint is rarely significant in a quantitative sense, its consequential effects (both qualitative <u>and</u> quantitative) need not themselves be negligible as is generally assumed. Manifestations of structural instability can, of course, be <u>extremely</u> sensitive to the existence of even minimal restraint.

---

* Although ideal membranes, having infinite flexibility and infinitesimal thickness, which would not behave in this manner might supposedly be conceived of, neither would these hypothetical entities fulfill the "desired" role of specimen isolation.

(Obvious instances of this extreme sensitivity abound in regard to the various phenomena of elastic column buckling: e.g. compare the usual order of magnitude of the central restraint necessary to suppress the first mode of buckling of an axially loaded pin-ended column* to the relative degree of change in the ultimate load carrying capacity which the presence of such slight restraint produces.)

Mention is made above of the characteristic interference concomitant to the "protective" use of typical flexible membranes because the jacketed cylindrical specimen/confining cell (fluid) pressure techniques of geomechanics have also been employed in the field of concrete testing. Rather surprisingly, however, there are very few records of attempts having been made to obtain direct measures of equal biaxial compressive strength from concrete specimens by such methods. (The element of surprise expressed derives principally from the parallel observation that biaxial strength "results" yielded by other much more dubious loading arrangements - these being often beset by glaring inadequacies - are to be found reported in great quantity and detail in the relevant concrete literature.) For the most part, where cylindrical concrete specimens have been subjected to membrane-modified cell pressure (p), this regime has been supplemented by a hard solid-to-solid axial component (σ_a); i.e. the confining action of the lateral fluid pressure, transmitted to the specimen via the membrane, has generally comprised but the biaxial fraction of an overall regime of applied triaxial compression. Whilst the results from "normal" triaxial compression tests (σ_a>p) conducted in such a manner offer little reliable indication of what the corresponding specimen behaviour might be if σ_a were absent, inferences <u>can</u> be drawn in this regard from the results of those triaxial compression tests for which σ_a<p. (The latter are commonly designated as being of the "extension" variety**.) Inferred or indirect biaxial strength measures (i.e.

---

* It is pertinent to note here that, for the ideal case of a perfectly straight column (as per the classical Euler approach), the suppression of the first mode only requires the mere "existence" of a positional restraint mechanism at mid-height. No systematic demand for the provision of a finite force by the mechanism itself actually prevails!

** The term triaxial extension test is something of a misnomer. Although test specimens may in fact experience "tensile strain" in the direction of σ_a during the application of σ_a<p regimes, axial extension (with respect to the "original" longitudinal dimension) is neither a necessary nor an automatic general feature of the test per se. Of course, if p>>$\sigma_a \simeq 0$, relative elongation in the direction of minimum load will almost certainly occur as a consequence of the Poisson's ratio effect.

maximum p for $\sigma_a = 0$), determined by extrapolating sets of compatible $\sigma_a < p$ triaxial compressive strength data obtained from protected concrete specimens, are typically of around the same magnitude as the appropriate uniaxial compressive (cylinder) strength values, ρ_c (where σ_c = maximum σ_a for p = 0). Hobbs[132] has shown that this approximate equality, which can also be arrived at via interpolation, is borne out by the results of more direct measurement.

Unlike the majority of their imitators, the early pioneers of jacketed triaxial testing in the concrete field, Richart, Brandtzaeg, and Brown[124], realised (<u>and</u> were prepared to comment on the fact) that the various compressive strength results which they produced were necessarily influenced to some extent by membrane restraint, and that the effects of such restraint might well be especially significant in certain instances. Thus, in the context of equal biaxial compression (p only), their stated conclusions are limited to the somewhat reticent observation that *"the strength of the concrete in two-dimensional compression was at least as great as the strength in simple compression"*. The cylindrical specimens tested by Richart et al. were sheathed by thin metallic jackets. Subsequent investigators, working along similar lines of research into the quantitative nature of concrete strength, have been able to take advantage of pertinent technological developments in the manufacture and mechanical properties of synthetic rubber materials. It would seem, however, that the experimental convenience fostered by the changeover to synthetic rubber as the "standard" medium for protective membranes has brought with it a largely unwarranted sense of complacency regarding the possible significance of any secondary restraint derived therefrom. Very few "modern" workers, for example, have seen fit to formally reiterate the relevant statements of qualification offered by Richart et al.. Whether this omission indicates that the rather indeterminate (and potentially troublesome) question of secondary restraint and its effects is no longer considered problematic, or that such uncertainty is now deemed an "embarrassment" which should be suppressed rather than admitted, is unclear and must therefore remain a matter for conjecture only.

In the absence of a structural view of material/specimen strength, the neglect of membrane restraint might seem a trifling oversight. The actual forces of restraint likely to be generated are extremely small in a relative sense. However, where the systematic property of composite strength is governed by local stability requirements, the mere existence of these forces can exert an influence on structural behaviour out of all proportion to their quantitative magnitude. Important evidence* substantiating the structural view in the biaxial strength context (and hence contrary to the sizeable body of opinion which would have it believed that the effects of membrane restraint are trivial) has been reported by Langan and Garas(319). As part of a study on the failure of concrete specimens under complex (non-uniform) regimes these workers subjected cylinders to uniform biaxial compression through a technique which involved prestressed wire-winding. Like the surface loading provided by a pressurised (fluid activated) membrane, the "wired" biaxial confinement imposed by circumferential stressing is of the inherently soft solid-to-solid type; but, unlike the former, a wire-would regime is fundamentally incapable of supplying the specimen with a source of continuous longitudinal restraint. The specimens tested by Langan and Garas yielded biaxial strength values of approximately 0.44 - 0.55 σ_c, the failure mode being in each case a near-planar <u>separation</u> of the concrete cylinder via a <u>single</u> cleavage fracture "perpendicular" to its longitudinal axis. It is worth noting here that the various standardised tests employed throughout the world to obtain uniaxial compressive strength measures, σ_c, have one thing in common: viz, <u>all</u> are based upon a hard system of load application.

The soft biaxial strength results of Langan and Garas are obviously at variance with those derived from would-be equivalent regimes involving jacketed specimens subject to lateral fluid pressure. This very noticeable difference illustrates the practical futility of trying to formulate broad strength criteria based solely upon a rationale which appeals to the concept of elemental stresses "at a point". (Of course, localised cracking - the most which such limited criteria can ever be expected to predict - is far from a generally reliable indicator of impending systematic failure; if it were, and in the light of available knowledge concerning microcracking phenomena, there would be no alternative but to interpret concrete in its "unloaded" or regime-free state as having already failed!) The single-cleavage (macrocrack) fracture reported as a consistent physical characteristic of the wire-wound failure mode clearly mirrors that exhibited by "unprotected"

---

* Although the mainstream of concrete technology appears to have taken little heed of this evidence (presumably because it conflicts with other accepted or preconceived notions of how materials "should" perform) the writer considers it one of the most significant pieces of experimental information on concrete behaviour to emerge in recent times.

concrete specimens to which biaxial fluid (cell) pressures have been applied[320, 321]. In both cases, the first occurrence of a "complete" separation constitutes a systematic instability, and hence precludes any further increase in the intensity of the external loading regime. By way of contrast, the "corresponding" strength measure obtained from a hard regime relates typically to a much greater degree of specimen fragmentation; i.e. the physical nature of a hard regime may often provide a significant contribution to overall systematic stability, in which case the ultimate strength measure derived therefrom represents, in effect, the result of a progressive (internal) series of tests and "re-tests".

Fumagalli[320] has presented biaxial concrete strength data, drawn from tests on unprotected cylindrical specimens under lateral hydrostatic pressure, which lies within the range 0.2 - 0.4 σ_c. Similar results but of a slightly lesser magnitude - also pertaining to the use of water as the biaxial cell medium - have been recorded by others[322]. Some would undoubtedly claim that, because of pore pressure effects, such regimes impose indirect tension and are therefore not of a "true" biaxial character. Indeed, this view finds formal expression in several published works, e.g. that of Kupfer et al.[112]; many have chosen to promulgate the same sentiments less explicitly. However, the arguments commonly underpinning the "fundamental" objections (like the continuum-orientated notions from which these invariably spring) rely for their validity upon the implicit acceptance of a rather idealistic (and by no means irrefutable) interpretation of material systems. According to the rationale of the objections, a distributed pore fluid - despite its patently "internal" physical location - should not be treated as an actual component of a saturated material specimen but rather as a mere extension of the pressurised external loading environment to which the latter is subject. The potential ambivalence endemic to this particular modified continuum approach needs no further amplification here, bearing in mind comments made earlier on the question of hierarchical order within composite solid/fluid systems. (Had the idealistic "stresses at a point" philosophy led to any <u>consistent</u> understanding of the general material strength topic, the case for effectively depriving a pore fluid of its internal connotations might have been a little more plausible!) There is, of course, <u>no</u> strictly logical reason why a pore fluid can not be considered as an integral part of a material (specimen) system per se. In the author's opinion, it is the conceptual divorce suggested above which merits the greater suspicion. Given this "alternative" view, the would-be objections to Fumagalli's results cease to have immediate relevance and hence may simply be ignored; i.e. porous specimen behaviour under direct lateral fluid loading is open to interpretation as a "true" biaxial response.

Investigations into the effects of pore pressure on concrete have mostly been directed towards quantifying the potentially dangerous phenomenon of hydrostatic uplift in gravity dams[323-326]. These studies, motivated in the main by a need for emprical information with which to implement preconceived safe-design philosophies based upon elementary "limiting stress" notions, tend to adopt a somewhat superficial approach as regards the mechanics of failure. A clear and unambigous understanding of material breakdown processes is rarely sought. In fact, some recorded uplift data actually embodies assumed strength criteria. Nevertheless, despite such obvious shortcomings - a case for which might quite reasonably be argued on the grounds of practical necessity in the design context - results accumulated from the various uplift investigations do give valuable insight into the many local factors likely to influence concrete strength measures obtained via biaxial cell-pressure techniques. These include both specimen characteristics (size, aspect ratio, porosity, permeability, initial degree of saturation, etc.) and loading details (fluid type, exact form of increasing pressure regime, period of test, etc.). For example, work by Carlson[325] indicates that the smooth (moulded) external surfaces of concrete specimens can be significantly less permeable than their internal regions; as a consequence, some researchers[321] have taken deliberate steps to remove this "protective" layer through brief specimen immersion in dilute acid. Recent studies[322] nominally unconnected with the uplift topic but of some relevance thereto have demonstrated that sizeable differences in the measured values of concrete strength may be wrought by a change of biaxial cell fluid from water to nitrogen; in the tests referred to the use of pressurised nitrogen as a lateral loading medium on cylindrical concrete specimens yielded consistently lower strength measures than those derived from the more conventional water-activated cell. It is worth noting that this result would not be forecast by the majority of pore pressure/uplift theories. The sense of the recorded difference is, however, totally compatible with that which prevails between the results of Fumagalli and those of Langan and Garas; of course, a circumferential wire-wound regime imposes a loading condition upon a cylindrical concrete specimen which is essentially equivalent to that provided by a pressurised biaxial cell fluid to which the specimen is impermeable.

In practice, soft biaxial regimes suffer from one major disadvantage - viz, the principal load components are invariably of equal magnitude. A satisfactory technique for overcoming this unfortunate deficiency has yet to be devised. Hard biaxial regimes typically offer a high degree of versatility with regard to applied load combinations, but are often diminished in worth (and relevance) by inherent secondary restraint problems. As was mentioned previously, a failure to take into account the effects of induced restraint can render would-be "simple" strength data extremely suspect, if not totally invalid. The literature of concrete research contains many examples of such oversight. In certain instances - the biaxial compressive arrangement employed by Rosenthal and Glucklich[(131)] provides a particular case in point - the sources of additional systematic restraint are so conspicuous that these might almost be classified as primary features of the test procedure! The various problems associated with the presence of superfluous restraint are by no means peculiar to hard regimes (see earlier remarks on the secondary effects concomitant to the use of protective membranes). In fact, the previous comment regarding specific aspects of that work cited immediately above pertains to a "mixed" scheme of applied biaxial loading in which one component was soft (and indirect), the other hard (and direct). However, such is their physical nature that hard regimes are especially susceptible to the development of induced friction effects and the like. These effects need not, of course, be "secondary" in a quantitative sense; their contribution to the total loading imposed on a specimen might well constitute a relatively significant fraction. Unless precautions are taken to minimise boundary friction phenomena and other allied manifestations of mechanical interference, the behavioural traits exhibited by tested specimens are more than likely to reflect the influence of the latter to some non-trivial extent. Because of this fact, many superficial investigations into the performance of concrete specimens under hard biaxial compression have yielded what can best be described as "apparent strengths"; i.e. the ultimate data recorded relates to a nominal loading condition which did not prevail alone during the actual tests, its practical attainment as a sole regime being precluded by induced subsidiary loading of an indeterminate (but far from negligible) magnitude.

The hard "biaxial" results of Iyengar et al,[(327)] are typical of the high apparent strengths attributable to the presence of unrecognised secondary restraint within a testing arrangement. As has been emphasised by Vile and Sigvaldason[328], great care and attention to detail is required in the design of biaxial test apparatus to ensure that the concrete specimen is not inadvertently subject to a complex triaxial regime via the influence of boundary effects. A set of hard test data due to Fumagalli[329] illustrates most forcibly the extent to which superfluous secondary restraint can distort nominal strength measures based only upon primary (known) forces. This data actually prompted Fumagalli's subsequent work with the direct fluid regimes already mentioned. It derives from an experimental test series in which a number of square-plate type (L x L x t) concrete specimens were compressed biaxially under conditions approximating to a state of plane strain. The source of direct surface (L x t) loading comprised two pairs of solid steel platens activated by linked hydraulic jacks (equal jack forces). Throughout this series the characteristic specimen dimension, L, was held constant, but t varied. For t = L, i.e. for cubical specimens, apparent biaxial compressive strengths within a range 3 - 4 σ_c were obtained. (Here, the "uniaxial" reference quantity σ_c is a standard "cube strength" - itself the rather nominal product of an inherently complex, restraint-affected test.) Iyengar et al., working exclusively with cubes, recorded similar biaxial results when equal jack loads were employed*. However, Fumagalli found that as the specimen thickness, t, decreased ($L > t > 0.29L$) so too did the apparent biaxial strength; for the lowest value of t within the spectrum of thickness covered by the test series, the biaxial regime yielded specimen strength measures inferior to σ_c in quantitative magnitude. While these changes in apparent strength were without doubt "caused" by alterations in specimen geometry, the interdependence manifest is related far more to consequent variations in the systematic influence of that secondary restraint which necessarily obtained at the specimen/platen interfaces than it is to immediate differences in specimen geometry as such. The latter is certainly not a trivial aspect of the material/specimen strength and behaviour topic, but the order of the changes alluded to above is well beyond that which might reasonably be attributed to the direct influences of geometry alone. This assertion is confirmed by the hard biaxial results of others[330].

---

* Since the test programme of Iyengar et al. was aimed at the establishment of a complete biaxial compressive strength envelope, this also included many jack load ratios other than unity.

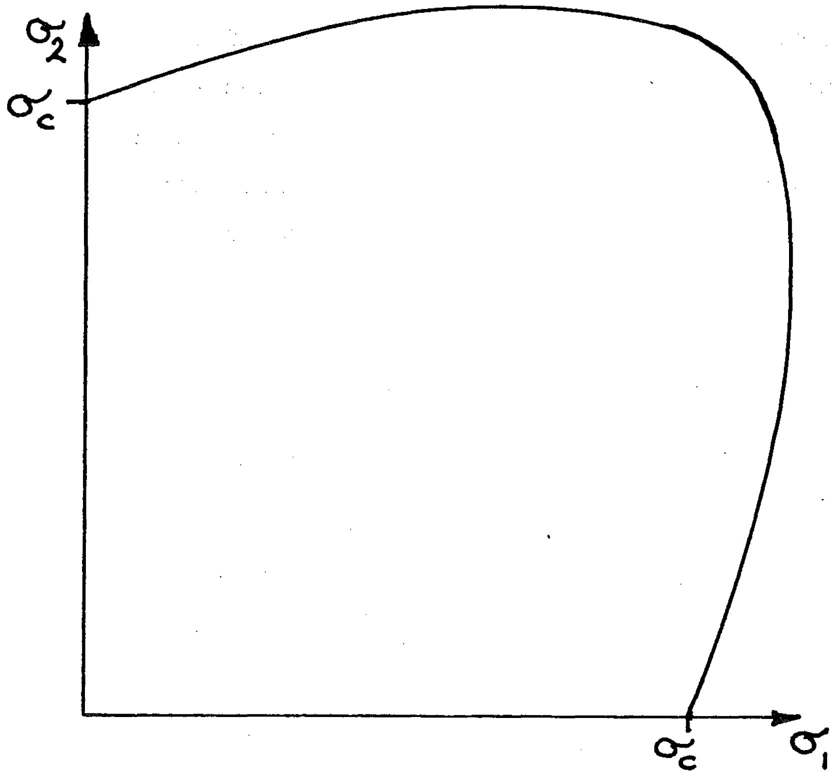


FIGURE 4.30: Typical Biaxial Strength Envelope Obtained Via "Hard" Regimes

Where attention has been paid to minimising the effects of induced secondary restraint during the testing of concrete and mortar specimens, nominal biaxial strengths derived from hard compressive regimes lie typically[330] within the range 1.0 - 1.4 σ_c. When the principal load components are of different magnitudes the greater of these is normally employed to compute the nominal strength value associated with the ultimate failure condition. It is well worth noting that the case of equal principal loads rarely yields a maximum biaxial strength - see Figure 4.30 The appearance of concrete specimens which have failed under hard regimes of unequal biaxial compression varies somewhat with the degree of applied load inequality. The designation unequal biaxial compression is of course infinitely flexible; it embodies all possible load component combinations between (but not including) the two local "extremes" of uniaxial compression and equal biaxial compression. The ordered variations in fracture pattern exhibited by concrete specimens under different forms of unequal biaxial loading simply reflect the physical consequences of this transitional aspect.

4.3.5 The Question of Shear

It may now be seen that there is more than ample experimental evidence in support of the claims to suitability of the diphase model presented as regards the breakdown of concrete specimens subject to uniaxial and biaxial compression. However, before proceeding to examine the model in the context of other loading regimes, the question of shear must be broached.

Apart from those references to superfluous boundary restraint, little mention has been made above of shear mechanisms as such. The main reason for this "oversight" is that many practical investigations have revealed local separation, rather than relative slip, to be the most prevalent feature of the internal disintegration process within concrete systems under applied regimes of uniaxial and biaxial compression, where the latter do not give rise to significant secondary shear effects. Theories of concrete failure based upon a limited shear capacity employ a rationale which is largely unsubstantiated by experimental observation in the "true" uniaxial/biaxial compression context. Although contrary to most forms of primitive expectation (including that of would-be "crushing") the physical predominance of the cleavage-type fracture mode is now well established. (Unfortunately, the situation as regards formal recognition of this phenomenon is less clear cut; somehow, despite the wealth of factual information available to act as a check on the logical consistency of preconceived "understandings", a number of plainly fallacious interpretations continue to defy total dismissal*.)

Current literature in the concrete field, when dealing with the strength topic, tends to place much emphasis on modern research developments; great store is set by the discovery/identification of microcracks and by the results of studies to detect their formation and subsequent growth characteristics. From this it might wrongly be inferred that an appreciation of cleavage failure modes of other than the single ("tensile") type is relatively recent. In fact, long before the term microcrack was ever coined many workers had seen the misnamed "uniaxial crushing" phenomenon in its proper light. The following quotation derives from the conclusions of Richart, Brandtzaeg, and Brown contained in their classic paper[124] of 1928:

> *In this stage* (nearing specimen breakdown under uniaxial compression) *there was a very great increase in the lateral deformation, which finally produced an increase in volume under continued loading, indicating by this lateral bulging that a splitting failure was taking place throughout the material on the surfaces parallel to the direction of the applied compressive stress.*

The accentuation afforded the word "splitting" is supplementary to the original text, its purpose being to highlight an old description devoid of crushing or primary shear connotations. Of course, no implied denegration of modern microcrack studies is intended here; in the absence of the detailed information made available by the latter, an element of reasonable doubt might still surround the physical realities of those local breakdown processes within concrete systems which culminate in macroscopic failure.

There can be little question that shear mechanisms do play some role in the ultimate disintegration of concrete specimens subject to true uniaxial compression. It would appear, however, that this role is not of a primary nature and that it is largely a reflection of the local heterogenuity imparted to concrete systems by the distributed presence of

---

* Notions which are demonstrably false, but "intuitive", are among the most difficult to dispel - regardless of the weight of contradictory evidence. The difficulty is compounded when such intuition engenders a degree of theoretical convenience. For example, those sentiments expressed in an earlier quote attributing the multiple cleavage associated with the uniaxial compressive falure of rock specimens to a macroscopic "wedging" action have since been re-echoed by Hobbs(132) in connection with concrete.

aggregate particles. Cement paste specimens display what might be termed a purer form of multiple parallel cleavage than do equivalent mortar or concrete specimens[19, 331]; i.e. the structural "falling apart" of longitudinal elements alluded to previously is far more immediate in the visual sense. To quote Spooner[331],

> *It is interesting to observe that apart from the obvious* (longitudinal) *cracks in the cement paste specimens the material appears undamaged. It is similar to a china plate which has been dropped, the plate is broken but each piece appears undamaged.*

As was highlighted much earlier, normal aggregate particles - especially of the coarse variety - can impede the progress of a spreading (separation type) crack and cause this to deviate from its previous trajectory. The composite crack patterns exhibited by mortar specimens subject to regimes of increasing uniaxial compression tend to assume, once established*, a near-parallel configuration: any deviations from the overall trend are generally slight. With concrete specimens, such local deviations may be somewhat more pronounced, requiring the near-parallel designation of the established crack pattern as a whole (which still obtains) to be interpreted a little more loosely. (This need for lattitude does not impair the fundamental aptness of the composite description offered.)

Figures 4.26 embody clearly discernable evidence of the characteristic departures from a strictly parallel crack configuration manifest by degenerate concrete cube specimens under "friction-free" uniaxial compression; concrete prisms tested likewise display similar traits[88]. When the degenerate state is attained, through the multiple proliferation and coalescing of individual cracks, the once-whole concrete specimen is ultimately reduced to a near-parallel system of discrete, partially contiguous structural elements. Unlike the case referenced above pertaining to cement paste specimens, such elements generally show obvious signs

* Pre-existent microcracks in the "unloaded" state are unlikely to be preferentially directed unless special conditions prevail. Only upon their subsequent load-induced growth does a consistent pattern emerge. Similarly, the direction in which newly formed microcracks first extend during the initiation stage may not always correspond to that of the applied axial loading; with propagation, however, comes the tendancy towards preferential alignment(15). These comments are of equal relevance in the context of systems which contain coarse aggregates - the interfacial regions between the latter and the mortar phase typically sustaining the bulk of "early" microcrack activity.

of accumulated prior damage; also, because the physical presence of coarse aggregate particles is likely to have "forced" local inclinations and angular diversions on the paths of their formative cracks, the resulting surfaces of these longitudinal concrete elements are rarely possessed of relative smoothness (see Figure 4.27). It is at this stage, where the retention of any load-carrying capacity depends very much upon the elements' remaining stable, that shear transfer mechanisms across adjacent surfaces (or, indeed, a lack thereof) can exert considerable influence on subsequent structural performance, providing circumstances permit. The behaviour of concrete specimens in the "descending branch" of a strain-controlled compressive test regime bears excellent testimony to such potential influence(19); so too (by default) does the fact that the load/deformation curves of cement paste specimens do not exhibit a stable descending branch under stiff incremental strain conditions.

In view of the relatively "late" stage of uniaxial compression at which shear mechanisms may* make a significant contribution to the ultimate forms of behaviour manifest by concrete specimens, their omission from the diphase model as so far presented is quite justifiable in that context. (A similar argument applies in the case of biaxial compression; the high degree of similarity is effectively such that the latter does not require detailed elaboration here.) The diphase model makes no hard forecasts regarding the exact mode of specimen collapse. It merely predicts that increasing uniaxial compression will induce a degenerate state through progressive multiple cleavage and that this process can not continue indefinitely because some form of composite instability must eventually emerge. Specimen failure, in terms of the model, corresponds to a loss of further load-carrying capacity; i.e. the attainment of maximum specimen strength (which need not, of course, be synonomous with complete collapse) is seen as a critical point in the degeneration process, beyond which the specimen system as a whole loses at least part of its previous identity.

For that broad class of nominally brittle materials conforming to a statistically isotropic designation, the cleavage predictions offered

---

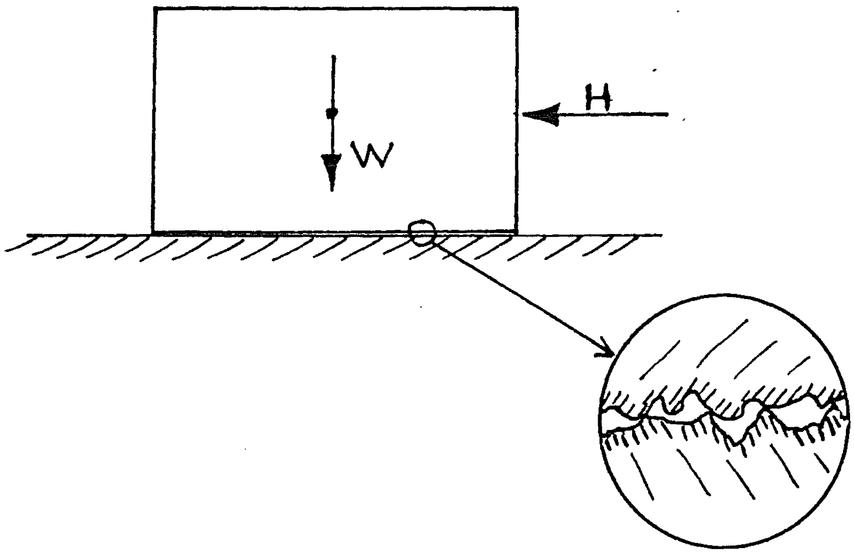
* The extent to which progressive shear degredation is allowed to occur is not insensitive to the exact manner of testing. Thus, for example, the very nature of many controlled regimes automatically precludes all possibility of a "descending branch".

by the diphase model - within the range of specimen behaviour it purports to represent - are completely borne out by experimental findings. (The apparent concentration above upon members of the concrete family is largely prescribed by the relative abundance of available information pertaining to cement pastes, mortars, and concretes.) With reference to such materials there is little or no valid evidence to suggest that the general process of systematic specimen breakdown prior to the maximum load condition under applied uniaxial or biaxial compression advances via local shear displacements. Would-be evidence in this regard which derives from regimes imposing appreciable secondary restraint must naturally be discounted. It is interesting to note that, even where uniaxial/biaxial compressive loading regimes permit concrete specimens to exhibit a "descending branch", the bulk of any progressive shear degradation which occurs therein typically involves slip mechanisms across elemental surfaces created by "earlier" cleavage-type cracks[19]. The additional formation and propagation of predominantly shear-type cracks is rarely observed as a characteristic phenomenon. Of course, the aspect of time sequence, which has not always been fully appreciated, is very important here. Impressions gained solely from the appearance of a collapsed specimen can be misleading in the extreme, since the manner of the final instability may reflect a distorted picture of those contributory factors which preceed it. Thus, for example, the degenerate specimen shown in Figure 4.26(c) displays among its many signs of impending collapse a horizontal crack! Without the knowledge that this crack occurred after the onset of composite instability, its interpretation would be rendered a most challenging exercise. Several workers have seen fit to emphasise particular traits of concrete specimen collapse; the following passage is taken from a paper by Kupfer et al.[112]:

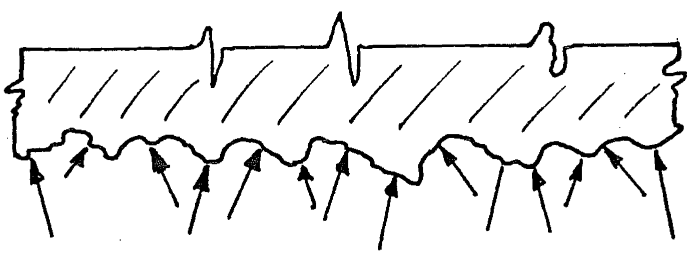
> *In the* (strain-controlled) *tests under uniaxial compression numerous microcracks parallel to the direction of the applied load were formed. Complete collapse of the specimen was accompanied by the formation of one major crack which has an angle of approximately 30 deg with respect to the direction of the externally applied load. Specimens subjected to* (equal) *biaxial compression showed similar microcracks parallel to the free surfaces of the specimens. At failure an additional major crack developed which had an angle of 18-27 deg to the free surfaces of the specimen.*

In the writer's opinion, such emphasis on the details of "major" cracks is somewhat misplaced. (Naturally enough, the author has no complaint regarding the stated microcrack information.) Prior to the emergence of

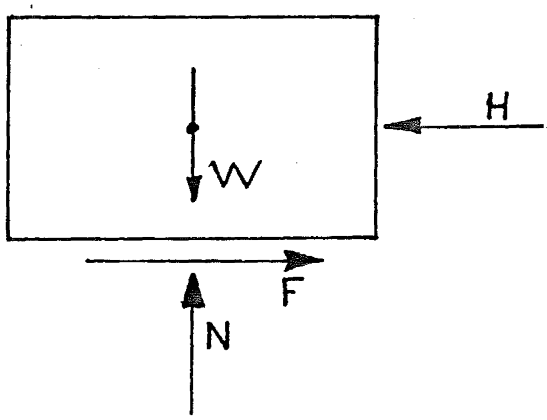
(a)



(b)



(c)



FIGURES 4.31: Matters of Friction

(a) Block of weight W subject to horizontal force, H

(b) Forces on the lower block surface

(c) Conventional free-body

these fractures the specimens had already sustained progressive structural damage: their mechanical strengths had been surpassed. Whether the complete systematic instability epitomising the final collapse of the specimens was attributable to "major" cracks, or vice versa, is therefore of relatively minor concern.

Despite the (now justified) lack of consideration so far given to shear mechanisms within the framework of the diphase model, the concept of shear per se does have a formal place there. Indeed, having "predicted" regional separation of the quasi-solid components as the prime mode of systematic local breakdown under applied uniaxial/biaxial compression, an adequate resistance to any potential shearing actions such loading regimes might provide has been implicitly assumed throughout. None of the internal surfaces as yet examined has required an appreciation of shear. Surfaces can, however, be identified in the two loading cases studied for which the absence of net shear transfer would preclude composite structural stability.

Perhaps the simplest means of understanding the inherent capacity for shear transfer embodied within the diphase model via a very brief contemplation of the important physical aspects of friction phenomena. Figure 4.31(a) shows a block of weight W resting upon a horizontal surface. If a horizontal force, H, is applied to the block, slip (shear displacement) will occur unless the nature of the block/surface interface is such that a frictional equilibrant can be generated there. Where it exists, static resistance to the shearing force*, H, derives from interfacial roughness; i.e. the equilibrant is generated through mechanical interference between the "mating" surfaces. Figure 4.31(b) shows a selective representation of the many discrete normal contact forces likely to be experienced by the base of the block. (Of course, if either the base of the block or the horizontal bearing surface were "perfectly" plane - see earlier comments in relation to ideal planes - these compressive contact forces would all be parallel to the vertical direction: such an interface would have no capacity to resist shear.) Before the

* The present discription of H as a "shearing force" does not conform to the more conventional use of this term. However, although normal to the surface on which it is shown to act, the applied compressive force obviously supplies the immediate potential for any block slippage which might take place.

application of H the resultant of the contact forces must be vertical if equilibrium is to prevail: i.e. the only force to be balanced thereby is W. Now consider the consequences to a rough interface wrought by a gradually increasing H. In the "early" stages, the magnitude of those contact forces with horizontal components which oppose H must increase, while those with horizontal components effectively complementing H must decrease. Since any change in an individual horizontal force component is accompanied by an appropriate change in the corresponding vertical force component, the progressive alterations are naturally governed by the overall stability requirement that the vertical resultant of the various contact forces continue to equilibrate the weight. This process of compensating changes (increases/decreases) can not, however, proceed indefinitely. Through gradual quantitative elimination, the number of contact forces with finite components complementing H will steadily diminish. A situation will therefore eventually arise where an increase in the net horizontal component of the contact forces would produce (if it were physically possible) a net vertical component in excess of the weight. At this point the system becomes unstable. As H can no longer be balanced a dynamic condition obtains, manifesting itself through slippage of the block relative to the bearing surface.

The conventional applied mechanics treatment of friction phenomena tends to adopt a rather axiomatic approach and hence rarely accentuates the physical aspects of interfacial roughness to any great extent: a conceptual association of smoothness with a lack of friction is generally mentioned, but little else. Figure 4.31 (c) illustrates the manner in which the frictional force, F, and the "normal" component, N, pertinent to the case just cited, would be typically represented on the relevant free-body diagram. While this "picture" is quite satisfactory as regards the forces involved it does have certain implicational deficiencies. Providing these are recognised the convenience it offers justifies its use; all-too-often, however, such convenience is unfortunately mistaken for physical realism. Despite its linear depiction in Figure 4.31(b) the lower surface of the block can not be interpreted as possessing the qualities of an ideal plane. (If the surface were indeed plane no frictional resistance could be generated.) The common assumption that friction forces act tangentially on a plane is therefore totally nonsensical.

At first sight it might appear as if the immediate criticism offered verges on the unnecessarily pedantic: i.e. the difference between interpreting forces as acting parallel to a hypothetical plane and considering the same as acting on that surface might seem slight. Not so! Since it effectively contravenes the "understanding" from which it derives, the latter constitutes more than mere loose description; all logical consistency is automatically forfeit by any realistic acceptance thereof. It is not suggested, of course, that shear stress parameters are bereft of usefulness in an applied sense - only that their links with physical "reality" are somewhat tenuous. As a concept in its own right (rather than as a quantitative parameter) shear stress is quite meaningless. Bearing in mind the considerable emphasis bestowed upon shear stress by the practitioners of continuum mechanics, the author would contend that the potential for logical inconsistencies contained therein fully merits the attention it has been given here. (The potential is alas realised* much more frequently than it is recognised.)

Hypothetical planes, in the role of reference quantities, are of course by no means foreign to the diphase model. The concept of partial pressures, for example, is framed specifically in terms of gross projected areas which may well be visualised as pertaining to "equivalent" ideal planar surfaces. Although such ideal planes are generally amenable to some form of quantitative determination - hence their usefulness - there is no suggestion that these "exist" in other than a notional sense (Unless the significance of measurement is somehow misconstrued - see earlier comments on this topic - the fact that hypothetical entities can often be quantified by practical means does not constitute a fundamental paradox.) The natural element of uncertainty which epitomises the real world only perplexes those who, whether by design or ignorance, fail to recognise it; i.e. uncertainty only assumes the status of an enigma within those philosophies which effectively deny it meaning from the outset. Much of the ambivalence endemic to the continuum view of matter stems from this implicational premise of complete determinism. That real surfaces are perforce "rough" is not a trivial assertion. Indeed,

---

* Other than by convenient (?) definition, it is difficult to imagine the manner in which an ideally plane surface might ever resist tangentential forces. That such surfaces do possess a capacity for shear resistance is, however, the mysterious connotation which clearly emerges from most "practical" continuum mechanics texts.

compared to the formal recognition of such inherent roughness, the extent to which real surfaces depart from conforming to the mathematical abstractions of absolute planeness and/or smoothness is relatively unimportant. This non-crucial aspect, the essence of which enables continuum mechanics to operate as a useful model of the real world despite its deficiencies, is consequential rather than merely fortuitous; it arises because deviations from the ideal are themselves beyond the realms of obtainable certain knowledge.

Consider now the application of the partial pressure approach to the static friction example described recently. The force N of Figure 4.31(c) is the vertical resultant of the interfacial contact forces acting on the block: it is, however, only a "normal" force insofar as its direction is perpendicular to the notional plane associated with the cross-sectional area, A_n, the latter corresponding to the gross horizontal projection of the contact surface. The vertical partial pressure on the block arising from inter-solid contacts on its base may therefore be written as,

$$p'_{Sn} = \frac{N}{A_n}$$

This quantity is totally analagous to those p'_{Sx}, p'_{Sy} and p'_{Sz} terms employed during the earlier development of diphase equilibrium/stability relations. But, unlike the various internal surfaces then concentrated upon, the interface of current concern also sustains a net (resultant) "tangential" force, F. Since the presence of F as a systematic reaction to H owes its very existence to interfacial roughness, it would seem somewhat inappropriate to express the relevant partial pressure, p'_{St}, in terms of A_n. Accordingly, a notional quantity, A_t, will be defined representing some manner of gross interfacial reference area, orthogonal to A_n. Thus,

$$p'_{St} = \frac{F}{A_t}$$

It may be seen that while p'_{Sn} is equivalent to the conventional "mean normal stress", σ_n, there is no such complete interchangibility between p'_{St} and the "mean shear stress", τ. The latter is, of course, defined as,

$$\tau = \frac{F}{A_n}$$

In contrast to the axiomatic treatment of friction phenomena which defines the no-slip condition ($F<\mu N$) via a "coefficient of friction", μ, the physical rationalisation offered previously develops the pertinent stability requirements from the elementary concepts of balanced force equilibrium. The vertical force, N, is independent of F (N must equal W throughout if vertical stability is to be maintained). However, since the "resultant" forces F and N are themselves components of a total contact resultant, it would not seem unreasonable to partition N such as to reflect the contribution, N_F, of those local changes responsible for the generation of F,

i.e. $$N = N_F + B$$

, where B is the necessary "balance" of effect. Constitutive definition can now be used to enlarge upon the meaning of A_t;

viz, $$N_F = F.\frac{A_n}{A_t}$$

As N_F increases with increasing H (H = F for horizontal equilibrium) B decreases towards its limiting lower-bound value of zero. The relevant no-slip condition therefore emerges as,

$$N_F \ (= F.\frac{A_n}{A_t}) < N \qquad \ldots\ldots 4.16$$

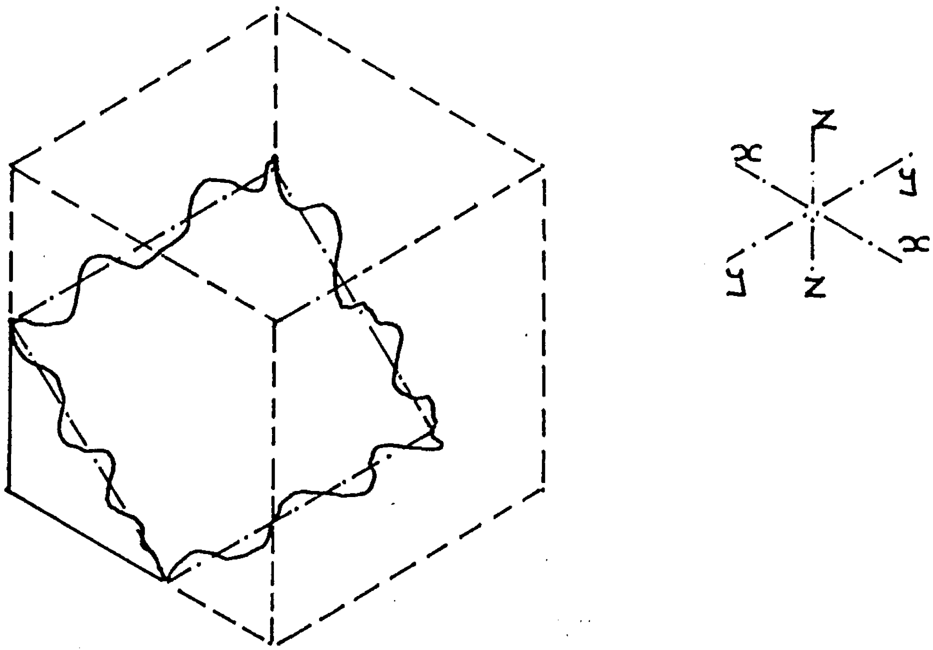
The conditional inequality 4.16 can be expressed in various forms. Two particular re-arrangements are of special interest:

viz, $$F < \frac{A_t}{A_n}.N \qquad \ldots\ldots 4.16(a)$$

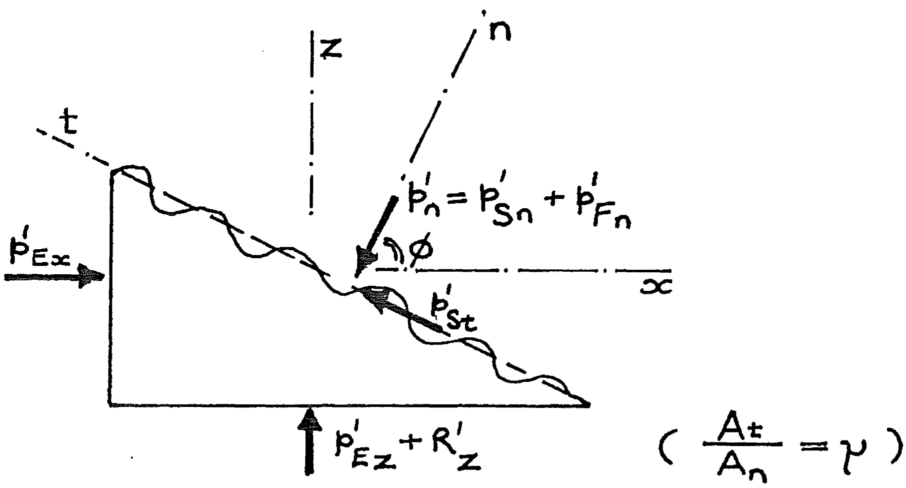
and $$p'_{St} < p'_{Sn} \qquad \ldots\ldots 4.16(b)$$

A brief comparison of 4.16(a) with the more conventional $F<\mu N$ reveals that a knowledge of A_n and μ enables the quantity A_t to be determined ($A_t = \mu A_n$); i.e. A_t is measureable! Thus, although it may have appeared otherwise when p'_{St} was initially introduced, the parameter itself is eminently practical. (If desired, a conversion between p'_{St} and τ can be effected using $p'_{St} = \tau/\mu$.) The partial pressure approach modifies the "in-limit" connotations typically bestowed upon μ so that it becomes thereby a quite general descriptive parameter for rough surfaces, rather than merely a relative indicator of ultimate frictional load-carrying

(a)



(b)



FIGURES 4.32: Wedge-Shaped Element

(a) General view showing reference axes

(b) End-elevation showing partial pressures associated with uniaxial compression R'_z

capacity. As a consequence of this alteration in both the meaning and the status of μ, the no-slip condition 4.16 (b) assumes an extremely simple form.

The incorporation of frictional aspects within the conceptual framework of the diphase model solid is a straightforward matter. By virtue of the particulate (skeletal) quasi-solid substructure prescribed as a constitutive feature of the model, the roughness of its "internal surfaces" is assured. Roughness is not, of course, the sole proviso for partially contiguous surfaces to manifest a finite resistance to shearing actions; some degree of forced contact (p'_{Sn}) is also required. However, this too is built into the diphase model as a primary facet. Accordingly, the diphase model has no need for an additional concept of "intrinsic cohesion"*.

Figure 4.32 shows a general view and an end elevation of a wedge-shaped composite element isolated from the cubical statistically isotropic diphase specimen of previous discussions. For the sake of argument, the parent specimen is considered subject to a regime of uniaxial compression, R'_Z. On the exposed internal surface two partial pressure components - one "normal" (p'_n), the other "tangential" (p'_{St}) - are identified; p'_n is itself the sum of two components, p'_{Sn} and p'_{Fn}, which derive from the interacting quasi-solid (S) phase and the presence of the complementary quasi-fluid (F) phase, respectively. (The fluid character initially afforded to F precludes the emergence of a significant p'_{Ft} term.) Both p'_n and p'_{St} are amenable to solution as functions of p'_{Ex}, p'_{Ey}, and $\emptyset$ via a conventional continuum-type equilibrium analysis ($p'_n \equiv \sigma$, $p'_{St} \equiv \tau/\mu$). To guarantee the structural stability of the element in relation to its remaining an integral part of the composite specimen system, two quite separate conditions must be satisfied;

viz, $\quad p'_{Sn} > 0 \quad$ (as previously)

and $\quad p'_{St} < p'_{Sn}$

---

* It is interesting to note that, in its conventional form of presentation, the Coulomb-Mohr equation, $\tau = \mu\sigma + c$, tends to suggest that the cohesion term, c, be directly associated with shear strength. Such an interpretation is widely inferred. The equation can, however, be expressed as $\tau = \mu(\sigma + a)$, in which case the quantity $a(= c/\mu)$ might reasonably be termed the "intrinsic compression".

The former governs an absence of complete S-component separation on the particular surface concerned, the latter a lack of relative slip potential there.

Having specified a uniaxial regime, R'_z, the likelihood of the first condition being violated on an "inclined" surface such as that shown in Figure 4.32 is remote; i.e. surfaces for which $\emptyset \approx 0°$ are inherently more prone to separational tendancies. As regards the possible occurrence of a slip mechanism, surfaces for which $\emptyset = 45°$ sustain the maximum intensity of p'_{St}*. Whether, with increasing load $(p'_{St})_{max.}$ ever attains a critical value $(p'_{St} = p'_{Sn})$ before the specimen degenerates via S-component separation on other surfaces obviously depends upon the exact nature of the pertinent systematic interactions and the relative order of μ. Thus, although the simple principles of statics are sufficient to enable the functional determination of p'_{St} and p'_n, they are powerless to differentiate between the respective contributions to the latter of p'_{Sn} and p'_{Fn}.

The dual stability conditions developed above are equally valid for composite elements which do not conform to the convenient (quasi-biaxial) geometry of Figure 4.32. (Ease of pictorial illustration was the main factor motivating the choice of prismatic wedge-shaped element shown therein.) Elements for which the reference n direction associated with the internal surface is not orthogonal to y are governed by the same general principles of equilibrium. Neither is the line of reasoning employed above restricted to the case of uniaxial compression: the response of the model solid to regimes of biaxial compression can be treated likewise. In the context of applied uniaxial/biaxial compression it may therefore be seen that the statistically isotropic diphase model actually encompasses two possible modes of systematic breakdown. Expressed in an alternative fashion, two classes of diphase model can be envisaged - one (henceforth known as Class A) which degenerates primarily via progressive cleavage-type separation, the other (Class B) which loses its initial regime-free composite integrity through the emergence of elemental slip mechanisms. While, by implication, the two classes are mutually exclusive this need only be so at a particular level: cf.

---

* This only applies because of the statistically isotropic feature. If μ exhibits directional sensitivity there need be no correspondence between the directions of $(p'_{St})_{max.}$ and $\tau_{max.}$.

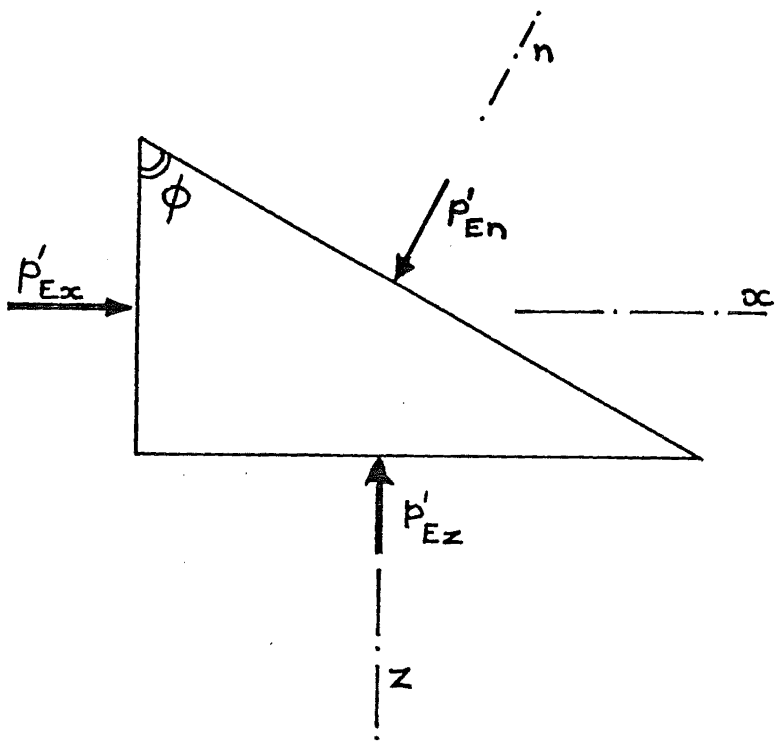


FIGURE 4.33: A Wedge-Shaped Body

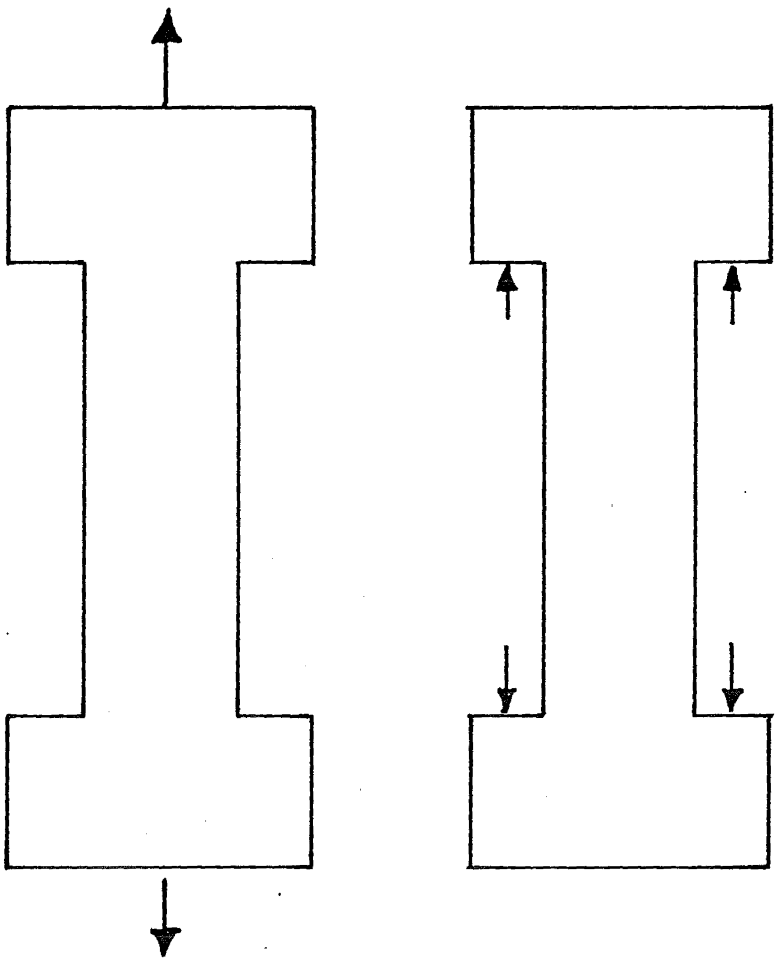
earlier comments on solids and fluids, etc. Thus, within an extended hierarchical scheme, the degenerate elements of a Class A system may themselves constitute Class B systems upon their emergence, and behave accordingly.

Experimental evidence gained in relation to the breakdown of concrete specimens subject to uniaxial/biaxial compression indicates that elemental separation is the prime characteristic of the failure process - hence the somewhat late introduction of the Class B model. Of course, not all material systems display the same propensity towards the cleavage mode in such circumstances. Having now demonstrated that the diphase model can accommodate shear mechanisms its scope of potential application has been broadened considerably. Although no attempt will be made to explore this potential in any great detail, it should be apparent that the existence of a shear rationale opens up many avenues to the diphase model: these include the behaviour of soil systems, ductile metals, etc., etc.

Before departing from the shear topic and proceeding to examine the response of the diphase solid to applied regimes of other than uniaxial/biaxial compression, a previous "loose end" must now be tied. Figure 4.33 shows an elevational view of a wedge-shaped body in the regime-free state. The angle ϕ is arbitrary. Because the background environment (E) is envisaged as a quasi-fluid no net "tangential" partial pressure components can derive therefrom; i.e. despite any external surface roughness, the appropriate partial pressures p'_{Ex}, p'_{Ey}, p'_{Ez}, and p'_{En} are each of the "normal" variety. (The reference y direction is orthogonal to the plane of Figure 4.33.) A simple application of elementary mechanical principles is sufficient to reveal that, for static equilibrium to prevail, p'_{Ex}, p'_{Ez}, and p'_{En} can not exhibit quantitative independence; due to the absence of any net surface shearing actions, a common magnitude is required. By re-orientating the wedge-shaped body with respect to the reference x, y, and z axes, this need for equality may easily be shown to include p'_{Ey}. Thus, in the context of static equilibrium, p'_E is a directionally insensitive parameter. (Such insensitivity obtains regardless of whether the body is isotropic.) As a consequence, p'_E will not be subscripted henceforth.

4.3.6 Tensile Regimes

The diphase model does not, of course, embrace the concept of absolute tension (pull). It is for this reason that whenever the term tension has been used in connection with the diphase model it has been



(a) (b)

FIGURES 4.34: Alternative Views of a Tensile Test

(a) Tension as "pull"

(b) Tension via applied compression

included within inverted commas. The hierarchical approach to material/structural systems only recognises tension in a relative (secondary) sense, i.e. as a lessening of pre-existent compression. (An analogous situation prevails in the rational study of thermodynamics where heat is treated as the primary concept and "cold" is simply interpreted as a secondary notion signifying a state of less heat than some arbitrary reference; negative heat states are thereby deprived of associative meaning.)

Like "cold", the terms "pull" and "suction" belong to a familiar everyday vocabulary. This does not, however, guarantee their suitability as regards the formulation of a consistent understanding of physical phenomena. The extent to which the common language of everyday experience is governed by the strict dictates of formal logic is quite minimal. The language of science must be much more precise: while <u>some</u> lattitude can be tolerated, this should not be such as to promote ambivalence (see earlier comments).

Conventional mechanics texts generally attempt to effect a clear (and almost fundamental) distinction between direct and indirect tension. That this distinction can not be substantiated in practice is seldom realised. It is therefore interesting to note that the two most prevalent indirect tensile test methods for concrete specimens, beam bending and cylinder splitting, both involve the application of <u>compressive</u> line loads. Mention was made earlier that many would consider the subjecting of an unprotected porous specimen to a <u>compressive</u> regime of biaxial fluid pressure as an indirect tensile test. The typical reasoning behind the indirect interpretation is that the disruptive "push" of the internal pore pressure may be seen as statically equivalent to an external "pull" in the unloaded direction. Were it the case that so-called direct tensile tests involved external "pull", the as-if philosophy underlying this argument would be less vulnerable. However, a close examination of the physical aspects of such tests reveals otherwise - viz, that <u>all</u> tension is indirect.

In practice, specimen systems subject to nominal "direct tension" are invariably <u>pushed</u> (rather than pulled) apart; i.e. the applied loads are transmitted to the specimen system via surface <u>compression</u>. Figure 4.34(a) shows the simple tensile test as it is commonly depicted for illustrative purposes. In common with the conventional representation of shearing actions recently discussed, the implications inherent to this idealised abstract view are somewhat misleading. (Although rarely asked

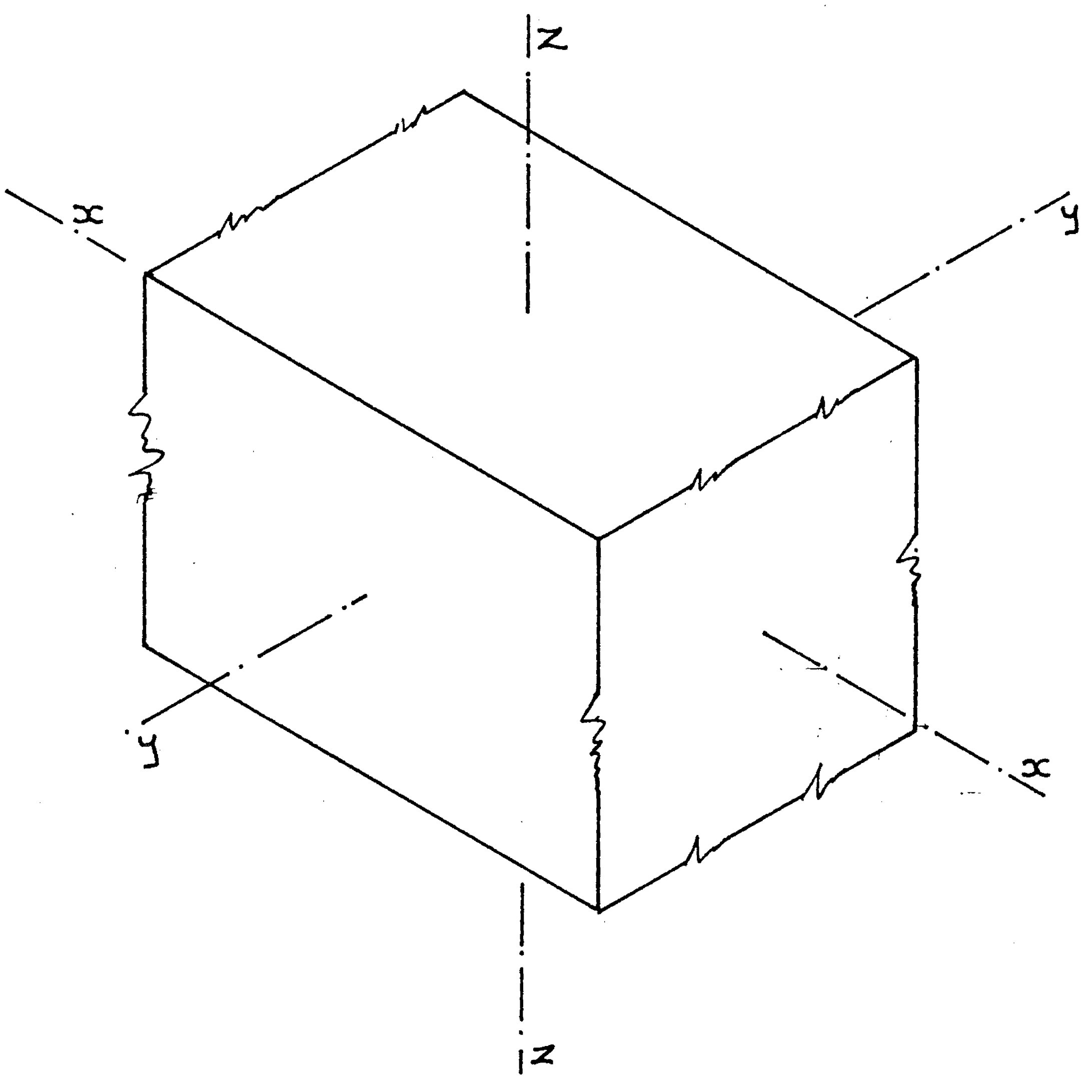


FIGURE 4.35: Part of a Specimen Subject to Uniaxial Tension, $-R'_x$

the pertinent question as to the exact manner by which an external boundary surface of a material body might ever be "pulled" in practice is nonetheless valid.) An alternative view of the same simple tensile test regime is shown in Figure 4.34(b). While the latter is also an extreme idealisation, it embodies a distinct (and previously absent) element of realism as regards the local compressive nature of the applied loads.

Within the context of the diphase model, tension on a material body is seen merely as a mechanical action by which the overall degree of external confinement in one or more directions is effectively reduced from that which prevails upon the body in the regime-free state (p'_E). As such, tensile regimes are quite amenable to quantitative description via the partial pressure approach. Also, providing the relative geometry of the specimen system allows the convenient implementation of a St.Venant-type principle*, meaningful internal/external distributed force balances can be deduced along similar lines to those adopted for compressive regimes.

Consider, for example, a statistically isotropic diphase specimen subject to uniaxial tension - R'_x. (Having now elaborated upon the diphase understanding of tensile actions and upon the disparities between this and the more conventional view, the earlier need for the terms tension and tensile to be repeatedly enclosed within inverted commas has passed.) As before, the suitability of a rigid body approximation will be presumed.

Equilibrium conditions (see Figure 4.41)

x $$p'_E - R'_x = [p'_{Sx}]_0 - \Delta p'_{Sx} + [p'_{Fx}]_0 - \Delta p'_{Fx}$$

y $$p'_E = [p'_{Sy}]_0 + \Delta p'_{Sy} + [p'_{Fy}]_0 - \Delta p'_{Fy}$$

z $$p'_E = [p'_{Sz}]_0 + \Delta p'_{Sz} + [p'_{Fz}]_0 - \Delta p'_{Fz}$$

---

* In essence, the St.Venant principle asserts that "distant" internal regions of a loaded body (well removed from the actual points of boundary load application) are relatively free from the extreme local variations of effect likely to obtain in immediate vicinity of the latter. By design, most practical tensile test specimens are proportioned in a waisted form which renders the end regions non-critical as regards the overall retention of composite integrity. Of course, in many instances, it is only through an implicit acceptance of the St.Venant principle that the mechanical performance of the central (gauge length) portion of a waisted specimen can be interpreted as representing the response to a near-uniform regime; were it not for the St.Venant principle the monitoring of such performance would become a rather questionable exercise.

Stability conditions

(i) separation $\quad p'_{Sn} > 0$ (general)

$$\rightarrow \quad \Delta p'_{Sx} < [p'_{Sx}]_0$$

(ii) shear $\quad p'_{St} < p'_{Sn}$ (general)

It is probably apposite to remark here that the important stochastic quality imparted to the "internal" partial pressure terms during the developmental stage of the diphase rationale still obtains. Mention is made above of the prerequisite (but not, of course, necessarily governing) condition for sustained shear stability because, as was the case for applied uniaxial/biaxial compression, two quite separate classes of diphase model may be envisaged. However, again drawing upon a wealth of accumulated experimental evidence, the Class A behavioural model, which exhibits a defined preference towards composite degeneration via the cleavage mode must be deemed the more appropriate for amorphous, nominally brittle material systems such as concrete in the present context of uniaxial tension. While the alternative option - the Class B model - with its inherent (prescribed) relative "weakness" in shear is far from trivial*, its immediate relevance to the main topic in hand is somewhat marginal.

For a statistically isotropic diphase specimen subject to a steadily increasing uniaxial tensile regime, $- R'_x$, the Class A model forecasts the "gradual" loss of forced composite integrity between individual S components in the x direction ($p'_{Sx} \rightarrow 0$). It may be recalled that this is exactly the same prediction as emerged for the case of equal biaxial compression, $R'_y = R'_z$. Thus, Figure 4.25(a), relating to the latter, also serves to illustrate the likely manifestations of "early" microcracking on a specimen under uniaxial tension! The suitability of this picture as regards the microcrack patterns exhibited by concrete

* There do exist, of course, many real material systems which ultimately respond to "simple" tension in a manner ideally suited to the descriptive potential of the Class B model. The performance of concrete specimens in such circumstances does not, however, generally lend itself to an interpretation whereby systematic failure is seen as a prime consequence of a limited capacity to resist associated shearing actions.

specimens subject to uniaxial tension is borne out in all respects by experimental findings[117-119].

For the previously examined case of equal biaxial compression, $R'_y = R'_z$, the Class A model indicated that macroscopic failure of the diphase specimen system might be characterised by either single or extensive multiple cleavage, which is the more likely depending on the precise physical nature of the applied loading regime. A brief comparative reflection should be sufficient to verify that this "option" of response between alternative ultimate breakdown modes at the phenomenological level does not prevail in the present context of uniaxial tension. Here, as in the case of biaxial fluid loading, the first complete loss of quasi-solid contiguity on an erstwhile internal x surface is undoubtedly critical; such a stage in the dissociation process heralds immediate dynamic instability; specimen failure occurs instantly, without further degradation, as the external quasi-fluid environment supplying a partial pressure, p_E, gains (or forces) access to the "end" surfaces of the two new composite sub-systems thereby created; cf. the box analogy mentioned earlier. Again, the prediction offered by the diphase model agrees very well with experimental information pertinent to the behaviour of nominally brittle materials, gross tensile failure via a single cleavage mode being perhaps the latters' most oft-noted feature.

The constant threat to overall composite stability posed by the external environments' inherent potential for forced entry to the interior of the model specimen under test may be seen as "explaining" many of the distinctive traits which epitomise brittle tensile response in the real world. Thus, for example, the typical suddeness of macroscopic brittle fracture in those situations where the applied tensile regime steadily intensifies clearly mirrors the manner of final demise "expected" of a Class A diphase system under such loading conditions. Similarly, because the disruptive consequences to the Class A model of significant environmental intrusion are likely to be both severe and "instant", the diphase rationale also forecasts the considerable practical difficulties associated with the controlled tensile testing of brittle material systems. An obvious case in point is the problem of sustaining a satisfactory degree of control over strain-regulated regimes; of course, any testing regime designed to encompass quasi-static load/deformation interactions beyond the stage at which the maximum load-carrying capacity of the specimen is exceeded must be sufficiently stiff to inhibit uncontrolled disintegration of the degenerate material body upon which it acts (i.e. the physical make-up of the regime itself must effectively supress, or even counter, the inherent instability of "falling branch" behaviour if this is to be monitored as a relevant fraction of the controlled response). It is worth noting here that, from a diphase viewpoint, the falling branch is strictly unrepresentative of overall specimen performance, even when this is rendered pseudo-stable by a stiff regime. Beyond the peak load capacity the once-composite material system undergoes a process of transition whereby previous hierarchical boundaries are altered (redefined) via breakdown, and hence the specimen no longer functions as a "statistical" whole. Regardless of the degree of external control exercised, the spread of local dissociation ceases to become a distributed bulk phenomenon; degenerative tendancies are focussed in particular regions as the new sub-systems emerge. Accordingly, a diphase interpretation of mechanical response must equate nominal specimen "failure" with the attainment of the peak load condition. While more conventional mechanistic descriptions often attempt to model the falling branch as a precursor of total break-down (an alternative "failure" definition), there can be little doubt that the continuum orientated parameters stress and strain typically employed therein are quite unsuited for this purpose. The relevance of stress and strain to a physical state of which the lack of continuity is perhaps the most salient feature is, to say the least, highly suspect.

The parallel forecasts offered by the diphase model for the nominally dissimilar cases of uniaxial tension and biaxial compressive fluid loading are without general counterpart in the conventional theories of material behaviour. Only with respect to certain materials in particular circumstances has any link between the two regimes been seriously suggested. For example, the response of "unprotected" porous specimens which are naturally permeable to a biaxial loading fluid would fall into this special category. However, even there, a clear and consistent "understanding" has repeatedly eluded formulation - see earlier comments and criticisms regarding the traditional notions of pore pressure effects. Some might claim that such understanding does prevail, but the available evidence indicates otherwise. A prime instance of this evidence (and of the manner by which unconscious preconception and/or an over-reliance on familiar but untested theories - "established" through longevity rather than proven suitability - can actually hinder its recognition) may be found in a reported study[321] due to Elvery et al.*. These workers subjected saturated concrete specimens to effective biaxial compression in a water-activated cell. Failure modes of the single cleavage (*tensile*) type were observed under conditions of increasing load. In accordance with the simple notion of critical hydrostatic uplift pertinent to the background (design-orientated) theory of gravity dams, failure was attributed to the disruptive influence of internal pore water pressure giving rise to indirect tension. The *average pore pressure causing failure* was then compared with a representive experimental measure of uniaxial tensile strength, the latter having been ascertained from *direct* tests on companion specimens cast from the same batch of concrete. Unfortunately, although this comparison revealed an approximate equality, the embarrassing fact that the quantitative critical pore pressure/tensile strength ratio was numerically less than unity (0.96) - and hence inconsistent with the cause of biaxial failure presumed - appears to have escaped the notice of Elvery et al. Thus, lured into error by prior expectation, they mistakenly quote the ratio so derived as the appropriate uplift area coefficient, whereas the "correctly" calculated - but nonetheless impossible - value thereof is actually 1/0.96, i.e. 1.04! (The uplift area coefficient is simply a notional relative continuum parameter representing that fraction of the gross cross section upon which the pore water pressure exerts an influence; by definition this can not exceed unity.) Of course, from a diphase standpoint, the results referred to above are neither paradoxical nor embarrassing: while the internal quasi-fluid environment (F) envisaged as a hierarchical complement to the model's quasi-solid substructure (S) may well include pore water where appropriate, there is certainly no reason to suppose that, if this is the case, $\Delta p'_F$ must become synonomous with changes in the partial pressure contribution of the latter alone.

In the context of biaxial fluid regimes the diphase model does not rely upon any aspect of inherent specimen permeability to the compressive loading medium to sustain its fundamental rationale; i.e. the model embodies an internal fluid (F) from the outset and hence whether this is supplemented during loading makes no difference to the basic

---

* No slight is intended upon the authors of the paper concerned; their research is of a high quality. Momentary irrational thought, born of preconception, numbers all thinking beings among its victims at some point or other.

equilibrium/stability principles involved. Accordingly, the special (pervious) connotations endemic to the conventional "explanation" of equivalent tensile effects outlined above must be seen as somewhat superfluous from the diphase viewpoint. Although the traditional pore pressure approach marks a refreshing departure from the norm in that it recognises (albeit indirectly) local compression as the physical source of tensile effects, there is little else to commend it. The quantitative discrepancies already mentioned are small in terms of relative magnitude but ultimately damning in terms of sense. What then of the extended generality implied by the diphase model? It transpires that experimental information consistent with the diphase forecasts has actually existed for some time. Writing in 1912, Bridgman[(332)] drew first attention to the fact that material specimens subject to biaxial fluid loading behaved *"as if being pulled apart at their ends"*. None of the materials to which Bridgman referred could be considered susceptible to interstitial intrusion by the loading fluid. The high degree of apparent tensile correspondence made a lasting impression on Bridgman and he alluded to it often in his subsequent works. Thus, from his now-classic text, *"The Physics of High Pressure"*[(333)], comes the following passage*:

> *When the pressure rises to a value approximately equal to the breaking stress in pure tension the cylinder parts as though it had been pulled apart by a tensile pull applied to the projecting ends. If the rod is of a brittle material like glass or a glass-hard tool steel the rupture takes place on a perfectly clean plane perpendicular to the axis, but if the rod is of a material that can yield before rupture like soft steel there is a considerable contraction at the area of the break which looks like the break in an ordinary tensile test.*

Bridgman's great interest in the phenomenon he had observed was undoubtedly fired by his realisation that conventional notions and theories of failure were powerless to either predict or explain it. (Evidence indicative of Bridgman's natural willingness to highlight discrepancy was of course presented earlier.) Probably for the very same reason, others more sensitive to conceptual embarrassment closed their minds to such "anomolous" behaviour, preferring the fabled ostrich approach to an honest admission of ignorance. In consequence, the extreme

---

* This quote is also given some prominence by Grimer and Hewitt[(110)]. However, such is its impact that its repitition here was felt to be warranted.

similarities of physical effect which exist between the actions of biaxial fluid loading and uniaxial tension regimes on solid specimens are among the least known facets of material response. Few modern introductory texts in material science cite Bridgman's findings, while the perpetuation in print of long-since obsolete failure theories, having little save historical or curiosity value, is almost standard. Bridgman himself sought repeatedly but in vain for a comprehensive rationale of failure which would involve the descriptive parameters of applied mechanics and yet be consistent with reality. His later attempts[334] to reconcile the notion of gross macroscopic strain with that of discrete changes at the atomic level are fairly plausible but, in the context of systematic breakdown, lack the total conviction required of a truly satisfactory basis for associative understanding. By this point (1938) an element of pessimism regarding the viability of a simple rationalisation via cause-and-effect type principles can be detected in Bridgman's work. Nevertheless, certain of his enlightened comments are as pertinent to current trends within material science as they once were to the state-of-the-art which then existed, more than four decades since; thus, for example,

> *In conclusion and summary, the problem of rupture is essentially a problem in stability, this has been too little appreciated in mathematical deductions of the conditions of rupture.*

The fundamental principles of associated external/internal partial pressure changes embodied within the conceptual framework of the diphase model are unaffected by considerations of nominal "class". The latter is only crucial insofar as it "selects" the dominant mode of composite instability (separation or slip) characterising systematic breakdown under applied load. In general, a system which has undergone physical transformation via some breakdown process need not itself be rendered unstable thereby. Subsequent (load-induced) degeneration of the "new" (emergent) system may well be required before total failure - in a gross phenomenonological sense - is manifest. It should be apparent therefore that the various phenomena which typify "real" ductile behaviour are no less amenable to a diphase interpretation than are the alternative traits of brittle response.

Although a rigorous development of the model in relation to ductile performance is beyond the scope of the present work, the descriptive potential which it offers in this area is worthy of mention. Experimental observations have shown, for example, that the spread of ductile yield is indeed a process whereby the material body concerned undergoes progressive structural alteration at some microscopic level; i.e. the so-called "yield plateau" which epitomises the classical form of ductile response may be seen in the light of a systems approach as corresponding to an effective transitional state. Likewise, the diphase scheme is well disposed to accommodate the occurrence of an ultimate instability via necking, both under uniaxial tension and biaxial fluid compression regimes. In retrospect, it is rather strange (notwithstanding the powerful influence of preconception) that necking under biaxial fluid compression should have been considered anomolous by so many; even the most primitive physical intuition would seem to render this behaviour less disturbing to the intellect than necking under uniaxial tension! Of course, because the diphase representation includes the stipulated external presence of a non-trivial quasi-fluid environment (E) acting as an ambient source of "additional" partial pressure influences, such similarity of effect becomes an expected feature. The necking of ductile specimens subject to uniaxial tension (or biaxial fluid compression) is not an indefinite phenomenon. At some stage in the process local fracture takes place, the subsequent separation-type failure being generally of a somewhat "sudden" nature. This too finds consistent explanation within the predictive capacity of the diphase rationale. (Since separation allows the external fluid complete access to the previously internal fracture surfaces there follows an immediate local change in prevailing partial pressures - cf. the box analogy of Section 4.2.1.) Bearing in mind the ubiquitous role envisaged for the "unrecognised" environment, it is significant (and, to say the least, highly encouraging) that recent experimental studies on a diverse range of metals[335] have revealed the occurrence of transient electromagnetic effects in the aftermath of tensile fracture. The very existence of these effects (which are detectable regardless of whether specimens exhibit necking or similar ductile traits) may be seen as vindicating the extended generality of the hierarchical diphase "picture".

Comments directed to material system response under tensile regimes of other than the "simple" uniaxial form will be offered in the next section.

4.3.7 Inferences and Predictions

The diphase model does not purport to forecast specific quantities such as numerical strength values. Fine detail, in a quantitative sense, is beyond its present powers. Nevertheless, it may be usefully employed as an indicator of likely performance under a wide variety of different circumstances. By this means a consistent basis for comparative forecasting in the real world can be established.

Consider a near-rigid isotropic diphase specimen system, characterised via initial (regime-free) parameters p'_E, p'_{So}, and p'_{Fo} and convenient orthogonal reference axes, x, y, and z. The behavioural traits (including maximum strength) such a specimen will manifest under the action of any particular loading regime which effectively supplements or counteracts p'_E (in one or more directions) are necessarily dictated by a number of factors. Among the more obvious of these is included the physical attributes of the additional loading medium (or media) involved and the degree of statistical variation inherent to the internal "set" of partial pressure terms.*

A further aspect, especially crucial in the context of ultimate strength, concerns the relative partitioning between S and F fractions of those internal changes, $\Delta p'_S$ and $\Delta p'_F$ respectively, induced by the presence of an external regime. Thus, for example in the case of a Class A system subject to hard uniaxial compression, say R'_x, the greater is the induced change in p'_{Sx} ($\Delta p'_{Sx} = R'_x - \Delta p'_{Fx}$), the greater is likely to be the maximum stable load-carrying capacity; it is, of course, the "complementary" increase in p'_{Fx} which is seen as giving rise to multiple splitting tendancies, since the "fluid" transmission of this change effectively reduces p'_S in all directions orthogonal to x. Two limiting possibilities may be identified, corresponding to a total transfer of the external change, R'_x, to either of the internal component phases. At one extreme, where $\Delta p'_{Sx} = R'_x$ ($\Delta p'_F = 0$), the model specimen must be considered to possess infinite compressive strength; this somewhat unrealistic "upper bound" only obtains because breakdown of the S components per se is not encompassed within the Class A rationale. Alternatively, if the manner of external/internal change interaction

---

* As was emphasised much earlier, directional equations of internal/external equilibrium for the system as a whole are not endowed with quantitative uniqueness. By virtue of the formal recognition given to the "existence" of an underlying structure, a notional spectrum of gross internal surfaces must be associated with any one reference direction. Accordingly, it would be quite unreasonable (and self-defeating) to treat the parameters, p'_S and p'_F, pertaining thereto as having other than a ranged stochastic nature.

conforms to the opposite extreme and $\Delta p'_{Fx} = R'_x$ ($\Delta p'_{Sx} = 0$) then a lower bound compressive strength potential, consistent with the same initial degree of systematic restraint (p'_{So}), is forecast.

The converse applies in the case of a Class A specimen subject to uniaxial tension, $-R'_x$. Here, the relative extent to which p'_{Sx} is reduced is the governing factor as regards the likely order of composite strength displayed. Accordingly, if $\Delta p'_{Sx} = R'_x$ a lower bound tensile strength is predicted; the appropriate upper bound strength again emerges as an unattainable quantity but, in this instance, corresponds to an extreme condition whereby $\Delta p'_{Sx} = 0$ ($\Delta p'_{Fx} = R'_x$).

Of the many traits which have been used by material scientists to typify "brittleness" in real material systems, none is more commonly quoted than a marked disparity between customary uniaxial tensile and compressive strength measures. Translated into the language of the Class A diphase model, such relative weakness in "simple" tension implies that those mechanical testing regimes conventionally employed to obtain quantitative information on specimen strengths must impart a predominant fraction of their immediate effect to the skeletal quasi-solid phase. Taking into account both the hard (solid-to-solid) physical aspects of most customary uniaxial strength-testing regimes and the contiguous form of substructure envisaged for the model composite, an ensuing bias towards the S phase as regards the internal transfer of external effect is quite within the bounds of reason (and expectation). Indeed, had an inference to the contrary been required for the sake of sustaining "realism" (tailored compatibility), this could have proved a somewhat contentious issue!

It is important to note at this juncture that the "normal" ranking of ultimate compressive and tensile load capacities in the context of brittle response is very much a conditional feature of the Class A scheme. The latter embraces the possibility of uniaxial compressive/tensile strength ratios exhibiting a drastic departure from usual trends if appropriate local circumstances prevail. Thus, for example, even relative weakness in simple compression - the complete antithesis of most conventional "understanding" with respect to brittleness - can not be ruled out as a matter of general principle; if the peculiarities of a particular system and/or loading regime favour a predominant transfer of the applied external effect to the internal F phase, such a reversal from the norm must be considered likely. As will presently become apparent (if not already so), this characteristic element of predictive flexibility embodied within the Class A diphase model is far from being redundant in a practical sense; the mechanical performance under load of many real material systems (including concrete) can be demonstrated to display extreme sensitivity to the manner by which a nominal loading condition is achieved; two regimes of equal quantitative intensity need not produce the same effect!

Because the diphase model treats the action of any loading regime as supplementary to that provided by an ambient background environment, it is essentially triaxial by nature. Hence, the presented arguments pertaining to uniaxial regimes need very little modification and/or embellishment to enable their use in more "complex" loading situations. Consider, for example, the case of applied triaxial compression, $R'_x > R'_y = R'_z$, a regime commonly utilised for mechanical test purposes. Here, two alternative (but, in essence, complementary) approaches to the rational description of systematic change are possible. The first, and perhaps most obvious, would treat the regime-free condition as an appropriate reference (base) state and thus would associate subsequent internal changes $\Delta p'_S$ and $\Delta p'_F$ with all three components of the additional external regime. This particular option merely involves extending the conceptual development from uniaxial to biaxial loading, undertaken earlier, one stage further. The second approach dispenses with the would-be "convenience" of a notionally fixed external reference state. Instead, a "floating" basis for comparison is adopted which includes not only p'_E but also the isotropic fraction of the applied regime within the "background influence" designation. As will emerge, the benefits to be gained from such origin flexibility far outweigh any element of inconvenience which might seem to derive from the sacrifice of a static reference.

To facilitate this alternative means of description, a special symbol, $\bar{R}'$, will henceforth be employed to denote the effective increase in ambient isotropic (environmental) partial pressure due to the action of an applied compressive loading regime; deviatoric components will be ascribed the symbol D', the latter being subscripted where necessary to take account of directional variations. Thus, for the triaxial case* under immediate consideration ($R'_x > R'_y = R'_z$) the total external loading

* In all previous cases examined, $\bar{R}' = 0$.

TABLE 4.1 Descriptive Parameters Characterising a "Solid" Diphase (S-F) Specimen Subject to:

(a) Uniaxial Compression, $R'_x > R'_y = R'_z = 0$

(b) Triaxial Compression, $R'_x > R'_y = R'_z > 0$

		(a) Uniaxial Compression		(b) Triaxial Compression
Regime:		$R'_x > R'_y = R'_z = \overline{R}' = 0$		$R'_z > R'_y = R'_z = \overline{R}' > 0$
Reference states: (partial pressures)				
(i) external		p'_E	$<$	$p'_E + \overline{R}'$
		$=$		$=$
(ii) internal		p'_{s_o}	$>$	p'_{s_o}
		$+$		$+$
		p'_{F_o}	$<$	p'_{F_o}
Deviatoric Load: (partial pressure)		$D'_x \; (= R'_x)$		$D'_x \; (= R'_x - \overline{R}')$
Internal Changes:				
	x	$\left.\begin{array}{l} + \Delta p'_{Sx} \\ + \Delta p'_{Fx} \end{array}\right\}$	$\Sigma = D'_x$	$\left\{\begin{array}{l} + \Delta p'_{Sx} \\ + \Delta p'_{Fx} \end{array}\right.$
	y	$\left.\begin{array}{l} - \Delta p'_{Sy} \\ + \Delta p'_{Fy} \end{array}\right\}$	$\Sigma = 0$	$\left\{\begin{array}{l} - \Delta p'_{Sy} \\ + \Delta p'_{Fy} \end{array}\right.$
	z	$\left.\begin{array}{l} - \Delta p'_{Sz} \\ + \Delta p'_{Fz} \end{array}\right\}$	$\Sigma = 0$	$\left\{\begin{array}{l} - \Delta p'_{Sz} \\ + \Delta p'_{Fz} \end{array}\right.$

in the three orthogonal reference directions becomes:

$\underline{x}$ $$p'_E + R'_x = (p'_E + \overline{R}') + D'_x$$

$\underline{y}$ $$p'_E + R'_y = (p'_E + \overline{R}')$$

$\underline{z}$ $$p'_E + R'_z = (p'_E + \overline{R}')$$

It may be seen that the association of $\overline{R}'$ and p'_E as joint environmental components (hence the employment of "combining" brackets) leads to a view of the situation in which there exists a very strong parallel with the descriptive basis adopted for the case of uniaxial compression, $R'_x > R'_y = R'_z = 0$. Here, D'_x assumes the "additional" role previously held by R'_x, with the combined environmental partial pressure $p'_E + \overline{R}'$ replacing the action of p'_E alone. A similar rationale can therefore be implemented with regard to forecasting the likely manifestations of composite breakdown, both local and macroscopic. Table 4.1 serves to compare the two regimes as either would affect any particular diphase specimen. (The convenient stipulation of a common specimen system for the purposes of regime comparison renders p'_E a local constant.) From this emerges clearly an indication of the extensive similarity which obtains; in all aspects other than degree a sustainable basis for critical distinction is patently absent.

Several inferences concerning likely orders of relative strength can be drawn from Table 4.1. Since the skeletal substructure of a "solid" diphase specimen system derives both its composite integrity and inherent strength potential from the element of forced contact between individual S components (p'_S), the finite isotropic fraction, $\overline{R}'$, of the triaxial regime cited effectively increases the system's "basic" capacity to sustain applied deviatoric load; i.e. $\overline{R}'$ enhances the "initial" degree of background (environmental) restraint, p'_{So}, which must be overcome or at least altered before conditions conducive to systematic breakdown via instability can prevail. Of course, this increase in p'_{So} over that which originally obtains in the uniaxial case (viz, in the regime-free state) is not equivalent to the magnitude of $\overline{R}'$ itself: part of the latter is "taken up" by a corresponding increase in p'_{Fo} - see previous remarks on the consequences of applied isotropic compression given in Section 4.3.3. However, in view of more recent arguments and inferences, a pronounced bias towards the S phase as regards S-F load-sharing under conventional (solid-to-solid type) mechanical testing regimes must be expected, unless appropriately "unusual" means of load application are employed. (Although the operating schemes of many practical strength-testing systems incorporate the use of pressurised fluids as an effective source of load generation and/or transfer, it is relatively rare for the material specimen under test to be subject to direct physical interaction with such media.)

A reiteration of previous comments concerning the phenomenological (and ultimately quantitative) importance which attaches to the physical attributes of any particular loading regime is apposite at this point. Consider, for example, the triaxial "reference states" mentioned in Table 4.1. It will be recalled that the original designation of a diphase specimen system as statistically isotropic alluded specifically to the regime-free condition. While a lack of directional sensitivity with regard to the ranged partial pressure parameters characterising the state of force equilibrium within the system may also prevail under an appropriate combination of applied load*, some care must be exercised in presuming such to be the case in particular instances. Thus, although it is indeed tempting to suppose that a statistically isotropic specimen system subject to an isotropic loading regime must automatically retain the feature of directional invariance, the realisation of potential is only strictly arguable if the regime per se is isotropic in every sense. Quantitative isotropy ("equal stresses") is a necessary condition for this, but not a sufficient one. (In the past, many investigators interpreting mechanical test results have failed to appreciate that the intensity of a loading regime is not the sole determinant of "material" response. Unfortunately, there are still those who in the face of all evidence to the contrary - e.g. the demonstrable influences of specimen geometry and loading form - would appear to prefer conceptual self-deceipt by clinging to the mythical supposition of unique response rather than accomodate the "inconvenience" of a concrete reality (the very existence of which serves to indict the erroneous nature of such an untenable premise). Qualitative anisotropy is a common feature of practical test arrangements, even where the applied loading condition

---

* Of course, this excludes any regime which incorporates a deviatoric component. Similarly, any specimen system that is statistically anisotropic in the regime-free state is unlikely to be rendered otherwise upon subsequent loading.

attempts to represent an all-round "equal stress" state*. Different forms of surface loading can never be expected to give rise to equal effects - in the fullest sense - especially if associated "secondary" influences vary markedly. In consequence, the quantitative reference terms p'_{So} and p'_{Fo}, corresponding to a finite $\overline{R}'$, need not be statistically isotropic simply because the specimen system to which these allude so qualifies when $\overline{R}' = 0$. Formal recognition of this possibility (rather than its detailed examination) is the key issue here. As was emphasised much earlier, statistical anisotropy in no way interferes with the fundamental principles of the diphase model; it merely limits the degree to which, in the absence of specific information concerning the manner of directional variation involved, likely modes of systematic breakdown can be forecast.

Providing the respective means of deviatoric (D'_x) load application are similar in form (and are thus suited to valid comparison), the diphase model predicts that the resistance offered to such actions by a material specimen will be increased by the presence of a finite $\overline{R}'$. It may be further inferred that if $\overline{R}'$ is qualified as deriving from one particular physical source, and if a consistent loading pattern is employed throughout, then the greater is the quantitative magnitude of $\overline{R}'$ the greater is likely to be the effective enhancement of deviatoric strength capacity.

Of course, the case of uniaxial compression, $R'_x > R'_y = R'_z = 0$, constitutes a lower boundary to the infinite set of triaxial regimes which comply with the designation $R'_x \geqslant R'_y = R'_z \geqslant 0$. However, while this fact engenders an "expected" overall similarity of response insofar as induced partial pressure changes are concerned (allowance being made for origin flexibility), it does not follow that the set itself necessarily gives rise to a family of similar failure modes.

To illustrate the immediate significance of the above non-sequitur, consider a near-rigid statistically isotropic specimen system which behaves in the Class A manner (eventual loss of composite integrity via cleavage-type degeneration) when subjected to increasing "hard" uniaxial compression. This particular breakdown mode represents a

---

* The term qualitative anisotropy espouses all aspects other than that pertaining directly to conventional stress measures. Although the extent to which such potential sources of variation are significant is largely indeterminate from a strict applied mechanics viewpoint, there is of course no suggestion being advanced that these are beyond any form of quantitative description.

systematic preference under prescribed conditions. In no way should it be interpreted as a general (unqualified) characteristic of the system per se. Expressed in an alternative - and broader - fashion, brittleness does not represent any intrinsic "material property" in the classical sense but rather a structurally orientated description of interactive performance to which the detailed character of both the specimen system (structure) and the applied loading environment make important contributions*. Neither is a marked preference towards Class A (or Class B) behaviour an arbitrary facet, this despite the extreme versatility of possible response seen from the diphase viewpoint as being embodied within any composite system. The emergence of a Class A failure pattern merely indicates that the potential for other forms of degeneration has not been realised in that set of prevailing circumstances.

Now consider an identical specimen system but subject instead to a triaxial regime, $R'_x > R'_y = R'_z > 0$, where R'_x takes the same form (in respect to physical source , rate of application, etc.) as that pertaining to the uniaxial case just described, and the intensity of the deviatoric component, D'_x, has reached a level where it equals the corresponding strength capacity then attained. For reasons already outlined, further increment of the applied deviatoric load would be required before the system suffers a complete loss of composite integrity. The eventual manner in which such a loss would occur is, as previously, controlled by the exact nature of local test conditions. A knowledge of the ultimate specimen response under alternative circumstances - regardless of any specific similarities which might obtain - is unreliable as an indicator of probable performance. Thus, while a manifestation of compressive breakdown via multiple cleavage and consequent instability of degenerate columnar elements is again possible, the likelihood thereof is not effectively enhanced by virtue of the earlier uniaxial failure mode having conformed to the Class A designation. Indeed, as will now emerge, a strong case can be argued for "expecting" a general trend towards suppression of Class A failure characteristics under conventional applied triaxial loading which falls within the quantitative category, $R'_x > R'_y = R'_z > 0$.

---

* Brittleness and ductility (like colour) are "spectrum" properties that can vary much according to the "light" in which observations are made!

For a diphase specimen system to exhibit degenerative tendancies of the Class A type, it must be capable of resisting any shear potential associated with the deviatoric component of an applied loading regime; naturally, were such resistance to prove inadequate, alternative (Class B) failure traits would be displayed. The fundamental question of shearing actions and of the manner by which a capacity to withstand these might reasonably arise in the diphase context was examined in a previous section. It will be recalled that, having formally recognised "interfacial" roughness as a simple consequence of the quasi-solid (S) particles which comprise the skeletal substructure being endowed with a finite - although unspecified - size, the diphase model needed no additional concept of "cohesion" or the like to sustain a consistent understanding; the other main physical attribute usually seen as an essential requirement for the generation of "internal friction" - viz, forced (compressive) mutual contact - is of course a primary feature of the diphase rationale.

In the triaxial regime situation of current interest, $R'_x > R'_y = R'_z > 0$, the shear demand imposed by a general (blanket) Class A response is likely - under certain combinations of applied load - to attain levels that are many orders higher than those (including the maximum) for the corresponding uniaxial case. This "probability" (relating as it does to a strictly hypothetical retention of Class A characteristics) arises because the structural stability of columnar elements plays such a major role in determining the ultimate degree to which the original specimen system can undergo multiple cleavage without sustaining at least a partial collapse and hence a loss of further load-carrying capacity. For obvious reasons, a strong analogy can not be drawn here with the classic phenomenon of elastic strut buckling; however, insofar as the latter amply demonstrates the dramatic differences in stable load-carrying capacity which may be wrought by the provision of even minimal restraint, it serves as a useful parallel. Thus, not only does the presence of a finite $\overline{R}'$ increase the "initial" precompression of the quasi-solid phase, p'_{So}, which must be overcome before significant separation can occur, but also has the potential to act as an effective source of lateral (y - z) stabilisation should the structural system actually tend to degenerate via the multiple cleavage mode. Where $\overline{R}'$ derives from solid-to-solid type isotropic loading (whether "hard" or "soft"), as is predominantly the case in testing practice, the additional restraint factor can be expected to exert considerable influence and especially so if $\overline{R}'$ is "hard". For any particular finite $\overline{R}'$, the greater is the deviatoric load, D'_x, applied beyond the uniaxial (Class A) compressive strength then the greater becomes the likelihood of subsequent Class B behaviour (or, in conventional terminology, the greater is the intensity of all associated shear stresses with respect to some critical value). Although the shear capacity of the diphase specimen system will naturally increase with increasing $\overline{R}'$ (through an enhancement of the "initial" forced contact between S components), so too will its inherent resistance to separation. If, as seems eminently reasonable under most conventional testing conditions, the systematic transfer of external effect (load) is primarily to the internal quasi-solid phase, the latter increase can be expected to exceed the former. In such circumstances, a complete transition from Class A to Class B response, due to a reversal in the previous order of the respective associated failure loads, becomes a viable consequence. While some degree of uncertainty must obviously prevail with regard to the minimum relative intensity of R' necessary to secure a transition of this type, there can be no doubt concerning its status as a non-trivial possibility. Regimes in which the manner of application of R' contributes significantly to lateral stability (via some form of "beneficial" structural interaction) would appear especially suited to favour an "early" promotion of the potential Class A failure load to a level which is beyond that realisable before alternative Class B breakdown characteristics are exhibited.

The above forecasts and inferences (both qualitative and, in the relative sense, quantitative) are totally compatible with documented experimental information pertaining to the physical strength-testing of many nominally "brittle" material systems - concrete included. It is interesting to note that the logical train developed in recent arguments may be applied equally well to specimen systems which behave in the "brittle" Class B fashion (macroscopic shear breakdown) under uniaxial compression*. The consequent "expectation" of such characteristics being retained, in this instance, under triaxial compression $R'_x > R'_y = R'_z > 0$ is again borne out fully by recorded experimental evidence.

---

* Concrete specimens subject to unconfined uniaxial compression seldom display Class B failure patterns unless some aspect of their constitution or preparation (mixing, compaction, curing, etc.) has given rise to "planes of weakness". Rock specimens are, however somewhat prone to exhibit the traits of Class B degeneration in "simple" compressive tests.

Before extending the examination of triaxial regimes, it should perhaps be emphasised that the isotropic component of load, $\overline{R}'$ (= R'_{min} as defined above), is <u>not</u> synonomous with the "octahedral normal" or "hydrostatic stress", σ_o, commonly referred to in conventional terminology with respect to the triaxial context. Neither should any deviatoric component, D' (= $R' - \overline{R}'$), be confused with the "octahedral shear" or "deviatoric shear stress", τ_o. (Both σ_o and τ_o are referenced in Chapter 3, Sect. 3.5.) Apart from that aspect concerning the applied load being partitioned into two distinct categories, the present approach differs radically from the classical elastic theories of failure involving distortional strain energy and/or octahedral stresses; even the manner of notional load partitioning is quite different!

Consider now the case of a diphase specimen system subject to a triaxial compressive regime, $R'_y = R'_z > R'_x > 0$. The specimen which undergoes this "triaxial extension test*" will be assumed similar in nature to that previously described; i.e. inter alia, it would behave in the Class A fashion were it to be loaded under uniaxial compression. As with the earlier triaxial case (and indeed with all loading cases) the ultimate mechanical performance of the specimen will not be insensitive to the physical attributes of the applied regime. However, several inferences of a fairly general nature may be drawn, upon which subsequent comparisons of likely strength orders can be based should the compatibility of local conditions justify this step.

Two alternative (but complementary) options would appear open with regard to partitioning a triaxial regime, $R'_y = R'_z > R'_x > 0$, into isotropic, $\overline{R}'$, and deviatoric, D', components; viz,

(i) $\overline{R}' = R'_x$
$D'_y = D'_z = R'_y - R'_x \; (= R'_z - R'_x)$
$D'_x = 0$

, or (ii) $\overline{R}' = R'_y \; (= R'_z)$
$D'_x = R'_x - R'_y \; (= R'_x - R'_z)$
$D'_y = D'_z = 0$

As before, a notional association of $\overline{R}'$ and p'_E as joint components of

* Comment has already been offered concerning the somewhat misleading implications endemic to this conventional term.

an "initial" external effect deriving from a combined "background" environment can be envisaged - thus giving rise to a convenient "floating" reference state. Having adopted the latter, option (i) may be seen to mirror the case of equal biaxial compression examined previously. By the same token, option (ii), which merely implements a manner of partitioning that departs from the earlier choice of $\overline{R}' = R'_{min}$, reflects the case of uniaxial tension. Here, D'_x (a quantitative change parameter) is negative, signifying a decrease in effective external compression on the body of the specimen in the x direction. Of course, the inherent degree of general systematic equivalence which exists between regimes of uniaxial tension and equal biaxial compression (and highlighted formerly in Sect. 4.3.6 of this Chapter) renders options (i) and (ii) mutually compatible. Any potential for either option to serve as a more suitable basis for "realistic" description than the other depends largely on the actual loading sequence employed during the "triaxial extension test". Option (i) would certainly seem preferable where the test consists of first establishing an isotropic load state and then increasing only the biaxial components thereof. Where, instead, the corresponding second stage involves effecting a gradual load decrease solely in one direction option (ii) appears more appropriate. Both forms of the "triaxial extension test" alluded to immediately above have found use in practice, as have continuous proportional loading techniques. In the latter circumstances neither option can be claimed to possess special relevance; each is viable in its own right without prejudicing the status of the other.

If prevailing conditions are conducive to the valid drawing of comparisons, relative orders of strength (deviatoric ranking) can be forecast along similar lines to those adopted for the previous configuration of applied triaxial load. Again, the greater is $\overline{R}'$ (either option), the greater is likely to be the deviatoric strength capacity manifest by the specimen system, since the former influences the "initial" precompression of the quasi-solid phase. This expectation is consistent with experimental data derived from "triaxial extension tests" on mortar and concrete specimens[330]. By further implementing that rationale developed earlier, which afforded condition-sensitivity to systematic breakdown preferences, the diphase model also predicts a transition to Class B degenerative response should the magnitude and/or nature of $\overline{R}'$ be such as to effectively suppress the emergence of primary Class A traits, whether of the single or multiple cleavage type. Unfortunately, as regards the results of concrete testing, experimental evidence in this context (either supportive or to the contrary) is both sparse and rather inconclusive, the range of practical $\bar{R}'$ values typically employed being somewhat limited in quantitative extent; apparent shear mechanisms <u>have</u> been reported on occasions but it is often difficult in these instances to gauge the contribution to such behaviour made by indeterminate boundary restraint conditions (potentially "high" secondary shear effects). In other areas, however, where regimes involving relatively large $\bar{R}'$ components have been applied to would-be "brittle" material systems, complete behavioural transitions to a classically ductile manner of response have been observed[314]. Rock specimens, for example, have been demonstrated to exhibit severe necking prior to ultimate fracture under appropriate quasi-tensile load combinations[336].

It will presently transpire that one particular (and unusual) form of the "triaxial extension test" offers the opportunity to check upon the merit of an important inference drawn much earlier. Firstly, however, a little background discussion is required.

At the beginning of this section the question of internal load-sharing between the S and F components of a diphase specimen system subject to either uniaxial compression or uniaxial tension was broached. Logical arguments were advanced in relation to Class A performance which showed that the greater was the share of the external load change "taken up" by the quasi-solid fraction <u>in the direction of loading</u> the greater was likely to be the exhibited compressive strength but the lesser the ultimate tensile load-carrying capacity of the specimen. (For both the "simple" cases, $\bar{R}' = 0$, the relevant deviatoric component, D, for each being equivalent to the corresponding applied R'.) It was then suggested that solid-to-solid type loading, the form most commonly adopted in practice, naturally favoured a predominant transfer of immediate external effect (change) to the internal quasi-solid phase, this being especially so for hard regimes. The Class A corollary thereto, that the "normal" low/high ranking of uniaxial tensile and compressive strengths is a direct (and hence alterable) consequence of those physical attributes with which conventional loading techniques tend to be endowed rather than an intrinsic property of "brittle" material systems per se, was also aired, but without detailed amplification.

The documented results of equal biaxial compression tests which are to be found widely in the expansive literature of concrete research lend considerable credence to the prior assertion that the tensile strength of "brittle" specimens may indeed attain comparatively high levels under appropriate circumstances. It will be recalled that, according to the diphase rationale, regimes of equal biaxial compression, say $R'_y = R'_z$, effectively reduce the degree of forced contact between S components in the third principal direction, x. While such an outcome will also derive from the alternative application of a uniaxial tension regime, $-R'_x$, the critical aspects of internal (S - F) load sharing alluded to above influence each case quite differently*. Thus, the capacity of a diphase specimen to resist Class A degeneration under equal biaxial compression, $R'_y = R'_z$, is likely to be enhanced, the greater is the proportion of the external change transferred directly to the internal quasi-solid phase (i.e. the greater are the ratios $\Delta p'_{Sy}/R'_y$ and $\Delta p'_{Sz}/R'_z$). For a tensile regime, $-R'_x$, the converse applies; in limit, a total transfer of external change directly to the S fraction corresponds to the condition (i.e. $\Delta p'_{Sx}/R'_x = 1$) for minimum strength potential. Experimental findings in relation to the performance and strength of concrete specimens subject to equal biaxial compression confirm both the suitability and consistency of the diphase model as developed. Previous implications with regard to the "internal" consequences of differing external regime characteristics are fully sustained. As predicted by the diphase model, a broad spectrum of behavioural response emerges, bounded by two distinct extremes. The first of these, associated with hard, solid-to-solid biaxial loading is epitomised by multiple cleavage breakdown patterns and relatively high failure loads: specimen strength values obtained from hard biaxial compression invariably exceed equivalent (hard) uniaxial measures. At the other extreme, where "unprotected" concrete specimens are subjected to the very soft, potentially penetrative action of biaxial (cell) fluid regimes, macroscopic failure occurs via the single cleavage mode at comparatively low orders of applied pressure. It is interesting to note that the use of "jacketed" specimens constitutes an intermediate condition (soft solid-to-solid type loading) yielding medium/high failure loads and single cleavage[124, 337]; in the context of the diphase model, any protection from a biaxial

---

* The various similarities which exist between the two regime designations as viewed from the diphase standpoint act to establish a general systematic equivalence. This feature, highlighted previously in some detail, does not of course imply total identity. Neither is the extreme significance attached thereto diminished in any way by factors which serve to preclude the latter.

cell fluid regime afforded to a specimen by way of an imbervious membrane or the like is naturally interpreted as restricting the potential for direct communication of effect to the internal F phase. The "wire-wound" regime of Langan and Garas[(319)] alluded to earlier (giving low/medium biaxial strength values) represents another intermediate condition*. Of special significance here, bearing in mind recent arguments, is the experimentally recorded similarity between conventional (hard) tensile response and that pertaining to the soft biaxial extreme (fluid media). If the converse applies, as is also forecast by the diphase model, "softer" tension should induce behavioural traits (<u>including</u> failure loads) more akin to those which are typically manifest under hard biaxial compression.

The hierarchical understanding of tensile regimes is, of course, strictly relative; i.e. "simple" tension is seen as a state whereby a system is effectively subject to <u>less</u> external (environmental) compression in one principal direction than exists in the regime-free state (p_E'). How then might such a reduction be effected so that, in the direction concerned, the corresponding internal change be "taken-up" predominantly by the interstictial quasi-fluid (F) phase? Drawing upon experience gained in the applied biaxial compression context, some form of fluid loading arrangement would appear to offer the greatest hope of success. Perhaps the most obvious possibility (in a notional sense, at least) is to partially "shield" a material specimen from the influence of the external quasi-fluid, E, and hence reduce the ambient intensity of p_E' in one direction. Unfortunately, although the aether-like background environment has been accorded formal recognition, no suggestion as to a practical means of securing a direct diminution of its influence, for strength-test purposes, is immediately forthcoming. (Had such a means been apparent from the outset, the task of establishing the non-trivial status of the would-be "void" need not have relied so heavily upon philosophical argument.) Under normal circumstances p_E' naturally includes a contribution from atmospheric (air) pressure. While this component - and its effect - may be removed to varying degrees, the maximum possible external change to be derived therefrom is slight in

---

* Circumferential wire-winding imposes a solid-to-solid type loading which is inherently soft. This feature renders the form of biaxial strength envelope suggested by Langan and Garas (319) very questionable, referring as it does to a rather incongruous mixture of hard and soft results.

quantitative terms. However, by altering the "initial" environment so that p_E' includes a more sizeable contribution from the external presence of a "real" pressurised fluid (whether liquid or gas) this deficiency can be easily overcome; i.e. by physically creating a new base reference state, the total background environment, E, becomes more amenable to control, as regards subsequent change. It should now be evident therefore that the requirements for "soft tension", communicative with the interstitial (F) phase, are not beyond the realms of practicality. If a material system is permeable, appropriate "triaxial extension tests" (as per option (ii) mentioned earlier) in which the applied compressive loading to the specimen derives exclusively from fluid media are likely to impose such action. Tests of this type have, in fact, been performed on concrete specimens[338], with the results thereof fully confirming diphase forecasts of relatively high deviatoric ($D' = R' - \overline{R}'$) load capacity*.

A slight (and brief) digression from the main track is now fitting. Subsequent upon the early pioneering studies of Bridgman, workers in the experimental field of high pressure physics have long since come to recognise the general lessening of "brittleness" (or, from a complementary stance, the increased ductility) manifest in material systems under applied deviatoric load wrought by the presence of a superimposed background (isotropic) pressure. However, unlike the diphase approach, conventional physics offers little in the way of theoretical understanding and/or consistent explanation of such changes; as a consequence, empiricism prevails widely and derivative tautology flourishes. It is not uncommon, for example, to read in physics texts that the capacity for material deformation increases under hydrostatic-type background conditions because (?) such circumstances promote more ductile response! (cf. the equally naive assertion - bearing in mind the definitive meaning of strength - that a material system fails because its strength has been exceeded.) Notwithstanding the plight of the gullible or the mentally unwary, it is indeed fortunate that frequency of promulgation does not diminish the logical unsoundness of circular argument but merely secures its rather thin disguise.

Although unrelated directly to the principal topic of this work

---

* Having previously drawn attention to certain fundamental inadequacies inherent to conventional "indirect tension via pore pressure" theories, it is pertinent to note here that the latter are also quite powerless to predict this particular manner of quantitative response.

- viz, the mechanical performance of concrete systems - one particular instance where the above tautology is often proffered serves to further illustrate the sustained predictive capacity of the diphase model. Thus, it has been found[314] that metal extrusion processes can, if certain circumstances obtain, give rise to final products thereof which exhibit distinct fracture patterns characterised by multiple cleavage perpendicular to the direction of forced motion. In view of the relatively large changes in the degree of applied confinement (especially as regards longitudinal restraint) that a material system undergoes in the latter stages of a compressed extrusion process, such quasi-tensile manifestations pose no real challenge to the diphase model. By way of contrast, conventional physics - if it ignores self-deciept - can but record the phenomenon among its reference list of empirical facts. Some workers have sought to compensate for (disguise) the "embarrassing" lack of applied tensile actions (to which separation-type fracture is normally attributed) through an appeal to an implied failure criterion based upon the concept of limiting "tensile" strain; however, this simplistic notion - transposing cause and effect almost at will - is demonstrably tenuous, if not unsound (see earlier comments). The diphase model not only forecasts the possibility of parallel fracture but also suggests a practical solution to the extrusion/cleavage problem where it exists; viz, decrease the relative change in confinement experienced during the transition process from billet to final product. Two extreme alternatives present themselves. The first simply involves decreasing the degree of internal (pre-extrusion) confinement to which the material system is subject. (Tendancies towards post-extrusion cleavage do in fact[314] increase the greater is the level of distortion imposed on a billet by a die arrangement.) The second approach attacks the required decrease in relative change from the opposite quarter and involves increasing the effective longitudinal restraint provided by the external environment into which the material specimen is finally extruded; i.e. through replacing the minimal quantitative influence of normal atmospheric air conditions with the stabilising potential of a more highly pressurised fluid medium, the interactive aspect of forced contact between S components - a requirement essential to the continued systematic integrity of the quasi-solid substructure - may be retained. In recent times, the practical implementation of this second option ("back-pressuring") has broadened considerably the viable scope of extrusion technology. Although the benefits of back-pressuring as a means of

TABLE 4.2: Groupings of "Similar" Test Regimes

(i) One deviatoric component - increasing:

uniaxial compression	$R'_x > R'_y = R'_z = 0$	$(\bar{R}' = 0)$
triaxial compression (a)*	$R'_x > R'_y = R'_z > 0$	$(\bar{R}' = R'_y = R'_z > 0)$
equal biaxial tension	$R'_x = 0 > R'_z = R'_z$	$(\bar{R}' = R'_y = R'_z < 0)$
compression - equal biaxial tension	$R'_x > 0 > R'_y = R'_z$	$(\bar{R}' = R'_y = R'_z < 0)$
triaxial tension (a)	$0 > R'_x > R'_y = R'_z$	$(\bar{R}' = R'_y = R'_z < 0)$

(ii) One deviatoric component - decreasing:

uniaxial tension	$R'_x < R'_y = R'_z = 0$	$(\bar{R}' = 0)$
equal biaxial compression	$R'_x = 0 < R'_y = R'_z$	$(\bar{R}' = R'_y = R'_z > 0)$
triaxial compression (b)	$0 < R'_x < R'_y = R'_z$	$(\bar{R}' = R'_y = R'_z > 0)$
tension - equal biaxial compression	$R'_x < 0 < R'_y = R'_z$	$(\bar{R}' = R'_y = R'_z > 0)$
triaxial tension (b)	$R'_x < R'_y = R'_z < 0$	$(\bar{R}' = R'_y = R'_z < 0)$

(iii) Two deviatoric components - both increasing:

biaxial compression	$R'_x = 0 < R'_y < R'_z$	$(\bar{R}' = 0)$
triaxial compression (b)	$0 < R'_x < R'_y = R'_z$	$(\bar{R}' = R'_x > 0)$
general triaxial compression	$0 < R'_x < R'_y < R'_z$	$(\bar{R}' = R'_x > 0)$
triaxial tension (b)	$R'_x < R'_y = R'_z < 0$	$(\bar{R}' = R'_x < 0)$
tension - biaxial compression	$R'_x < 0 < R'_y < R'_z$	$(\bar{R}' = R'_x < 0)$
general triaxial tension	$R'_x < R'_y < R'_z < 0$	$(\bar{R}' = R'_x < 0)$
uniaxial tension	$R'_x < R'_y = R'_z = 0$	$(\bar{R}' = R'_x < 0)$

(iv) Two deviatoric components - both decreasing:

biaxial tension	$R'_x = 0 > R'_y > R'_z$	$(\bar{R}' = 0)$
triaxial compression (a)	$R'_x > R'_y = R'_z > 0$	$(\bar{R}' = R'_x > 0)$
general triaxial compression	$R'_x > R'_y > R'_z > 0$	$(\bar{R}' = R'_x > 0)$
triaxial tension (a)	$0 > R'_x > R'_y = R'_z$	$(\bar{R}' = R'_x < 0)$
compression - biaxial tension	$R'_x > 0 > R'_y > R'_z$	$(\bar{R}' = R'_x > 0)$
general triaxial tension	$0 > R'_x > R'_y > R'_z$	$(\bar{R}' = R'_x < 0)$
uniaxial compression	$R'_x > R'_y = R'_z = 0$	$(\bar{R}' = R'_x > 0)$

(v) Two deviatoric components - one increasing, one decreasing:

general triaxial compression	$R'_x > R'_y > R'_z > 0$	$(\bar{R}' = R'_y > 0)$
biaxial tension	$R'_x = 0 > R'_y > R'_z$	$(\bar{R}' = R'_y < 0)$
biaxial compression	$R'_x = 0 < R'_y < R'_z$	$(\bar{R}' = R'_y > 0)$
general triaxial tension	$0 > R'_x > R'_y > R'_z$	$(\bar{R}' = R'_y < 0)$
tension - biaxial compression	$R'_x < 0 < R'_y < R'_z$	$(\bar{R}' = R'_y > 0)$
compression - biaxial tension	$R'_x > 0 > R'_y > R'_z$	$(\bar{R}' = R'_y < 0)$

*Note: Triaxial compression and triaxial tension are broad (blanket) terms. Three different versions of these regimes are referred to above - hence the discrimination via (a), (b), and "general".

fracture inhibition were originally discovered[339] without the aid of diphase principles, there can be little doubt that the latter "explain" its success in a more rational manner than does the cloaked empiricism (masquerading as would-be theory) typically advanced by conventional physics.

Returning to the concrete scene, it could not be claimed that the various applied loading cases examined to date in the context of the diphase model are exhaustive. Nevertheless, these constitute an adequate basis from which predictions pertaining to most other regime types follow without great difficulty via the flexible origin ($p_E' + \overline{R}'$) approach. As has already been shown with the cases of uniaxial tension and equal biaxial compression, inter alia, nominally dissimilar regimes may be rendered "equivalent" (in a general systematic sense) by giving prime consideration to the relative changes involved; i.e. through the adoption of a floating base reference state which conveniently includes $\overline{R}'$ as part of the "initial" external environment (complementing or counteracting p_E'), and to which "subsequent" changes D', $\Delta p_S'$, $\Delta p_F'$, etc. relate, whole families of comparable regimes - perhaps embracing several conventional designations* - can be identified. Table 4.2 lists examples of possible groupings: although reasonably comprehensive, the various categories described and their subsidiaries could, if desired, sustain further additions. Individual members of particular regime families are linked by the viability of a common interpretation with regard to the deviatoric aspects of associated external/internal partial pressure changes. The expected similarity of systematic response so engendered need not, of course, extend to the final manner of composite breakdown at the macroscopic level. Initial (reference) states may vary markedly; also, the interactions between a specimen system and its changing external environment will not be insensitive to the structural form and/or physical character of the deviatoric loading scheme employed. The diphase rationale is, however, well suited to cope with local variations; its inherent versatility enables due allowance for such factors to be made as a simple matter of course.

Since change is a strictly relative concept, its "understanding" is tied to a base reference position. Where the latter is chosen on the

---

* That the differences implied by conventional regime nomenclature may often allude to distinctions of a more local than fundamental nature should come as no surprise, bearing in mind the arbitrary precepts upon which the traditional vocabulary of the mechanics of material response is based.

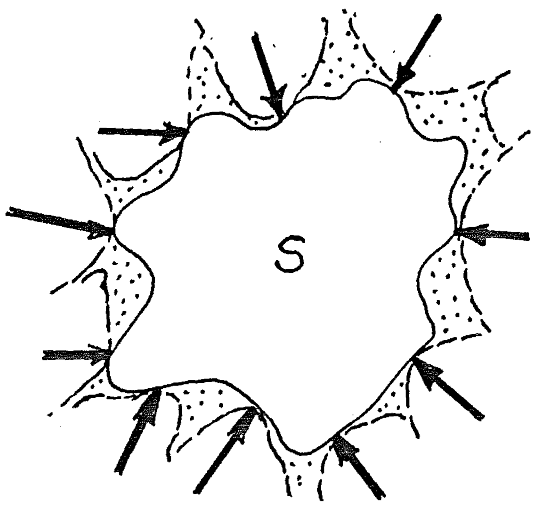
grounds of convenience, as has been advocated above in the regime family context, any interpretations which derive therefrom are necessarily lacking in uniqueness. In consequence, no one family can lay sole claim to its members. (The important qualification afforded by this statement is by no means limited to the families of current concern.) It may be seen that many of the regimes listed in Table 4.2 "belong" to more than one category. Despite first appearances, this element of diversity does not detract from the notion of regime families or its usefulness; indeed, the reverse is the case! In terms of comparative forecasting, it is extremely advantageous that family "links" do exist; these enable an overall perspective to be established. Another interesting feature of Table 4.2 is the limiting of consideration to a maximum of two deviatoric components. The would-be fundamental "need" for up to three deviatoric components implied by the conventional applied mechanics approach to general quantitative description of loaded material systems is quite illusionary. While the normal three-component view is perfectly viable (as one option among many), it can hardly be said to have been productive in other than a mathematical sense. As regards the rationalisation of material performance, its use - and that of the reference position upon which it is based - has yielded little in the way of valuable insight or consistent understanding. The fact that a simpler two-component view is both sufficient and equally viable continues to escape widespread notice because, as yet, few have seen fit to question the granting of unjustified privilege and spurious physical significance to one particular (but nonetheless arbitrary) reference position - the presumptiously title "unloaded state".

In the interests of brevity there will be no attempt made here to examine on an individual basis the diphase "understanding" with respect to the various regime types cited in Table 4.2. For each of the five families listed at least one member has already been considered in some detail: similar lines of argument are applicable to those which remain. Suffice it to say that experimental results and observations recorded in relation to the general mechanical performance of concrete (and similar) systems would appear to amply vindicate the forecasts of the diphase model in the vast majority of instances. (Only one manner of breakdown - to be focussed upon very shortly in the next section - defies adequate description via either the Class A or Class B options.) Since the diphase scheme interprets composite strength as an interactive structural property, influenced by both systematic and environmental factors, many of the seemingly contradictory results between the works of different researchers become amenable to rational (and simple) explanation; by way of contrast, these not-uncommon experimental "disagreements" act to confound conventional theories which interpret strength as an intrinsic material property. In Chapter 3, mention was made of Paul's dissatisfaction[202] with the use of friction-orientated failure criteria (whether based explicitly on shear/slip theories such as that governing the Coulomb-Mohr model or otherwise) in certain situations where applied tensile components obtain. Note might well be taken that this objection, although a valid indictment of conventional notions, has no immediate bearing on the diphase approach because the latter does not of course encompass the concept of tension in an absolute sense; i.e. applied tension is seen merely as a directional decrease ($D < 0$) in ambient compression and hence the physical basis for considering friction-orientated Class B response as at least a phenomenological possibility remains unimpaired.

4.3.8 In Closure - A Tying of Some Loose Ends

4.3.8.1 Equal Triaxial Compression - A Case Re-opened

For the most part, the diphase strength rationale as presented above attributes any tendancies towards systematic failure to the deviatoric fraction of an applied loading regime. The statistically isotropic near-rigid model is such as to effectively preclude Class B response under "pure" isotropic loading, $R'_x = R'_y = R'_z$ (cf. conventional failure theories involving distortional strain energy or limiting shear stress). Class A behaviour is forecast for the case of equal triaxial tension, $R'_x = R'_y = R'_z < 0$, but, for the opposite extreme of pure isotropic compression, $R'_x = R'_y = R'_z > 0$, the model would appear to predict (or, at least, imply) a complete lack of degenerative potential and thus, by default, infinite strength. While the latter (infinite strength) "forecast" is not itself unreasonable in the isotropic compression context (it is difficult, for example, to visualise a physical mechanism whereby a nominally solid material system might cease to sustain further increases in applied load), the parent prediction of no possible systematic breakdown lacks plausibility. Knowledge gained in the real world indicates that, under equal triaxial compression, the fundamental integrity of composite material systems (as distinct from their capacity to withstand continued isotropic load increases) may be seriously affected.



Key

= interactions from neighbours

= F phase

= boundaries of neighbours

FIGURE 4.36: An Individual S-Component

The source of this would-be anomalous feature of the diphase model lies in the latter's constitution; so too does a means of reconciling the associated discrepancy without detriment to its genuine predictive capacity as regards other forms of loading regime. It will be recalled that composite degeneration was originally conceived as a process which served, in effect, to identify (define) the "relevant" quasi-solid fraction of a notional diphase system: i.e. through an appropriate grouping of subordinate systems (selective discrimination), any need for an extended hierarchical "picture" in the context of material/structural breakdown was conveniently removed. Only one order of internal structure was ever considered to be at risk: potential damage - whether the result of overall Class A or Class B response - was seen to be restricted to a loss of composite stability (via structural alteration) within some prior arrangement of mutually interactive S - F components: degenerative "failure" of the relevant quasi-solid components, per se, was not contemplated. Insofar as the diphase model predicts that a material specimen subject to applied triaxial compression, $R'_x = R'_y = R'_z > 0$, will display no susceptibility to ultimate behavioural modes of either the systematic Class A or Class B types, its null-forecast does reflect experimental (real-world) findings. However, to rest the matter there, by merely skirting the issue of an apparent flaw in a rather negative fashion, would be somewhat less than satisfactory.

Figure 4.36 shows an individual S component isolated from within a diphase system. It may be seen that this internal "free-body" experiences environmental (locally external) loading from two quarters - viz, the surrounding F phase and its immediate quasi-solid neighbours. Thus, the mechanical status of the component differs little from that of a nominally solid material specimen subject to an irregular triaxial regime (in this case, soft all-round fluid loading plus hard discrete "point" loads in various orientations).

Although irregular loading patterns - lacking in both convenient directional characteristics and distributional uniformity - are a natural facet of the diphase scheme at the pertinent (S - F) level of internal discrimination, these have received no formal attention as regards macroscopic regimes. Neither is it intended that a rigorous development be undertaken here to demonstrate in an explicit manner the capacity of the diphase model to encompass predictions/descriptions of behavioural response under irregular regime types. That the diphase model has sufficient flexibility inherent to its fundamental principles and constitution to accomodate such an extension of scope should, it is hoped, be abundantly clear by now. A diphase version (or interpretation) of conventional stress analysis techniques is quite feasible*, resulting in meaningful descriptions of local systematic changes due to concentrated external load applications and the like. As previously, the Class A and Class B ultimate response options would prevail, which are the more likely to emerge and the probable manifestations thereof being determined (controlled) by the dictates of stability both local and total. Of course, real material systems display no less vulnerability to Class A and/or Class B type breakdown modes under irregular loading regimes than when subject to "purer" forms of applied action. In the Class A context, the ultimate behaviour of concrete specimens in standard "modulus of rupture" and "splitting" tests provides two obvious examples of such typical response; experimental studies on, inter alia, the "bearing strength" of concrete[340-344] serve to further confirm the generality of separation and/or slip as consistent mechanisms of systematic failure via disintegration.

Returning to the isolated quasi-solid particle of Figure 4.36, it therefore becomes apparent that the composite integrity of such "internal bodies" is at risk should the immediate environmental effects ever attain what might be termed critical levels. For the most part, the possibility of component breakdown (as distinct from system failure) can of course be eliminated in a notional sense by way of appropriate definition; i.e. through exercising convenient choice, the manifestations of any particular system failure under applied load may be seen in a purely structural light to reflect an emergence (via mechanical instability) of those quasi-solid elements having primary "relevance" as regards an equivalent diphase interpretation of the system itself. Justification for this selective manner of hierarchical grouping, which deliberately excludes the concept of component failure, has already been offered in relation to "understanding" degenerative response under a wide variety of regime designations. However, where the applied regime is one of equal triaxial compression, an approach which relies solely upon

* Weaknesses in the philosophical rationale of continuum mechanics - see earlier comments - can be allowed for by introducing statistical notions (or, at least, overtones) during any transformation of "stresses at a point" to equivalent partial pressure change parameters.

system failure traits to identify would-be relevant quasi-solid components must inevitably come to grief. Here, by its very nature, the regime provides "additional" confinement in all directions and hence the stability (under load) of the system as a whole is virtually assured. In consequence, load-induced systematic failure as previously understood will not occur unless special circumstances prevail; accordingly, the basis adopted earlier for establishing suitable quasi-solid components pertinent to a simple diphase picture is, by and large, rendered inoperative.

For the case of equal triaxial compression there exists, in general, no convenient alternative to an extended hierarchical approach, this involving a complete reversal of prior interpretations whereby quasi-solid components are considered prone to load-induced failure but the system as a composite entity is not! Such is the rather attypical character of material system response under equal triaxial compression that the departure from earlier treatments in no way jeopardises the rationale then developed. Component failure in this context may be seen as merely representing a non-critical form of systematic alteration through which the onset of local instabilities leads to internal hierarchical boundaries becoming redefined without detriment to overall forced integrity - the "new" system so created being able to sustain further external load increases until it too undergoes subsequent non-critical change, etc., etc.

The retention of composite stability under increasing triaxial load despite internal changes does not however guarantee that the system need maintain a mechanically viable regime-free condition; i.e. the internal alterations may be such as to produce a state which actually depends upon the presence of applied external compression to secure forced contact within the "new" quasi-solid substructure. Systems thus affected would therefore tend to disintegrate if the applied regime, having reached appropriate levels, were then removed or even lessened. Bearing in mind the relative orders of structure at which systematic change and local breakdown are envisaged here (somewhat lower than those contemplated previously) it should be apparent that a once-whole specimen system might well be reduced to an assortment of "fines" or "dust" through a process of loading and unloading. As regards the behaviour of nominally brittle material systems, this possibility of "non-critical" internal breakdown giving rise to subsequent regime-free instability has obvious parallel in the real world - viz, the phenomenon of crushing (in the proper sense of the word).

While the arguments advanced above have been framed specifically in terms of equal triaxial compression, it would be unreasonable to presume that the crushing option* pertained exclusively to this form of applied loading. Material systems subject to regimes of triaxial compression for which the deviatoric element(s), D', is (are) relatively small compared to the isotropic fraction, $\bar{R}'$, must also be deemed susceptible to at least some degree of component breakdown. Experimental observations substantiate the realism of the potential crushing forecast in such circumstances. (Some care must, of course, be exercised in examining the pertinent literature - especially in the realms of concrete research - to eliminate fallacious references to would-be crushing; the phenomenon is far less common than some would have it believed.)

4.3.8.2 The Diphase Model and the Nature of Concrete Systems

Concrete has been referred to on several occasions in this work - see especially Chapters 2 and 3 - as a multiphase material. In the various contexts within which the term was then employed, it was perfectly appropriate. Despite a superficial clash of nomenclature, the claimed aptness of a diphase model to represent such systems in no way compromises the validity of earlier descriptions. Of course, the diphase rationale selects its "relevant" (first-order) components on a quite different basis (mechanical convenience) from that governing the multiphase classification (immediate physical appearances at various levels of discrimination). Furthermore, the diphase "picture" is a purely local device; it does not attempt to deny the indefinitely continuous nature of hierarchical structuring but merely to condense this in a notional sense for the purposes of descriptive managability at a particular level. Any would-be element of seeming contradiction can therefore be reconciled without great difficulty. Nevertheless, in the interests of clarity, some brief comment on the diphase status of more conventional demarcations is apposite.

---

* Although loosely described as a separate "option" it is important to appreciate that, from an extended diphase viewpoint, crushing is seen as deriving from "lower level" Class and/or Class B response.

The first-order S fraction of a typical concrete system must be seen to comprise (or embrace) all material phases which, according to the traditional view based upon immediate physical appearances, would most often be classed as nominal solids. These include aggregates (both coarse and fine), the crystalline products of hydration, and any unhydrated cement. Individual components (or "particles") of the skeletal substructure can safely be presumed to vary somewhat in elemental constitution, this likelihood deriving from the overtly non-homogeneous nature of concrete systems. In certain instances, coarse aggregate/mortar interfaces (whether whole or in part) may be interpreted as potential boundary regions between adjacent S components: in others, such boundaries might well have to be taken as lying within coarse aggregate particles* and/or the mortar phase.

Neither can there be a unique diphase understanding with regard to the role of nominal (recognised) fluids such as air and water within concrete systems. Possible contributions in the context of a gross external quasi-fluid environment (E) pose little interpretive challenge. However, from an internal viewpoint, such media may be designated as generally contributing to <u>both</u> the F (interstitial) and S (skeletal) fractions. Some degree of associated uncertainty is therefore unavoidable; which proportion of which medium should be seen as "belonging" to which fraction will depend very much on the particular physical locations occupied and on the manner of incipient breakdown under prescribed loading endemic to the parent system itself. The first order S and F fractions are, of course, systems in their own right, each having hierarchical subordinates. Thus, for an S "particle" to contain fluid sub-fractions (within pores, interstices, or capillaries) is essentially no more problematic than it is for a fluid to contain solids (e.g. molecules - whether constituent or dissolved) at "lower levels".

It should not be overlooked here that the extended hierarchical philosophy, of which the diphase rationale functions as a local (grouped) application, rejects outright the conventional notion of total voids; the idea itself is treated as an essentially untenable premise - a conceptual fallacy born of somewhat naive presumption. In place of regions of would-be "nothing" (be these within concrete systems,

---

* It is not uncommon for the mechanical breakdown of mature concrete systems via cracking to involve some degree of coarse aggregate cleavage.

astronomical systems, or whatever) the hierarchical picture stipulates the active presence of non-trivial quasi-fluids. The latter are considered to complement, in a spatial sense, that which is normally designated as "real matter"; finite environmental influences or, more specifically, pressure distributions are seen to derive from such substantive fields. While this stance necessitates some degree of preliminary hypothesis as to the very nature of things physical, it allows for a logical development whereby bonding phenomena and the like become amenable to consistent "explanation"; i.e. the "existence" of a substantive non-matter complement manifests itself to the intellect rather than to man's more primitive senses*. By way of contrast, the established view of bonds must resort to various forms of axiomatic definition before it can "justify" (a posteriori) the primary raison d'etre of any composite system. In consequence, a rather unsatisfactory paradox emerges; viz, having first considered voids as regions of "nothing" if not explicitly, then at least by implication, the conventional approach is faced with a subsequent need for these "empty" spaces to be understood as possessing distinct physical characteristics - transmission of effect being perhaps the most obvious "property" required. Of course, the typically abstract form of associated mathematical presentations serves (whether by accident or design) to disguise this inherent conflict. Accordingly, in the current scientific climate, where little serious store is set by any physical argument which might detract from immediate mathematical convenience, the simple fact that a paradox exists is seldom recognised. Such is now the general ascendancy of applied mathematics over what was once termed natural philosophy that even the proper status of "empty space" - a mere postulate, neither invulnerable nor self-evident - is no longer widely appreciated; the elevation thereof to the rank of a truism is as fallacious as it is popular.

4.3.8.3 General Systematic Alteration

Notwithstanding the tenor of certain previous remarks with regard to the case of triaxial compression, applications of the diphase

---

* In a similar manner, many gaseous materials are beyond immediate physical observation but may, nevertheless, be "detected" via the powers of reason.

model have mainly concentrated upon degenerative tendancies as the precursors of some overall systematic failure induced by a "steadily" intensifying regime. There is, however, no explicit requirement that a regime need ever attain critical levels; i.e. the diphase model is not restricted in scope to failure considerations alone.

Those internal changes which occur within a system in the presence of any applied external regime may or may not be reversible. Whether a system sustains permanent (inelastic) damage under such "additional" loading will depend on both the nature of the system itself and that of the various associated changes which take place; in turn, the latter are likely to be influenced to some extent by the physical character of the regime actually causing the systematic alterations.

From a quite general viewpoint, the concept of an "elastic limit" is not precluded by the diphase philosophy; neither is it a necessary feature. For an elastic response to prevail, be this of a "linear" nature or whatever, it must be presumed that the potential for systematic degeneration is not realised. Conversely, although the application of a particular regime may not extend to the level of composite failure, it would be unreasonable to suppose that, where local dissociations of the S components do occur in the process, the system could retrace a behavioural path to the regime-free state showing no effects thereof. Thus, inelastic response under conditions of applied (supplementary) loading and unloading may be seen as corresponding to an altered regime-free state; in particular, a system which qualifies for description as statistically isotropic prior to applied loading, but subsequently degenerates to some extent as a consequence thereof, will not so qualify upon its return; the changes which have occurred in the interim will be reflected in a "new" set of regime-free parameters, p'_{So} and p'_{Fo}. Of course, in contrast to the connotations endemic to more conventional treatments, the diphase view of a regime-free state is not synonomous with that of a totally unloaded condition. Uniqueness is not demanded; different forms of "initial" composite integrity are perfectly viable.

Inelastic response, such as is obviously the more relevant "option" as regards the behaviour of concrete systems, may also manifest itself via systematic alteration under conditions where an applied regime is sustained at constant levels. Here again the diphase model appears well-suited to describing appropriate mechanisms of change.

Thus, although prior references to external/internal equilibrium states may perhaps have implied combinations of mechanical circumstance possessed of a rather static quality, an overtly dynamic interpretive approach (in which the convenience of the static "picture" still obtains in an "instantaneous" sense) is equally valid. Under an applied regime, R', of constant intensity, the partitioning of internal load between the S and F fractions need not exhibit temporal independence as a matter of principle. The dictates of force equilibrium only govern the quantitative aspects of stable systematic response insofar as the various sums of partial pressure parameters ($p'_S + p'_F$) are concerned. Complementary changes in the individual contributions of p'_S and p'_F to these aggregated sets are therefore quite "permissable"; i.e. it is perfectly conceivable that a system might alter in a continuous manner via interaction with its physical environment regardless of whether this is subject to temporal variation. Since the partitioning of internal load is presumably a reflection of systematic state, any quantitative change in the former may be interpreted as indicative (or even symptomatic) of progressive alterations in the latter.

The phenomenon alluded to above - generally classified as "creep" - can exhibit different forms. Thus, for example, in the context of concrete specimens subject to relatively high levels of uniaxial compression, short-term "creep-to-failure" can transpire in a matter of minutes[106]. Such is due to temporal spread of microcracks and the consequent emergence of an unstable degenerate state - a situation to which the particular diphase "understanding" as developed in this work can obviously relate. What then of long-term creep effects associated with the cement paste fractions of concrete systems? Here, the changes which occur concern processes of environmental interaction whereby local "imbalances" of effect diminish with time as the system seeks to establish some manner of steady-state condition. The simple (undifferentiated) diphase model is somewhat inadequate as a phenomenological descriptor in this context. However, the extended hierarchical view which embodies the concepts of nested environments and different levels of solid/fluid structuring is not similarly afflicted.

A recognition of the potential for "continuous" systematic alteration in response to environmental influences at different levels has important connotations with regard to the would-be constancy/ stability of the regime-free state. Thus, phenomena such as shrinkage and creep recovery can easily be brought within the scope of a generalised diphase understanding. So too can certain "anomolous" forms of behaviour which, although unrelated directly to the field of concrete performance, serve to confirm the rationale of the diphase approach. Mention has already been made of systems "returning" to the regime-free state in a different condition from that which prevailed prior to their "additional" external loading and unloading. Given that significant internal changes have occurred with respect to the distribution of partial pressures experienced by hierarchical components, a return to the regime-free state need not be seen as guaranteeing a return to relative stability; i.e. the "new" internal loading could now pose a threat to the system remaining viable as a composite entity. That being the case, the system could actually "creep" to failure. "Anomolous" failures of the delayed-action type have been observed with real material systems under certain circumstances. To quote Freudenthal[255]:

> *In one of the experiments performed by Bridgman a thick-walled, hardened steel cylinder was subjected to sufficient external pressure to produce radial plastic yield toward the center of the cylinder, resulting in a permanent decrease of the diameter of the inside hole. On release of pressure, and after standing for some hours, a radial crack developed at the inner wall and gradually spread outward, until it reached the outside of the wall. It is also known that glass cylinders, subject to sustained outside pressure, tend to crack spontaneously sometime after release of pressure. Although it might be concluded that the phenomenon is produced by the residual tensile stresses of the deformed cylinder, two of its aspects cannot be explained by residual stresses alone: the spreading of the crack through the entire thickness of the wall, and the delay in starting the crack.*

Bridgman[334] himself remarks,

> *The paradox in this situation is the development of rupture on release of the pressure that had produced the rupture.*

The explanations of delayed-action failure tendered by conventional sources (including Freudenthal and Bridgman) are far from convincing. These serve merely to disperse the air of paradox, not to resolve it. Indeed, satisfactory resolution is quite beyond the power of the normal view because the apparent anomaly is largely an ultimate product thereof, fostered by an overreliance on the suitability of arbitrary precepts. In their attempts to gain some degree of physical understanding, several workers have invoked notional datum shifts through which a material body is seen to sustain an internal stress state after previously applied external loading has been removed. Unfortunately, the potential embodied within this approach is never fully recognised; i.e. instead of its functioning as a starting point from which the spurious privilege normally afforded to the "unloaded" condition might be challenged, it is adopted on a strictly one-off basis - a contrived interpretation designed solely to cope with (save?) an embarrassing phenomenon. Thus, the same workers seem content to shun any need for consistency and quickly revert to conventional treatments ("stress-free" origins) when modelling other aspects of material behaviour.

4.3.8.4 The Significance of Strain

The diphase rationale associates the bulk deformations of near-rigid bodies with relative changes in the parameters p_S' and p_F', allowance being made for any directional variations which might obtain. Increased compression of the skeletal quasi-solid fraction is seen to give rise to condensation or contraction; conversely, dimensional elongation is interpreted as a consequence of decreased compression. Poisson's ratio effects are forecast by virtue of the interactive nature of the scheme envisaged*: i.e. the application of external load is understood to alter all directional values of p_S' and p_F', regardless of whether the load itself contributes to external partial pressure in more than one direction.

While the scope of the diphase model encompasses deformational traits, the latter do not constitute a particularly crucial aspect appropos the diphase understanding of general systematic breakdown. It will be recalled that the model attributes any loss of previous composite integrity to changes in prevailing external/internal partial pressure conditions; it attaches no direct causal significance to "corresponding" deformations. Accordingly, notional criteria of failure based upon limiting strain are quite foreign to the diphase model.

---

* Although no formal reference has been made to the possibility of load-induced structural rearrangement among the contiguous quasi-solid "particles", it should be apparent that positional change mechanisms could well exert considerable influence on the relative magnitude of Poisson's ratio effects in appropriate circumstances. Had the behaviour of overtly granular material systems been the main topic of this work, the role of physical mechanisms at the particulate level would have demanded greater attention.

Despite a wealth of experimental evidence to the contrary, the concept of limiting extensional strain continues to find adherents among concrete technologists as an "explanation" of failure under certain conditions. (In fairness, it must be said that concrete technologists are not alone in this; the premise pervades many areas of material science.) Lowe[337] has recently suggested that some of the "biaxial" evidence which acts to indict the limiting strain criterion itself merits suspicion and should perhaps be discounted as "unreliable". Other advocates of strain-based failure theories have expressed similar sentiments on occasions. Lowe's concern with experimental reliability is not without foundation; cf. earlier comments offered by the present writer in this regard. The would-be "results" of many recorded experimental programmes do lack valid substance and hence fully deserve unfavourable criticism. However, Lowe overlooks - or is unaware of - one manner of physical phenomenon which, beyond all doubt, exposes the vulnerability of the limiting extensional strain concept.

Mention has already been made of the similarities which can exist between the ultimate behaviour of material specimens under "simple" tension, $R'_x < 0 = r'_y = R'_z$, and that under triaxial regimes of the form $0 < R'_x < R'_y = R'_z$, the latter being commonly designated by the misnomeric term "triaxial extension tests". (The potential for similarity is of course forecast by the diphase model.) Thus, in both instances the classical manifestation of brittle response - bulk separation via a single cleavage-type (Class A) macrocrack - is possible. (Lowe actually alludes to experimental records highlighting this parallel in the context of concrete specimens.) The two regimes need not, however, share common characteristics as regards the sense of dimensional alterations. Single cleavage-type failures have been observed in concrete specimens subject to triaxial compression having components $R'_x < R'_y = R'_z$ such that all principal strains are compressive[338]. Further (and less recent) examples of material systems failing "against compression" as if in tension are to be found in the work of Bridgman[334]; Freudenthal[255], among others, has also made reference to this "complex" phenomenon.

Any appreciation of strain as a relative change parameter is naturally sensitive to the underlying reference length adopted. Thus, compressive strains as seen from "stress-free" origins may be rendered extensional ("tensile") through the choice of an alternative datum.

Arguments along these lines - aimed at "saving" the limiting extensional strain concept - have been advanced by several commentators. As with the case of delayed-action type failure discussed earlier, much of the reasoning typically offered is, in itself, extremely plausible. But, as before, the severe restricting of such arguments to special (troublesome) circumstances (cf. the inherent flexibility of the diphase approach) engenders inevitable ambivalence; for the most part, questions as to which makes the "new" origins more suitable or why coordinate datum shifts are not contemplated in other situations are rarely posed, much less answered. Those of a cynical disposition would find rich pastures here! Only Bridgman appears to accept the two "paradoxical" phenomena as an intellectual challenge rather than as something of an embarrassment; his thoughts on both topics reveal considerable insight and are relatively free of the logical "blindness" to which others seem (conveniently?) prone; that a satisfactory all-round understanding, such as the diphase model purports to provide, should have eluded his grasp (virtually by his own admission) is perhaps the saddest reflection on his continued endeavours during an otherwise fruitful lifetime.

Because the diphase model interprets any composite material body in the regime-free state as being effectively pre-loaded, it can not treat this condition as a suitable basis upon which to establish primary systematic origins whether in relation to load or deformation. Even its associative usefulness as a reference state for "appropriate" secondary (derivative) origins is open to challenge. Consider, for example, the case of increasing uniaxial compression, $R'_x > 0 = R'_y = R'_z$, as described previously. If the system exhibits Class A response, then as R'_x tends towards the "uniaxial strength value", $R'_{x_{max}}$ (i.e. as the total load in the x direction tends towards $p'_E + R'_{x_{max}}$) the degree of mutual forced contact within the quasi-solid phase, orthogonal to x, progressively decreases. It is, of course, the latter which is seen to govern the ultimate loss of stable load-carrying capacity in this instance; i.e. the onset of bulk disintegration ("falling apart") corresponds to a point where the overall effectiveness of p'_S in various directions is eliminated, mechanical viability of the system as a whole no longer obtains, and degenerate columnar elements "emerge". Notwithstanding its obvious practical importance, the uniaxial strength, $R'_{x_{max}}$, valued with respect to the regime-free state has no immediate bearing on the fundamental "cause" of structural breakdown envisaged here - viz, a lack of stabilising interaction between adjacent elements. The diphase model sees the "additional" presence of $R'_{x_{max}}$ as playing an instrumental role, rather than as that of the prime factor. In terms of systematic relevance, the point of uniaxial compressive failure may be interpreted as a lower limit of forced lateral integrity compatible with stable composite existence. Thus, the failure condition has much more to offer as a local phenomenological datum than has the "original" regime-free state.

That a switch to failure-based origins can lead to the emergence of simple quantitative interactions has been demonstrated by Grimer and Hewitt[110]. Thus, appropriate modification of normalised stress-strain data (derived from controlled short-term uniaxial compression tests to failure on concrete specimens having various material constitutions) allowed that data to be described by a family of "power-law" relationships, differences between individual concretes being reflected in the particular value of the power index which obtained for each. The degree of data correlation achieved thereby was to within the limits of experimental uncertainty. Bearing in mind that which has already been argued with respect to the possible interpretations of power-law formulations, such findings are strongly supportive of the diphase rationale.

4.3.8.5 Closure

The concepts of the diphase model, and of the extended hierarchical approach which lies behind it are potentially far-reaching. The writer is hopeful that he has been able to transmit some impression of that potential through this work.

REFERENCES

1. JOHNSON, R.P. Structural Concrete. London, McGraw-Hill, 1967. p.22.

2. LEA, F.M. The Chemistry of Cement and Concrete. London, Arnold, 1970.

3. CZERNIN, W. Cement Chemistry and Physics for Civil Engineers. London, Crosby Lockwood, 1962.

4. Hydraulic Cement Pastes: Their Structure and Properties. Cement and Concrete Association, London, 1976.

5. NEVILLE, A.M. Properties of Concrete. London, Pitman, 1972. pp.103-180.

6. NEWMAN, K. The effect of water absorption by aggregates on the water/cement ratio of concrete. Magazine of Concrete Research, Vol.11, No.33, November 1959. pp.135-142.

7. NEWMAN, A.J., and TEYCHNNE, D.C. A classification of natural sands and its use in concrete mix design. Proceedings of a Symposium on Mix Design and Quality Control of Concrete, London, May 1954. London, Cement and Concrete Association, 1955. pp.175-193.

8. POWERS, T.C. Physical properties of cement paste. Proceedings of the Fourth International Symposium on the Chemistry of Cement, Washington, 1960, Vol.2. pp.557-613.

9. NEWMAN, K. Discussion of Ref.6. Magazine of Concrete Research, Vol.12, No.35, July 1960. pp.115-118.

10. GILKEY, H.J. Water gain and allied phenomena in concrete work. Engineering News - Record, Vol.98, No.6, Feb.1927. pp.242-244.

11. HUGHES, B.P., and ASH, J.E. Water gain and its effects on concrete. Concrete, Vol.2, No.12, December 1969. pp.494-496.

12. RITCHIE, A.G.B. Stability of fresh concrete mixes. Journal of A.S.C.E., Vol.92, No.CO1, January 1966. pp.17-35.

13. HSU, T.T.C., SLATE, F.O., STURMAN, G.M., and WINTER, G. Micro-cracking of plain concrete and the shape of the stress-strain curve. Journal A.C.I., Proceedings Vol.60, No.2, February 1963. pp.209-224.

14. SLATE, F.O., and OLSEFSKI, S. X-rays for study of internal structure and microcracking of concrete. Journal A.C.I., Proceedings Vol.60, No.5, May 1963. pp.575-588.

15. ROBINSON, G.S. Methods of detecting the formation and propagation of microcracks in concrete. Proceedings of an International Conference on the Structure of Concrete, London, 1965. London, Cement and Concrete Association, 1968. pp.131-145.

16. KRISHNASWAMY, Z.I. Strength and microcracking of plain concrete under triaxial compression. Journal A.C.I., Proceedings Vol.65, 1968. pp.856-862.

17. SWAMY, R.N. Study of the micro-mechanical behaviour of concrete using reflective photoelasticity. R.I.L.E.M., Materials and Structures, Vol.4, No.24, 1971. pp.357-370.

18. DHIR, R.K., and SANGHA, C.M. Development and propagation of microcracks in plain concrete. R.I.L.E.M., Materials and Structures, Vol. 7 , No.37, 1974. pp.17-23.

19. SANTIAGO, S.D. Fracture mechanisms of concrete under compressive loads. Cement and Concrete Research, Vol.3, 1973. pp.363-388.

20. ASH, J.E. Bleeding in concrete - a microscopic study. Journal A.C.I., Proceedings Vol.69, April 1972. pp.209-211.

21. SLATE, F.O., and MATHEUS, R.E. Volume changes upon setting and curing of cement pastes and concrete from time zero to seven days. Journal A.C.I., Proceedings Vol.64, No.1, January 1967. pp.34-39.

22. HSU, T.T.C. Mathematical analysis of shrinkage stresses in a model of hardened concrete. Journal A.C.I., Proceedings Vol.60, No.3, March 1963. pp.371-390.

23. THEOCARIS, P.S., and KOUFOPOULOS, T. Photo-elastic analysis of shrinkage microcracking in concrete. Magazine of Concrete Research, Vol.21, No.66, March 1969. pp.15-22.

24. SMEATON, J. A narrative of the building of the Eddystone Lighthouse, 1791.

25. FERET, R. Sur la compacite des mortiers hydraulique. Ann. Ponts Chaussees, 1892. pp.1-184.

26. FERET, R. Studies on the internal constitution of hydraulic mortars. Bulletin de la Societe d'Encouragement pour l'Industrie Nationale, Vol.1, 1897.

27. ZIELINSKI, S. The development of the setting of Roman and Portland cements, in pastes, in mortars and in concrete. Proceedings of the International Association for Testing of Materials, Copenhagen, Vol.1, 1909. pp.1-55.

28. ABRAMS, D.A. Design of concrete mixes. Bulletin No.1, Structural Materials Research Laboratory, Chicago, 1918.

29. ABRAMS, D.A. Water-cement ratio as a basis of concrete quality. Proceedings A.C.I., Vol.23, 1927. pp.452-458.

30. ENTROY, H.C., and SHACKLOCK, B.W. Design of high strength concrete mixes. Proceedings of a Symposium on Mix Design and Quality Control of Concrete, London, May 1954. London, Cement and Concrete Association, 1955. pp.55-73.

31. KAPLAN, M.F. Flexural and compressive strength of concrete as affected by the properties of coarse aggregates. Journal A.C.I., Proceedings Vol.55, May 1959. pp.1193-1207.

32. WALKER, S., and BLOEM, D.L. Effects of aggregate size on properties of concrete. Journal A.C.I., Proceedings Vol.57, September 1960. pp.283-297.

33. GILKEY, H.J. Water-cement ratio versus strength - another look. Journal A.C.I., Proceedings Vol.57, April 1961. pp.1287-1312.

34. CORDON, W.A., and GILLESPIE, H.A. Variables in concrete aggregates and Portland cement paste which influence the strength of concrete. Journal A.C.I., Proceedings Vol.60, August 1963. pp.1029-1050.

35. BLOEM, D.L., and GAYNOR, R.D. Effects of aggregate properties on strength of concrete. Journal A.C.I., Proceedings Vol.60, October 1963. pp.1429-1454.

36. HUGHES, B.P., and CHAPMAN, G.P. The deformation of concrete and microconcrete in compression and tension with particular reference to aggregate size. Magazine of Concrete Research, Vol.18, No.54, March 1966. pp.19-24.

37. HUGHES, B.P., and BAHRAMIAN, B. Some factors affecting the compressive strength of concrete. Magazine of Concrete Research, Vol.19, No.60, September 1967. pp.165-172.

38. KAWAKAMI, H. Effect of gravel size on strength of concrete with particular reference to sand content. Proceedings of an International Concerence on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc.of Mat.Sc., Vol.4, 1972. pp.96-124.

39. POWERS, T.C., and BROWNYARD, T.L. Studies of the physical properties of hardened Portland cement paste. Part 6. Proceedings A.C.I., Vol.43, March 1947. pp.845-864.

40. POWERS, T.C. The physical structure and engineering properties of concrete. Cement Lime and Gravel, No.10, 1956.

41. POWERS, T.C. The non-evaporable water content of hardened Portland cement paste - its significance for concrete research and its method of determination. A.S.T.M. Bulletin, No.158, 1949.

42. POWERS, T.C. A discussion of cement hydration in relation to the curing of concrete. Proceedings of the Highway Research Board, Vol.27, December 1947. pp.180.

43. WISCHERS, G. Einfluss einer temperaturanderung auf die festigkeit von zementstein und zementmortel mit zuschlagstoffen verschiedener warmedehnung. Dusseldorf, Verein Deutscher Zementwerke, 1961. pp.117.

44. LAWRENCE, C.D. The properties of cement paste compacted under high pressure. London, Cement and Concrete Research Association, Research Report 19, 1969.

45. BAJZA, A. On the factors influencing the strength of cement compacts. Cement and Concrete Research, Vol.2, 1972. pp.67-78.

46. ROY, D.M., and GOUDA, G.R. High strength generation in cement pastes. Cement and Concrete Research, Vol.3, 1973. pp.807-820.

47. SEREDA, P.J., FELDMAN, R.F., and SWENSON, E.G. Effects of sorbed water on some mechanical properties of hydrated Portland cement pastes and compacts. Highway Research Board, Special Report 90, 1966. pp.58-63.

48. YUDENFREUND, M., HANNA, K.M., SKALNY, J., ODLER, I., and BRUNAUER, S. Hardened Portland cement pastes of low porosity v. compressive strength. Cement and Concrete Research, Vol.2, 1972. pp.731-743.

49. HARRIS, A.J. High strength concrete: manufacture and properties. The Structural Engineer, Vol.47, No.11, 1969. pp.441-452.

50. MacINNIS, C., and THOMSON, D.V. Special techniques for producing high strength concrete. Journal A.C.I., Proceedings Vol.67, December 1970. pp.996-1002.

51. POPOVICS, S. A model for the kinetics of the hardening of Portland cement. Highway Research Board, Report No.192, 1967. pp.14-35.

52. POPOVICS, S. Comparisons of various measurements concerning the kinetics of hydration of Portland cements. Proceedings of the Fifth International Symposium on the Chemistry of Cement, Tokyo, 1968.

53. POWERS, T.C. Concrete studies of the Bull Run Dam, City of Portland, Oregon. Journal A.C.I., Proceedings Vol.25, 1929. pp.388-431.

54. THAULOW, S. Discussion of Ref.32. Journal A.C.I., Proceedings Vol.57, March 1961. p.1234.

55. COLLIER, I.L. Variations of compressive strength of concrete within the water-cement ratio law. Proceedings A.S.T.M., Vol.30, 1930. pp.731-741.

56. ISHAI, O. Discussion of Ref.13. Journal A.C.I., Proceedings Vol.60, December 1963. p.1803.

57. HUGHES, B.P., and ASH, J.E. The effect of mix properties and aggregate dust upon the compressive strength of concrete. Magazine of Concrete Research, Vol.20, No.63, June 1968. pp.77-84.

58. SWAMY, R.N., and RAO, C.V.S.K. Fracture mechanism in concrete systems under uniaxial loading. Cement and Concrete Research, Vol.3, 1973. pp.413-427.

59. TE'ENI, M. Deformational modes and structural parameters in cemented granular systems. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.621-642.

60. GLUCKLICH, J. The strength of concrete as a composite material. Proceedings of an International Conference on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc.of Mat.Sci., Vol.4, 1972. pp.104-112.

61. ALEXANDER, K.M. Strength of the cement-aggregate bond. Journal A.C.I., Proceedings Vol.56, No.5, November 1959. pp.377-390.

62. ALEXANDER, K.M., and WARDLAW, J. Dependence of cement-aggregate bond strength on size of aggregate. Nature, Vol.187, No.4733, July 1960. pp.230-231.

63. HSU, T.T.C., and SLATE, F.O. Tensile bond strength between aggregate and cement paste or mortar. Journal A.C.I., Proceedings Vol.60, No.4, April 1963. pp.465-486.

64. ALEXANDER, K.M., WARDLAW, J., and GILBERT, D.J. Aggregate-cement bond, cement paste strength and the strength of concrete. Proceedings of an International Conference on the Structure of Concrete, London, 1965. London, Cement and Concrete Association, 1968. pp.59-81.

65. TAYLOR, M.A., and BROMS, B.B. Shear bond strength between coarse aggregate and cement paste or mortar. Journal A.C.I., Proceedings Vol.61, No.8, August 1964. pp.939-950.

66. le CHATELIER, H. Recherches experimentales sur la constitution des liants hydrauliques. Dunod, Paris, 1904.

67. WITTMANN, F. Experiments to determine van der Waals forces. Proceedings of an International Conference on Structure, Solid Mechanics and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.295-300.

68. PIHLAJAVAARA, S.E. History dependence, ageing and irreversibility of properties of concrete. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.719-741.

69. SWAMY, R.N. Aggregate-matrix interaction in concrete systems. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.301-315.

70. SHAH, S.P., and CHANDRA, S. Critical stress, volume change and microcracking of concrete. Journal A.C.I., Proceedings Vol.65, September 1968. pp.770-781.

71. NEPPER-CHRISTENSEN, P., and NIELSEN, T.P.H. Modal Determination of the effect of bond between coarse aggregate and mortar on the compressive strength of concrete. Journal A.C.I., Proceedings Vol.66, January 1969. pp.69-72.

72. LEZY, R., and PAILLERE, A.M. R.I.L.E.M. Symposium on Synthetic Resins in Building and Construction, Paris, 1967.

73. BERGER, R.L. Properties of concrete with cement clinker aggregate. Cement and Concrete Research, Vol.4, 1974. pp.99-112.

74. GALWEY, A.K., JONES, K.A., and BECKETT, R.S. Observations on the relatively high strength of a chalk-aggregate concrete. Journal of Applied Chemistry, Vol.16, 1966. pp.159-162.

75. MAYER, F.L. The effect of different aggregates on the compressive strength and modulus of elasticity of normal concrete. Beton, Vol.22, No.2, 1972. pp.61-62.

76. BENNET, E.W., and KHILJI, Z.M. The effect of some properties of the coarse aggregate in hardened concrete. Journal of the British Granite and Whinstone Federation. Part 1, Autumn 1963. pp.17-28. Part 2, Spring 1964. pp.17-24.

77. NEVILLE, A.M. Properties of Concrete. London, Pitman, 1972. p.251.

78. NATIONAL SAND AND GRAVEL ASSOCIATION, Joint Technical Information Letter No.155, Washington, April 1959.

79. BACHE, H.H. Strength of structural lightweight aggregate concrete. Proceedings of the Symposium of Testing and Design Methods of Lightweight-aggregate Concrete, Budapest, 1967. R.I.L.E.M., Vol.1, 1967. pp.9-43.

80. CZURYSZKIEWICZ, A. The effect of aggregate shape upon the strength of structural lightweight-aggregate concrete. Magazine of Concrete Research, Vol.25, No.83, June 1973. pp.81-86.

81. WILLS, M.H. Lightweight aggregate particle shape effect on structural concrete. Journal A.C.I., Proceedings Vol.71, March, 1974. pp.134-142.

82. SIGVALDASON, O.T. The influence of testing machine characteristics upon the cube and cylinder strength of concrete. Magazine of Concrete Research, Vol.18, No.57, December 1966. pp.197-206.

83. WRIGHT, P.J.F. Compression testing machines for concrete. The Engineer, Vol.203, No.5283, April 1967. pp.639-641.

84. COLE, D.G. Some mechanical aspects of compression testing machines. Magazine of Concrete Research, Vol.19, No.61, December 1967. pp.247-251.

85. SABNIS, G.M., and ARONI, S. Size effects in material systems - the state of the art. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.131-142.

86. SPOONER, D.C. Stress-strain-time relationships for concrete. Magazine of Concrete Research, Vol.23, No.75-76, June-September 1971. pp.127-131.

87. SANGHA, C.M., and DHIR, R.K. Strength and complete stress-strain relationships for concrete tested in uniaxial compression under different test conditions. R.I.L.E.M., Materials and Structures, Vol.5, No.30, 1972. pp.361-370.

88. SPARKS, P.R., and MENZIES, J.B. The effect of rate of loading upon the static and fatigue strengths of plain concrete in compression. Magazine of Concrete Research, Vol.25, No.83, June 1973. pp.73-80.

89. SCHICKERT, G. On the influence of different load application techniques on the lateral strain and fracture of concrete specimens. Cement and Concrete Research, Vol.3, 1973. pp.487-494.

90. PIHLAJAVAARA, S.E. A review of some of the main results of a research on the ageing phenomena of concrete: effects of moisture conditions on strength, shrinkage and creep of mature concrete. Cement and Concrete Research, Vol.4, 1974. pp.761-771.

91. HOBBS, D.W. The strength and deformation of concrete under short-term loading: a review. Technical Report, No.42.484. London, Cement and Concrete Association, September 1973.

92. BRITISH STANDARDS INSTITUTION. B.S.1881. Methods of testing concrete. Part 4. Methods of testing concrete for strength. London, 1970.

93. A.S.T.M. STANDARD C39-72. Standard method of test for compressive strength of cylindrical concrete specimens. Annual of A.S.T.M. Standards, 1974. pp.26-29.

94. NEWMAN, K., LACHANCE, L., and LOVEDAY, R.W. Strain measurements on saturated concrete specimens. Magazine of Concrete Research, Vol.15, 1963. pp.143-150.

95. DANTU, P. Etude des contraintes dans les milieux heterogenes: application au beton. Annales de l'Institut Technique du Batiment et des Travaux Publics, Vol.11, No.121, January 1958. pp.55-77.

96. McCREATH, D.R., NEWMAN, J.B., and NEWMAN, K. The influence of aggregate particles on the local strain distribution and fracture mechanism of cement paste during drying shrinkage and loading to failure. R.I.L.E.M., Materials and Structures, Vol.2, No.7, January-February 1969. pp.73-84.

97. JONES, R. A method of studying the formation of cracks in a material subjected to stress. British Journal of Applied Physics, No.3, 1952. pp.229-232.

98. YOKOMICHI, H., and FUJII, T. Instantaneous detection and mechanical behaviour of microcracking in concrete. Proceedings of an International Conference on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc. of Mat.Sc., Vol.4, 1972. pp.125-131.

99. NEWMAN, K. Criteria for behaviour of plain concrete under complex states of stress. Proceedings of an International Conference on the Structure of Concrete, London, 1965. London, Cement and Concrete Association, 1968. pp.255-274.

100. NEWMAN, K., and NEWMAN, J.B. Failure theories and design criteria for plain concrete. Proceedings of an International Conference on Structure, Solid Machanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.963-995.

101. NEWMAN, J.B. The complete deformational behaviour mechanics of failure and design criteria for plain concrete under various combinations of load. Criteria of Concrete Strength. Report No.2, Imperial College, 1973.

102. JONES, R., and KAPLAN, M.F. The effect of coarse aggregate on the mode of failure of concrete in compression and flexure. Magazine of Concrete Research, Vol.9, 1957. pp.89-94.

103. KAPLAN, M.F. Strains and stresses of concrete at initiation of cracking and near failure. Journal A.C.I., Proceedings Vol.60, July 1963. pp.853-879.

104. VILE, G.W.D. The strength of concrete under short-term static biaxial stress. Proceedings of an International Conference on the Structure of Concrete, London, 1965. London, Cement and Concrete Association, 1968. pp.275-288.

105. BRANDTZAEG, A. Failure of a material composed of non-isotropic elements. Det.Klg.Norske (Trodhjem), Vid.Sels.Skrifter No.2, 1927.

106. RUSCH, H. Researches towards a general flexural theory for structural concrete. Journal A.C.I., Proceedings Vol.57, July 1960. pp.1-28.

107. DHIR, R.K., and SANGHA, C.M. A study of the relationships between time, strength, deformation, and fracture of plain concrete. Magazine of Concrete Research, Vol.24, No.81, December 1972. pp.197-208.

108. GLUCKLICH, J. On the compression failure of concrete. T.&.A.M. Report, No.215, University of Illinois, March, 1962.

109. SPOONER, D.C. The stress-strain relationship for hardened cement pastes in compression. Magazine of Concrete Research, Vol.24, No.79, June 1972. pp.85-92.

110. GRIMER, F.J., and HEWITT, R.E. The form of the stress-strain curve of concrete interpreted with a diphase concept of material behaviour. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.681-691.

111. RUSCH, H. Versuche zur festigkeit der biegedruckzone. Deutscher Ausschuss fur Stahlbeton, No.120.

112. KUPFER, H., HILSDORF, H.K., and RUSCH, H. Behaviour of concrete under biaxial stresses. Journal A.C.I., Proceedings Vol.66, August 1969. pp.656-665.

113. CLARK, L.E., GERSTLE, K.H., and TULIN, L.G. Effect of strain gradient on the stress-strain curve of mortar and concrete. Journal A.C.I., Proceedings Vol.64, September 1967. pp.580-586.

114. STURMAN, G., SHAH, S.P., and WINTER, G. Effects of flexural strain gradients on microcracking and the stress-strain behaviour of concrete. Journal A.C.I., Proceedings Vol.62, No.7, July 1965. pp.805-822.

115. ABELS, P.W., and HU, C.H. Flexural microcracking in unreinforced concrete beams. Journal A.C.I., Proceedings Vol.68, October 1971. pp.779-786.

116. KAMLOS, K. Factors affecting the stress-strain relation of concrete in uniaxial tension. Journal A.C.I., Proceedings Vol.66, February 1969. pp.111-114.

117. JOHNSTON, C.D. Strength and deformation of concrete in uniaxial tension and compression. Magazine of Concrete Research, Vol.22, No.70, March 1970. pp.5-16.

118. HUGHES, B.P., and CHAPMAN, G.P. The complete stress-strain curves for concrete in direct tension. R.I.L.E.M. Bulletin No.39, March 1966. pp.95-97.

119. EVANS, R.H., and MARATHE, M.S. Microcracking and stress-strain curves for concrete in tension. R.I.L.E.M., Materials and Structures, No.1, January-February 1968. pp.61-64.

120. UPPAL, J.Y., and KEMP, K.O. The effect of longitudinal gradients of compressive stress upon the failure of concrete. Magazine of Concrete Research, Vol.23, No.74, March 1971. pp.11-22.

121. POPOVICS, S. Factors affecting the elastic deformation of concrete. Proceedings of an International Conference on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc. of Mat.Sc., 1972, Vol.4. pp.172-183.

122. HUGHES, B.P., and ASH, J.E. Some factors influencing the long term strength of concrete. R.I.L.E.M., Materials and Structures, Vol.3, No.14, 1970. pp.81-84.

123. SINHA, B.P., GERSTLE, K.H., and TULIN, L.G. Stress-strain relations for concrete under cyclic loading. Journal A.C.I., Proceedings Vol.61, February 1964. pp.185-211.

124. RICHART, F.E., BRANDTZAEG, A., and BROWN, R.L. A study of the failure of concrete under combined compressive stresses. University of Illinois, Eng.Expt.Stat.Bulletin No.185, 1928. pp.1-102.

125. SMITH, F.C., and BROWN, K.Q. The shearing strength of cement mortar. University of Washington, Eng.Expt.Stat.Bulletin No.106, 1941.

126. AKROYD, T.N.W. Concrete under triaxial stress. Magazine of Concrete Research, Vol.13, 1961. pp.111-118.

127. HANSON, J.A. Strength of structural lightweight concrete under combined stress. Journal P.C.A. Research and Development Laboratory, No.1, 1963. pp.39-46.

128. CHINN, J., and ZIMMERMANN, R.M. Behaviour of plain concrete under various high triaxial compression loading conditions. U.S. Air Force Weapons Lab.Tech.Report WL-TR64-163, 1965.

129. JOHNSON, R.P., and LOWE, P.G. Behaviour of concrete under biaxial and triaxial stress. Proceedings of an International Conference on Structure, Solid Mechanics and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.1039-1049.

130. GARDNER, N.J. Triaxial behaviour of concrete. Journal A.C.I., Proceedings Vol.66, February 1969. pp.136-146.

131. ROSENTHAL, I., and GLUCKLICH, J. Strength of plain concrete under biaxial stress. Journal A.C.I., Proceedings Vol.67, November 1970.

132. HOBBS, D.W. Strength of concrete under combined stress. Cement and Concrete Research, Vol.1, 1971. pp.41-56.

133. LIU, T.C.Y., NILSON, A.H., and SLATE, F.O. Stress-strain response and fracture of concrete in uniaxial and biaxial compression. Journal A.C.I., Proceedings Vol.69, May 1972. pp.291-295.

134. TAYLOR, M.A., and PATEL, B.K. The influence of path dependency and moisture conditions on the biaxial compression envelope for normal weight concrete. Journal A.C.I., Proceedings Vol.71, December 1974. pp.627-633.

135. PALANISWAMY, R., and SHAH, S.P. Fracture and stress-strain relationship of concrete under triaxial compression. Journal of the Structural Division, A.S.C.E., ST 5, May 1974. pp.901-916.

136. MILLS, L.L., and ZIMMERMANN, R.M. Compressive strength of plain concrete under multiaxial loading conditions. Journal A.C.I., Proceedings Vol.67, October 1970. pp.802-807.

137. KRISHNASWAMY, K.T. Discussion of Ref.131. Journal A.C.I., Proceedings Vol.67, May 1971. p.404.

138. SYNGE, T.L. Talking about Relativity. London, North-Holland, 1970. 193pp.

139. HASHIN, Z. Theory of mechanical behaviour of heterogeneous media. Applied Mechanics Reviews, Vol.17, No.1, January 1964. pp.1-9.

140. HASHIN, Z. The elastic moduli of heterogeneous materials. Journal of Applied Mechanics, Vol.29, No.1, March 1962. pp.144-150.

141. PAUL, B. Prediction of elastic constants of multiphase materials. Transactions of the Metallurgical Society of A.S.M.E., Vol.218, No.1, February 1960. pp.36-41.

142. HOBBS, D.W. Bulk modulus, shrinkage, and thermal expansion of a two phase material. London, Nature, Vol.222, 1969. pp.849-851.

143. HASHIN, Z., and SHTRIKMAN, S. A variational approach to the theory of elastic behaviour of multiphase materials. Journal of Mechanics and Physics of Solids, Vol.11, 1963. pp.127-140.

144. BUDEANSKY, B. On the elastic moduli of some heterogeneous materials. Journal of Mechanics and Physics of Solids, Vol.13, 1965. pp.223-227.

145. HILL, R. Elastic properties of reinforced solids. Some theoretical principles. Journal of Mechanics and Physics of Solids, Vol.11, 1963. pp.357-372.

146. BERAN, M.J. Statistical continuum theories. Monographs in statistical physics, Vol.2. London, Wiley-Interscience, 1965.

147. AXELRAD, D.R., and JAEGER, L.G. Random theory of deformation in heterogeneous media. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.571-578.

148. BROWN, C.B. Models for concrete stiffness with full and zero contiguity. Proceedings of an International Conference on the Structure of Concrete, London, 1965. London, Cement and Concrete Association, 1968. pp.3-15.

149. COUNTO, V.J. The effect of the elastic modulus of the aggregate on the elastic modulus, creep, and creep recovery of concrete. Magazine of Concrete Research, Vol.16, No.148, 1964. pp.129-138.

150. HIRSCH, T.J. Modulus of elasticity of concrete affected by elastic moduli of cement paste and aggregate. Journal A.C.I., Proceedings Vol.59, No.3, March 1962. pp.427-451.

151. HOBBS, D.W. The dependence of the bulk modulus, Young's modulus, creep shrinkage and thermal expansion of concrete upon aggregate volume concentration. R.I.L.E.M., Materials and Structures, Vol.4, No.20, 1971. pp.107-114.

152. HANSEN, T.C. Influence of aggregate and voids on the modulus of elasticity of concrete, cement mortar, and paste. Journal A.C.I., Proceedings Vol.62, No.2, February 1965. pp.193-216.

153. KOUFOPOULOS, T. On the Young's modulus of a two phase material. Athens, Greece, Control Laboratory of the Ministry of Public Works. Special Publication No.2, 1966.

154. La RUE, H.A. Modulus of elasticity of aggregates and its effect on concrete. Proceedings A.S.T.M., Vol.46, 1946. pp.1298-1309.

155. SWAMY, N., and RIGBY, G. Dynamic properties of hardened paste, mortar and concrete. R.I.L.E.M., Materials and Structures, Vol.4, No.19, 1971. pp.13-40.

156. BROWN, C.B. The effect of aggregate content on the properties of concrete. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.1015-1019.

157. KAPLAN, M.F. Effects of incomplete consolidation on compressive and flexural strength, ultrasonic pulse velocity, and dynamic modulus of elasticity of concrete. Journal A.C.I., Proceedings Vol.56, March 1960. pp.853-867.

158. POPOVICS, S. Effect of porosity on the strength of concrete. Journal of Materials, Vol.4, No.2, June 1969. pp.356-371.

159. HOFF, G.C. Porosity-strength considerations for cellular concrete. Cement and Concrete Research, Vol.2, 1972. pp.91-100.

160. LAWRENCE, P., MAJUMDAR, A.J., and NURSE, R.W. The application of the Mackenzie model to the mechanical properties of cements. Cement and Concrete Research, Vol.1, 1971. pp.75-99.

161. HANSEN, T.C. Cracking and fracture of concrete and cement paste. A.C.I. Publication SP-20, 1968. pp.43-66.

162. MARTIN, R.B., and HAYNES, R.R. Theoretical analysis of the effects of air voids in concrete. Journal A.C.I., Proceedings Vol.68, January 1971. pp.36-41.

163. HELMUTH, R.A., and TURK, D.A. Elastic moduli of hardened Portland cement and tricalcium silicate pastes. Effect of porosity. Symposium of Structure of Portland Cement Paste and Concrete. Highway Research Board, Special Report 90, 1966. pp.135-144.

164. MACKENZIE, J.K. The elastic constants of a solid containing spherical holes. Proceedings of the Physical Society, Vol.B63, No.2, 1950. pp.2-11.

165. ESHELBY, J.D. The determination of the elastic field of an ellipsoidal inclusion, and related problems. Proceedings of the Royal Society of London, Vol.A241,1957. pp.376-396.

166. HASHIN, Z. The moduli of an elastic solid containing spherical particles of another elastic material. Non-homogenuity in elasticity and plasticity. New York, Pergamon Press, 1959. pp.463-478.

167. POPOVICS, S. Method for developing relationships bteween mechanical properties of hardened concrete. Journal A.C.I., Proceedings Vol.70, December 1973. pp.795-798.

168. POWERS, T.C. Fundamental aspects of shrinkage of concrete. Revue des Materiaux, No.545, 1961. pp.79-85.

169. KINGERY, W.D., and COBLE, R.N. A review of the effect of micro-structure on mechanical behaviour of polycrystalline ceramics. Mechanical behaviour of crystalline solids, New York City, April 1962. National Bureau of Standards Monograph 59, 1963. pp.103-113.

170. HASSELMAN, D.P.H. Relation between the effect of porosity on strength and on Young's modulus of elasticity of polycrystalline materials. Journal of the American Ceramic Society, Vol.46, 1963. pp.564-565.

171. SCHILLER, K. Mechanical Properties of Non-metallic Brittle Materials. Walton, W.H.(Ed.). Butterworth, 1958. p.35.

172. AUSKERN, A. A model for the strength of cement-polymer and concrete-polymer systems. Brookhaven National Laboratory, BNL-1349312-2, 1969.

173. WRIGHT, P.J.F. Entrained air in concrete. Proceedings of the Institution of Civil Engineers, London, Part 1, Vol.2, No.3, May 1953. pp.337-358.

174. MANNING, D.G., and HOPE, B.B. The effect of porosity on the compressive strength and elastic modulus of polymer impregnated concrete. Cement and Concrete Research, Vol.1, 1971. pp.631-644.

175. REINER, M. Deformation, Strain and Flow. London, Lewis, 1960.

176. HANSEN, T.C. Creep and stress relaxation of concrete. Proceedings Swedish Cement and Concrete Research Institute, No.31, Stockholm, September 1960. pp.1-112.

177. GLUCKLICH, J. Rheological behaviour of hardened cement paste under low stresses. Journal A.C.I., Proceedings Vol.56, No.4, October 1959. pp.327-337.

178. BOZHINOV, G.M. On some aspects of application of the rheological models. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.527-532.

179. DESAYI, P., and SEN, B.R. An investigation into the prediction of creep and the true ultimate strength of concrete. Indian Concrete Journal, Vol.40, No.4, April 1966. pp.134-141. Vol.40, No.6, June 1966. pp.248-256.

180. TE'ENI, M., and STAPLES, G.E. Stress phase interaction models. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.469-485.

181. GRIFFITH, A.E. The phenomena of rupture and flow in solids. Philosophical Transactions of the Royal Society, Vol.221A, 1920. pp.163-198.

182. RADJY, F., and HANSEN, T.C. Fracture of hardened cement paste and concrete. Cement and Concrete Research, Vol.3, 1973. pp.343-361.

183. GOODIER, J.N. Concentration of stress around spherical and cylindrical inclusions and flaws. Journal of Applied Mechanics, April-June 1933. pp.39-44.

184. HILL, J.L. The stress distribution in an infinite elastic solid perfectly bonded to two unequal spherical inclusions. Report to Rhom and Haas, Co., 1965.

185. KAPLAN, M.F. Crack propogation and the fracture of concrete. Journal A.C.I., Proceedings Vol.58, No.5, November 1961. pp.591-610.

186. ROMUALDI, J.P., and BATSON, G.B. Mechanics of crack arrest in concrete. Journal of the Engineering Mechanics Division, Proceedings A.S.C.E., Vol.89, EM3, June 1963. pp.147-168.

187. MOAVENZADEH, F., and KUGUEL, R. Fracture of concrete. Journal of Materials, Vol.4, September 1969. pp.497-519.

188. NAUS, D.J., and LOTT, J.L. Fracture toughness of Portland cement concretes. Journal A.C.I., Proceedings Vol.66, June 1969. pp.481-489.

189. SHAH, S.P., and McGARRY, F.J. Griffith fracture criterion and concrete. Journal of the Engineering Mechanics Division, Proceedings A.S.C.E., Vol.97, EM6, December 1971. pp.1663-1676.

190. HARRIS, B., VARLOW, J., and ELLIS, C.D. The fracture behaviour of fibre reinforced concrete. Cement and Concrete Research, Vol.2, 1972. pp.447-461.

191. OKADA, K., and KOYANAGI, W. Effect of aggregate on the fracture process of concrete. Proceedings of an International Conference on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc.of Mat.Sc., Vol.4, 1972. pp.72-83.

192. KESLER, C.E., NAUS, D.J., and LOTT, J.L. Fracture mechanics - its applicability to concrete. Proceedings of an International Conference on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc.of Mat.Sc., Vol.4, 1972. pp.113-124.

193. GLUCKLICH, J. Static and fatigue fractures of portland cement mortar in flexure. Proceedings of the First International Conference on Fracture, Sendai, 1965, Vol.3. Japanese Society for Strength and Fracture of Materials, 1966. pp.1343-1382.

194. ISENBERG, J. A study of cracks in concrete by X-radiography. R.I.L.E.M. Bulletin No.30, 1966. pp.107-115.

195. GLUCKLICH, J., and COHEN, L.J. Strain energy and size effects in a brittle material. Material Research and Standards, Vol.8, 1968. pp.17-22.

196. GLUCKLICH, J., and COHEN, L.J. Size as a factor in the brittle-ductile transition and the strength of some materials. International Journal of Fracture Mechanics, Vol.3, 1967. pp.278-289.

197. GONNERMAN, H.F. Effect of size and shape of test specimen on the compressive strength of concrete. Proceedings A.S.T.M., Vol.25, 1925. pp.237-250.

198. RUSCH, H. Specimen size and apparent compressive strength. Journal A.C.I., Proceedings Vol.51, 1954. pp.803-804.

199. MEININGER, R.C. Effect of core diameter on measured concrete strength. Journal of Materials, Vol.2, 1968. pp.320-336.

200. MATHER, B., and TYNES, W.O. Investigation of compressive strength of molded cylinders and drilled cores of concrete. Journal A.C.I., Proceedings Vol.58, 1961. pp.767-778.

201. HARRIS, H.G., SABNIS, G.M., and WHITE, R.N. Small scale direct models of reinforced and prestressed concrete structures. Department of Structural Engineering, Report No.326, 1966. Ithica, New York, Cornell University.

202. PAUL, B. Macroscopic criteria for plastic flow and brittle fracture. Fracture, Vol.2, 1968. Leibowitz, H.(Ed.), New York, Academic Press. pp.313-496.

203. COTTERELL, B. On brittle fracture paths. International Journal of Fracture Mechanics, Vol.1, 1965. pp.96-103.

204. CEREPANOV, G.P. Cracks in solids. International Journal of Solids and Structures, Vol.4, 1968. pp.811-831.

205. ZAITSEV, J.W. Fracture of porous viscoelastic materials under multiaxial state of stress. Cement and Concrete Research, Vol.3, 1973. pp.389-395.

206. McLINTOCK, F.A., and WALSH, J.B. Friction on Griffith cracks in rocks under pressure. Proceedings of the 4th U.S. National Congress of Applied Mechanics. A.S.M.E. 1962. pp.1015-1021.

207. IRWIN, G.R. Analysis of stresses and strains near the end of a crack traversing a plate. Journal of Applied Mechanics, Transactions A.S.M.E., Vol.24, No.3, September 1957. pp.361-364.

208. TIMOSHENKO, S.P. History of Strength of Materials. New York, McGraw-Hill, 1953. 445pp.

209. NADAI, A. Theory of Flow and Fracture of Solids., Vol.1. New York, McGraw-Hill, 1950. 572pp.

210. BRESLER, B., and PISTER, K.S. Strength of concrete under combined stresses. Journal A.C.I., Proceedings Vol.55, September 1958. pp.321-345.

211. BRESLER, B., and PISTER, K.S. Failure of plain concrete under combined stresses. Transactions A.S.C.E., Vol.122, 1957. pp.1049-1059.

212. BREBBIA, C.A., and FREDERICK, C.O. A failure surface for concrete. Central Electricity Research Laboratory (U.K.), Note RD/L/N9/69, 1969. pp.1-13.

213. ROWE, P.W. The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. Proceedings of the Royal Society, London, 1962. pp.500-527.

214. CONSIDERE, A. Experimental Researches on Reinforced Concrete. New York, McGraw-Hill, 1906.

215. LEAN, A. Uber die scherfestigkeit des betons. Beton und Eisen, Vol.34, 1935. pp.130-135.

216. PAUL, B. A modification of the Coulomb-Mohr theory of fracture. Journal of Applied Mechanics, Vol.28, 1961. pp.259-268.

217. COWAN, H.J. The strength of plain, reinforced and prestressed concrete under the action of combined stresses, with particular reference to the combined bending and torsion of rectangular sections. Magazine of Concrete Research, Vol.5, No.14, December 1953. pp.75-86.

218. WU, H.C. Dual failure criterion for plain concrete. Journal of Engineering Mechanics Division, Proceedings A.S.C.E., Vol.100, EM6, December 1974. pp.1167-1181.

219. HANNANT, D.J., and FREDERICK, C.O. Failure criteria for concrete in compression. Magazine of Concrete Research, Vol.20, No.64, September 1968. pp.137-144.

220. HANNANT, D.J. Nomograms for the failure of plain concrete subjected to short-term multiaxial stress. The Structural Engineer, Vol.52, No.5, May 1974. pp.151-164 (including discussion).

221. McHENDRY, D., and KARNI, J. Strength of concrete under combined tensile and compressive stress. Journal A.C.I., Proceedings Vol.54, 1958. pp.829-839.

222. DRUCKER, D.C. A more fundamental approach to plastic stress-strain relations. Proceedings of the 1st U.S. National Congress of Applied Mechanics, A.S.M.E., 1952. pp.487-491.

223. PALMER, A.C., MAEIR, G., and DRUCKER, D.C. Normality relations and convexity of yield surfaces for unstable materials or structural elements. Transactions of A.S.M.E., Journal of Applied Mechanics, June 1967. pp.464-470.

224. PAUL, B. Generalised pyramidal fracture and yield criteria. International Journal of Solids and Structures, Vol.4, 1968. pp.175-196.

225. ILYUSHIN, A.A. Plastichnost. Moscow, Izdatelstvo Akademii Nauk, 1963.

226. HSU, C.T. Discussion of Ref.133. Journal A.C.I., Proceedings Vol.69, November 1972. pp.711-712.

227. BOKER, R. Die mechanik der bleibenden formanderung in kristallinish auf gebauten korpern. Mitteilungen uber Forcshungsarbeiten auf dem Gebiete des Ingenieurwesens, Vol.175 and 176, 1915.

228. ALEXANDER, K.M. Authors introduction to Ref.64. Proceedings of an International Conference on the Structure of Concrete, London, 1965. London, Cement and Concrete Association, 1968. pp.112-114.

229. TAYLOR, M.A. A theory for the deformation and failure of cement pastes, mortars, and concretes under general states of stress. Proceedings of an International Conference on Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc.of Mat.Sc., Vol.4, 1972. pp.21-31.

230. BRACE, W.F. An extension of Griffith theory of fracture to rocks. Journal of Geographic Research, Vol.65, 1960. pp.3477-3480.

231. MURRELL, S.A.F. A criterion for brittle fracture of rocks and concrete under triaxial stress and the effect of pore pressure on the criterion. Rock Mechanics, Fairhurst, C.(Ed.). Pergamon Press, 1963. pp.563-577.

232. NIWA, Y., and KOBAYASHI, S. Failure criterion of cement mortar under triaxial compression. Memoirs of the Faculty of Engineering, Kyoto University, Japan, Vol.XXIX, 1967. pp.1-15.

233. BUYUKOZTURK, O., NILSON, A.H., and SLATE, F.O. Stress-strain response and fracture of a concrete model in biaxial loading. Journal A.C.I., Proceedings Vol.68, August 1971. pp.590-599.

234. PALANISWAMY, R.G., and SHAH, S.P. A model for concrete subjected to triaxial stresses. Cement and Concrete Research, Vol.5, 1975. pp.273-284.

235. REINIUS, E. A theory of the deformation and failure of concrete. Magazine of Concrete Research, Vol.8, No.24, November 1956. pp.157-160.

236. BAKER, A.L.L. An analysis of deformation and failure characteristics of concrete. Magazine of Concrete Research, Vol.11, No.33, November 1959. pp.119-128.

237. BAKER, A.L.L. A criterion of concrete failure. Proceedings, Institution of Civil Engineers, London, Vol.45, February 1970. pp.269-278.

238. UPPAL, Y.J., and KEMP, K.O. An instability theory of failure for concrete. Magazine of Concrete Research, Vol.25, No.82, March 1973. pp.21-30.

239. VOLKOV, S.D. Statistical Strength Theory. New York, Gordon and Breach, 1962.

240. WEIBULL, W. The phenomenon of rupture in solids. Stockholm, Ing. Vetenskaps Akad., Handlingar No.153, 1939. 55pp.

241. DANIELLS, H.F. The statistical theory of the strengths of bundles of threads. Proceedings of the Royal Society, London, Vol.183, No.995, June 1945. pp.405-435.

242. JELLINEK, H.H.G. The influence of imperfections on the strength of ice. Proceedings of the Physical Society, Vol.71, No.461, May 1958. pp.797-814.

243. DESAYI, P. A model to simulate the strength and deformation of concrete in compression. R.I.L.E.M., Materials and Structures, No.1, January-February 1968. pp.49-56.

244. HOBBS, D.W. The compressive strength of concrete: a statistical approach to failure. Magazine of Concrete Research, Vol.24, No.80, September 1972. pp.127-138.

245. RAO, C.V.S.K., and RAO, R.K.S. Statistical aspects of strength and fracture behaviour of concrete. Proceedings of an International Conference on the Mechanical Behaviour of Materials, Kyoto, 1971. Japan, Soc.of Mat.Sc., Vol.4, 1972. pp.53-62.

246. RAO, C.V.S.K., and SWAMY, R.N. A statistical theory for the strength of concrete. Cement and Concrete Research, Vol.4, 1974. pp.669-681.

247. DOUGILL, J.W. A mathematical model for the failure of cement paste and mortars. Magazine of Concrete Research, Vol.19, No.60, September 1967. pp.135-142.

248. DOUGILL, J.W. Further consideration of a mathematical model for progressive fracture of a heterogeneous material. Magazine of Concrete Research, Vol.23, No.74, March 1971. pp.5-10.

249. MIHASHI, H., and IZUMI, M. A stochastic theory for concrete fracture. Cement and Concrete Research, Vol.7, 1977. pp.411-422.

250. GORDON, J.E. The New Science of Strong Materials. London, Penguin, 1968. p.21.

251. TRUESDELL, C. Essays in the History of Mechanics. New York, Springer Vertag, 1968.

252. KELLY, A. Strong Solids. Oxford, Clarendon Press, 1966.

253. FINNISTON, H.M. (Ed). Structural Characteristics of Materials. London, Elsevier, 1971.

254. RUOFF, A.L. Materials Science. New Jersey, Prentice Hall, 1973.

255. FREUDENTHAL, A.M. The Inelastic Behaviour of Engineering Materials and Structures. New York, Wiley, 1950.

256. CLAYTON, N., and GRIMER, F.J. The Diphase Concept with Particular Reference to Concrete. Developments in Concrete Technology. LYDON, F.D. (Ed.). London, Applied Science Publishing, 1979. pp.283-318.

257. RAMSEY, N.F. Thermodynamics and statistical mechanics at negative absolute temperatures. Physical Review, Vol.103, 1956. p.20.

258. TWEENEY, C.F., and HUGHES, L.E.C. (Eds.). Chambers Technical Dictionary. Edinburgh, Chambers, 1968.

259. ELLIS, B. Basic Concepts of Measurement. Cambridge University Press, 1968.

260. CAMPBELL, N.R. An Account of the Principles of Measurement and Calculations. London, Longmans, 1928.

261. BRIDGMAN, P.W. Dimensional Analysis. Yale University Press, 1931.

262. STEVENS, S.S. Measurement; Definition and Theories. New York, Wiley, 1959.

263. BRILLOUIN, L. Relativity Re-examined. London, Academic Press, 1970.

264. CAWS, P. The Philosophy of Science. Princeton, Van Nostrand, 1964.

265. KANT, E. A critique of Pure Reason, 1787. (Translated by MEIKLEJOHN, J.M.D.) London, Dent, 1934.

266. HESSE, M.B. Operational definition and analogy in physical theories. British Journal for the Philosophy of Science, Vol.2, 1952. p.281-302.

267. Metric Notes (and associated correspondence). New Zealand Engineering, Vols. 28-30, 1973-1975.

268. MACH, E. Die Principien der Warmelchre. Leipzig, 1896. pp.39-57. Translated in Reference 258, pp.183-196.

269. NADAI, A. On the creep of solids at elevated temperature. Journal of Applied Physics, Vol.8, 1937. pp.418-432.

270. WHITROW, G.J. The Natural Philosophy of Time. London, Nelson, 1961.

271. ARISTOTLE, Physics, Book 4. ROSS, W.D. (Ed.). Oxford, 1930.

272. SAMBURSKY, S. The Physics of the Stoics. London, Routledge and Kegan Paul, 1959.

273. ASPDEN, H. Physics Without Einstein. Southampton, Sabberten Publications, 1969.

274. MASSEY, Sir H.S.W. The New Age of Physics. London, Elek Books, 1960.

275. DIRAC, P.A.M. The quantum theory of the electron. Proceedings of the Royal Society, Vol.117, 1928. p.610.

276. BUSCOVICH, R.J. A Theory of Natural Philosophy, 1758. (Reprinted: Chicago, Open Court, 1922.)

277. MAXWELL, J.C. A dynamical theory of the electromagnetic field. Scientific Papers of James Clerk Maxwell. NIVEN, W.D. (Ed.), Vol.1, Cambridge, 1890.

278. CHRISTY, R.W. and PYTTE, A. The Structure of Matter: An Introdoction to Modern Physics. New York, Benjamin Inc., 1965.

279. EINSTEIN, A. Relativity: The Special and General Theory. New York, Crown Publishers Inc., 1961.

280. WHITTAKER, E.T. A History of the Theories of Aether and Electricity. Dublin, Longmans Green & Co., 1910.

281. BUILDER, G. Ether and Relativity. Australian Journal of Physics, Vol.11, 1958. p.279.

282. PROKHOVNIK, S.J. The case for an aether. British Journal for the Philosophy of Science, Vol.14, 1963. p.195.

283. DINGLE, H. Science at the Crossroads. London, Martin, Brian and O'Keefe, 1972.

284. DESCARTES, R. Discourse on Method. VEITCH, J.(Trans.). London, Dent, 1912.

285. HOPKINS, H.J. Opening Address, 6th Australasian Conference on the Mechanics of Structures and Materials. Christchurch, New Zealand, 1977.

286. LANDE, A. New Foundations of Quantum Mechanics. Cambridge, 1965.

287. THEOBALD, D.W. The Concept of Energy. London, E. & F.N. Spon, 1966.

288. RAVETZ, J.F. The representation of mathematical quantities in eighteenth century mathematical physics. Isis, Vol.52, 1961. p.7.

289. WHITTAKER, E. History of Theories of Aether and Electricity, Vol.2. London, Nelson, 1953.

290. WEINER, PP. (Ed.). Leibniz Selections. New York, Scribners, 1951.

291. LANDE, A. Foundations of Quantum Theory, A Study in Continuity and Symmetry. Yale, Yale University Press, 1955.

292. BORN, M. The Restless Universe. London, Blackie, 1935.

293. NEWTON, I. Principia Mathematica (1687). Revised Translation, CAJORI, F. Berkeley, University of California Press, 1962.

294. NEWTON, I. De Aere et Aethere. In Unpublished Scientific Papers of Isaac Newton. Cambridge, 1962.

295. CARDWELL, D.S.L. From Watt to Clausius. London, Heinemann, 1971.

296. KOYRE, A. Newtonian Studies. London, Chapman and Hall, 1965.

297. NEWTON, I. Optiks, London 1730. New York, Dover, 1952.

298. LEIBNIZ, G.W. Philosophical Writings. Translated - MORRIS, M. London, Dent, 1934.

299. ALEXANDER, H.G. (Ed.) The Leibnitz - Clarke Correspondence. Manchester University Press, 1956.

300. Von GUERICKE, O. Experimenta Nova Magdeburgica. Amsterdam, 1672. Aalen, Otto Zeller, 1963.

301. GUMBEL, E.J. The Statistics of Extremes. New York, Columbia University Press, 1960.

302. HANSON, N.R. Patterns of Discovery. Cambridge, 1958.

303. BONDI, H. Relativity. Rep.Prog.Phys., Vol.22, 1959. p.97.

304. HILL. T.L. Lectures on Matter and Equilibrium. New York, Benjamin, 1966.

305. FRESNEL, A. A note on the repulsion of warm bodies for one another at measurable distances. Annales de Chimie et de Physique, Vol.29, 1825. p.37.

306. FRESNEL, A. Additional observations on the repulsion of warm bodies. Annales de Chimie et de Physique, Vol.29, 1825. p.107.

307. BRIDGMAN, P.W. The Nature of Thermodynamics. New York, Harper, 1961.

308. TRUESDELL, C. The Tragicomedy of Classical Thermodynamics. International Centre for Mechanical Sciences, Udine. Courses and Lectures No.70, New York, Springer-Verlag, 1971.

309. DIN, F. (Ed.) Thermodynamic Functions of Gases. London, Butterworths, 1956.

310. REID, R.C., and SHERWOOD, T.K. The Properties of Gases and Liquids, Their Estimation and Correlation. New York, McGraw-Hill, 2nd Edition, 1966. Chap.3.

311. BRIDGMAN, P.W. The compressibility of water. International Critical Tables of Numerical Data, Physics, Chemistry and Technology. New York, McGraw-Hill, Vol.31, 1928. p.40.

312. BRIDGMAN, P.W. The compressibility of thirty metals as a function of pressure and temperature. Proceedings of the American Academy of Arts and Science, Vol.58, No.166, 1923.

313. TODHUNTER, I., and PEARSON, K. A History of the Theory of Elasticity and of the Strength of Materials from Galilei to Lord Kelvin. New York, Dover, 1960.

314. PUGH, H.L.I.D., (Ed.) The Mechanical Behaviour of Materials Under Pressure. London, Elsevier, 1970.

315. PANDIT, G.S. Discussion of: ATAN, Y., and SLATE, F.O. Structural lightweight concrete under biaxial compression. Journal A.C.I., Proceedings Vol.70, No.3, March 1973. Journal A.C.I., Proceedings Vol.70, No.9, Sept. 1973. pp.660-661.

316. PHIPPS, M.E. The strain capacity of compression-zone concrete subjected to short term loading. Magazine of Concrete Research, Vol.28, No.95, June 1976. pp.85-100.

317. NEWMAN, K., and LACHANCE, L. The testing of brittle materials under uniform uniaxial compressive stresses. Proceedings A.S.T.M., Vol.64, 1964. pp.1044-1067.

318. BERES, L. Investigation on structural loosening of compressed concrete. RILEM Bulletin, No.36, 1967. pp.185-190.

319. LANGAN, D., and GARAS, F.K. The failure of concrete under the combined action of high shearing forces and biaxial restraint. Proceedings of an International Conference on Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, 1969. London, Wiley-Interscience, 1971. pp.1061-1088.

320. FUMAGALLI, E. Caratteristiche de resistanzia dei conglomerati cementizi per stati di compressione pluriassiali. (Strength of concrete under multi-axial compression) Instituto Sperimentale Modelli e Structure, Bergamo, Vol.30, 1965. C. and C.A. Library Translation No.128, Cement and Concrete Assosiation, London, 1968.

321. ELVERY, R.H., COX, A.I., and BUTTER, J.E. Development of apparatus to investigate the influence of pore pressure upon the tensile behaviour of concrete. Magazine of Concrete Research, Vol.24, No.81, Dec.1972. pp.231-237.

322. CLAYTON, N. Fluid-pressure testing of concrete cylinders. Magazine of Concrete Research, Vol.30, No.102, March 1978. pp.26-30.

323. LELIAVSKY, S. Uplift in Gravity Dams. Constable, London 1958.

324. SERAFIM, J.L. The "uplift" area in plain concrete in the elastic range. Eighth International Congress on Large Dams, 1964.

325. CARLSON, R.W. Permeability, pore pressure, and uplift in gravity dams. Transactions A.S.C.E., Vol.122, 1957. pp.587-602.

326. ATTRI, N.S. Uplift in gravity type dams. Journal of the Structural Division, A.S.C.E., Proceedings, August 1967. pp.61-68.

327. IYENGAR, K.T.S.R., CHANDRASHEKHARA, K., and KRISHNASWAMY, K.T. Strength of concrete under biaxial compression. Journal A.C.I., Proceedings Vol.63, No.2, Feb.1965. pp.239-249.

328. VILE, G.W.D., and SIGVALDASON, O.T. Discussion of ref.327. Journal A.C.I., Proceedings Vol.63, No.9, Sept.1965. pp.1192-1196.

329. FUMAGALLI, E. Discussion of ref.327. Journal A.C.I., Proceedings Vol.63, No.9, Sept.1965. pp.1187-1188.

330. WASTIELS, J. Behaviour of concrete under multiaxial stresses - a review. Cement and Concrete Research, Vol.9, No.1, 1979. pp.35-44.

331. SPOONER, D.C. Progressive Damage and Energy Dissipation in Concrete in Uniaxial Compression. Ph.D. Thesis, Kings College, University of London, 1974.

332. BRIDGMAN, P.W. Breaking tests under hydrostatic pressure and conditions of rupture. Philosophical Magazine, Vol.24, No.139, July 1912. pp.63-80.

333. BRIDGMAN, P.W. The Physics of High Pressure. London, Bell, 1931. (Reprinted 1949) p.92.

334. BRIDGMAN, P.W. Reflections on rupture. Journal of Applied Physics, Vol.9, August 1938. pp.517-528.

335. MISRA, A. Electromagnetic effects at metallic fracture. Nature, Vol.254, No.3, March 1975. pp.133-134.

336. GRIGGS, D., and HANDIN, J. (Eds.) Rock Deformation. Geological Society of America, Memoir 79, 1960. pp.39-104.

337. LOWE, P.G. Deformation and fracture of plain concrete. Magazine of Concrete Research, Vol.30, No.105, Dec.1978. pp.200-204.

338. CLAYTON, N., GRIMER, F.J., and ARMER, G.S.T. Private communication, 1976.

339. PUGH, H.L.I.D., and GREEN, D. Progress report on the behaviour of materials under hydrostatic pressure. Report No.147. National Engineering Laboratory, East Kilbride, U.K., 1958.

340. HAWKINS, N.M. The bearing strength of concrete loaded through rigid plates. Magazine of Concrete Research, Vol.20, No.62, March 1968. pp.31-40.

341. HAWKINS, N.M. The bearing strength of concrete loaded through flexible plates. Magazine of Concrete Research, Vol.20, No.63, June 1968. pp.95-102.

342. HYLARD, M.W., and CHEN, W-F. Bearing capacity of concrete blocks. Journal A.C.I., Proceedings, Vol.67, No.3, March 1970. pp.228-236.

343. NIYOGI, S.K. Concrete bearing strength - support, mix, size effect. Proceedings A.S.C.E., Structural Division, Vol.100, No.ST8, August 1974. pp.1685-1702.

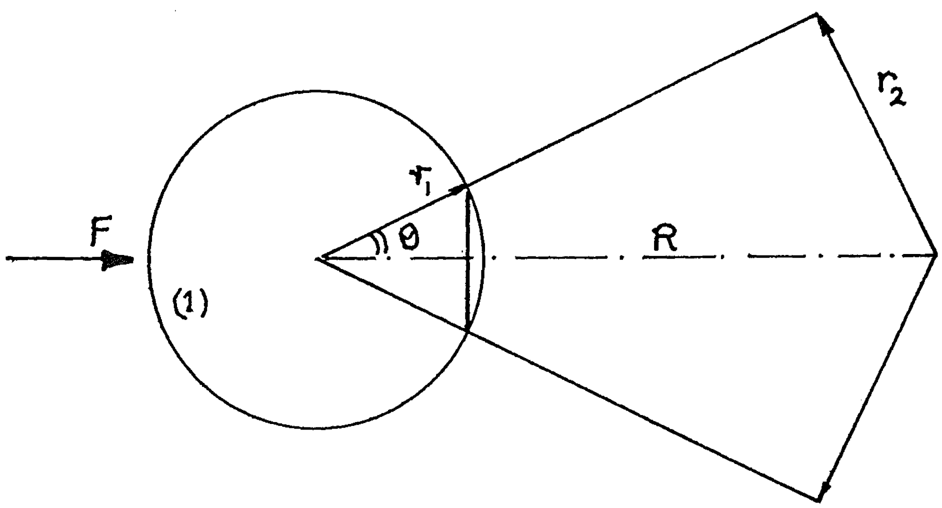
344. AU, T., and BAIRD, D.L. Bearing capacity of concrete blocks. Journal A.C.I., Proceedings, Vol.56, No.9, March 1960. pp.869-879.

APPENDIX 1: The Gravity Model

Let total atmospheric (near-ideal gas) pressure = p

Assume constant contribution from tail pressure = αp ($\alpha << 1$)

Consider body (1) of Figure 4.11 of p.101(a)



Force F = net pressure force

$$= (\alpha p) \times (\text{projected area of shielding})$$

$$= (\alpha p) \times \pi(r_1 \sin\theta)^2$$

$$= k\ (r_1 r_2/R)^2$$

where $k = \pi(\alpha p)$ = constant

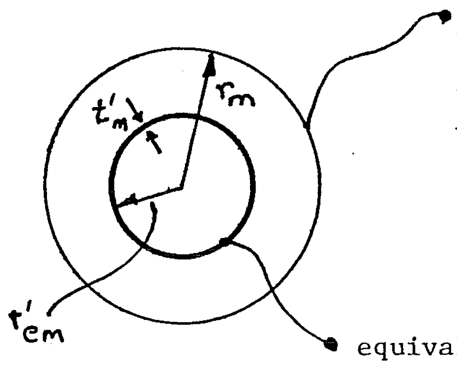
APPENDIX II: Response to Linear Pressure Variations

A body immersed within a fluid under conditions of a linear pressure gradient, $\frac{dp}{dx}$, experiences a force, F, in the direction opposite to that of the pressure increase, such that,

$$F = -\left(\frac{dp}{dx}\right)V_d$$

, where V_d is the "displaced volume" unavailable to the fluid by virtue of the pressure of the body. (Formal proof of the statement just offered - a simple corollary of Archimedes Principle - is to be found in most classical hydrostatic texts.)

(a) Single spherical molecule (radius r_m) within molecular/sub-molecular system.



gross molecular boundary (as "seen" by neighbouring molecular environment)

$V_d = V_{dm} = 4\pi r_m^3/3$

equivalent shell (as "seen" by sub-molecular environment)

$V_d = V_{ds} = 4\pi r'^2_{em}\ t'_m$

From molecular environment: $F = F_m = -\left(\frac{dp}{dx}\right)\ 4\pi r_m^3/3$

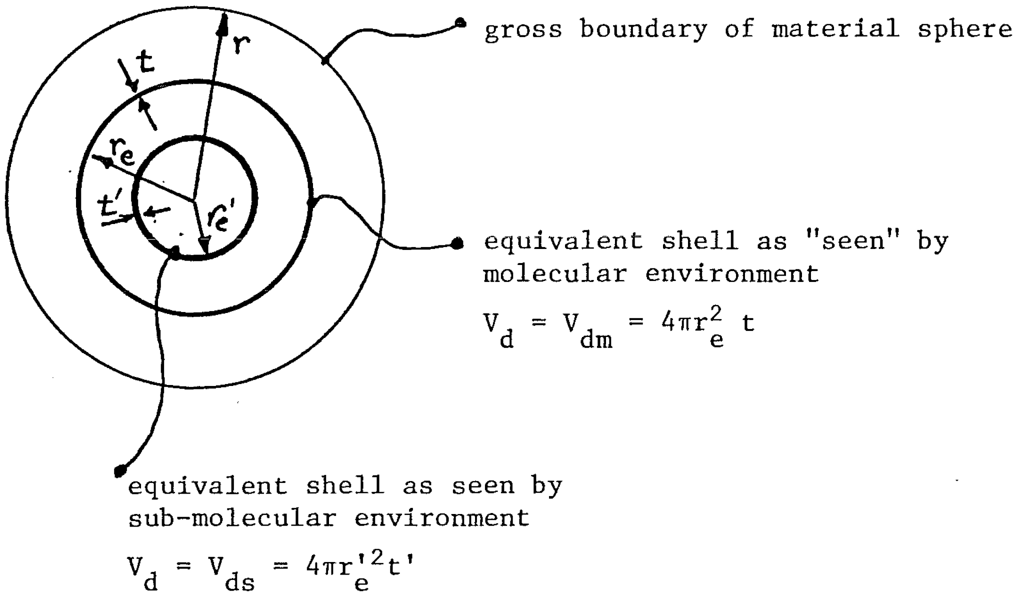
From sub-molecular system: $F = F_s = -\left(\frac{dp'}{dx}\right)\ 4\pi r'^2_{em}\ t'_m$

Balanced force condition: $F_m + F_s = 0$

$$\rightarrow \quad \frac{dp'}{dx} = -\Omega\frac{dp}{dx}$$

where $\Omega = r_m^3\ /3r'^2_{em}\ t'_m$

(b) Spherical body (radius = r) within molecular/sub-molecular environment



From molecular environment: $F = F_m = -\left(\frac{dp}{dx}\right) V_{dm}$

From sub-molecular environment: $F = F_s = -\left(\frac{dp'}{dx}\right) V_{ds}$

APPENDIX III: A Specific Gravitational Interaction

Consider an infinite volume of an Ideal Gas

Let original density $= \rho_o$

pressure $= p$

temperature $= T_o$

Now consider effects of presence of spherical inclusion having,

radius $= r$

mass density $= \rho_m > \rho_o$

Ideal Gas must respond to gravitational influence

Denoting x as radial distance from centre of sphere,

$$\rho = \rho(x) = \rho_o + \Delta\rho(x)$$

$$p = p(x) = p_o + \Delta p(x)$$

, where $\Delta\rho(x)$ and $\Delta p(x)$ are the induced changes in density and pressure, respectively at distance x.

Steady-state (quasi-static) analysis

Volumetric element of gas, $dV = dA.dx$, at distance x from centre of sphere:

effective gravitational mass $= \Delta\rho(x).dV$

gravity force on element $= F_g$ (via gravitational constant, G)

$$F_g(x) = 12\pi G.\Delta\rho(x).dV\ [r^3(\rho_m - \rho_o) + {}_r\int^x \Delta\rho(x).x^2 dx]/x^2$$

For $x >> r$, $F_g(x) = 4\pi G.\Delta\rho(x).dV\ [{}_o\int^x \Delta\rho(x).x^2 dx]/x^2$

For a steady-state condition ($\Sigma F_x = 0$),

$$dp(x).dA + F_g(x) = 0$$

whence, $-\frac{dp}{dx} = 4\pi G.\Delta\rho[{}_o\int^x \Delta\rho.x^2 dx]/x^2$ ..... (1)

Trial Solution $p(x) = p_o + C/x$, where C is a system constant

<u>Implications</u> (1) $\frac{dp}{dx} = -C/x^2$

(2) $\Delta\rho = b/x^{1.5}$ from (1)

where $b = (3C/8\pi G)^{0.5}$

(3) $T = T_o \rho_o p(x)/p_o \rho(x)$ Ideal Gas Law

<u>Check</u>

For quasi-static steady-state conditions (no net flow of Ideal Gas),

$$E = \text{energy/unit volume} = \text{constant.}$$

At $x = \infty$, all energy is kinetic (no gravitational potential)

For Ideal Gas,

$$p = \rho\overline{V^2}/3$$

where $\overline{V^2}$ = mean squared velocity of constituent molecules

Thus, <u>per unit volume</u>,

$$\text{kinetic energy} = KE = \rho V^2/2 = 3p/2 \qquad \ldots\ldots (2)$$

On basis of original analysis, the "loss" of gravitational potential energy per unit volume as x decreases from ∞ must equate with the gain in strain energy per unit volume, where the latter is given by,

$$\Delta SE = \int_o^{\Delta\rho} (\Delta p/\Delta\rho)d\Delta\rho$$

<u>If trial solution for p(x) is valid</u>, substitution for Δp and $\Delta\rho$ yields,

$$\Delta SE = 1.5C \int_x^{\infty} (1/x^2)dx$$

$$= 1.5\ C/x \qquad \ldots\ldots (3)$$

$$= 1.5\ \Delta p(x)$$

Since "strain energy" (continuum view) and "kinetic energy" (particulate view) represent alternative expressions of the same quantity in this context, it may be seen that the forecast given via equation (3), which is <u>conditional</u> upon the validity of the trial solution for p(x), is totally compatible with that of the <u>independent</u> equation (2).